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Generalized Syllogism Reasoning with the Quantifiers in Modern Square{*no*} and Square{*most*}

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Abstract

A modern Square{Q}={Q, Q¬, Q¬, ¬Q¬} is composed of a generalized quantifier Q and its three types of negative quantifiers: inner, outer and dual negative one. This paper mainly discusses the non-trivial generalized syllogisms reasoning with the quantifiers in Square{no} and Square{most}. To this end, this paper firstly gives formalizes generalized syllogisms, then proves the validity of the syllogism AMM-1 with the generalized quantifier most, and further deduces the other 24 valid syllogisms. The reason why these valid generalized syllogisms studied in this paper can be mutually reduced is because: (1) any of the four Aristotelian quantifiers in Square{no} can define the other three ones; (2) so can any of the four generalized quantifiers in Square{most}. This study is undoubtedly beneficial not only for the development of modern logic, but also for the development of inference machines in artificial intelligence.

Keywords

Aristotelian quantifiers, generalized quantifiers, generalized syllogisms, validity

1. Introduction

There are many generalized quantifiers in natural language (Barwise & Cooper, 1981). Generally speaking, noun phrases and their determiners are generalized quantifiers (such as *my book*, *most*, *fewer than half of the, both, infinitely many*). Aristotelian quantifiers (i.e., *all*, *no*, *some* and *not all*) are trivial generalized ones, and the latter is an extension of the former (Zhang & Wu, 2021). Aristotelian syllogisms characterize the semantic and inferential properties of Aristotelian quantifiers, and generalized syllogisms characterize those of generalized quantifiers. Thus, generalized syllogisms are extensions of Aristotelian ones.

There are many works on Aristotelian syllogisms (Łukasiewicz, 1957; Moss, 2008; Hao, 2023) and Aristotelian modal syllogisms (Thomason, 1997; Johnson, 2004; Malink, 2013; Zhang, 2020; Zhang,

2023) at home and abroad, but there are few works on generalized syllogisms (Moss, 2010; Endrullis & Moss, 2015). This paper attempts to promote the study of generalized syllogisms.

A non-trivial generalized syllogism contains at least one non-trivial generalized quantifier. A generalized quantifier Q has its three negative forms: its inner negative quantifier $Q\neg$, its outer negative one $Q\neg$, and its dual negative one $\neg Q\neg$. A modern Square{Q} is composed of these four quantifiers. In other words, Square{Q}={Q, $Q\neg$, $Q\neg$, $\neg Q\neg$}. For example, Square{*no*}={*no, all*, *some*, *not all*}. Any quantifier in Square{Q} can define the other three ones (Peters & Westerståhl, 2006).

Due to the abundance of generalized quantifiers in natural language, this paper focuses on the generalized quantifier *most* and its three negative quantifiers which are common generalized quantifiers in natural language. More specifically, the paper mainly discusses the non-trivial generalized syllogisms reasoning with the quantifiers in Square{*no*} and Square{*most*}, in which Square{*most*}={*most*, *fewer than half of the*, *at most half of the*, *at least half of the*}.

2. Preliminaries

In the following, let b, n and x be lexical variables, and D be their domain. The sets composed of b, n and x are respectively B, N, and X. Let δ, γ, π, and ψ be well-formed formulas (shortened to wff). Let Q be a quantifier, $\neg Q$, $Q\neg$ and $\neg Q\neg$ be its outer, inner and dual negative quantifier, respectively. '$|B\cap X|$' indicates the cardinality of the intersection of the set B and X. '$\vdash\psi$' represents that the wff ψ is provable, and '$\delta =_{def} \pi$' that δ can be defined by π. The others are similar. The operators (such as $\neg$, $\rightarrow$, $\wedge$, $\leftrightarrow$) in this paper are common symbols in mathematical logic (Hamilton, 1978).

The generalized syllogisms studied in this paper just involve the following 8 quantifiers: *all*, *no*, *some*, *not all*, *most*, *fewer than half of the*, *at most half of the*, *at least half of the*. Thus, these syllogisms only involve 8 types of propositions as follows: *all(b, x), not all(b, x), some(b, x), no(b, x), most(b, x), at least half of the(b, x), at most half of the(b, x), fewer than half of the(b, x),* and they are respectively shortened to: Proposition *A*, *O*, *I*, *E*, *M*, *S*, *H*, and *F*, which at least includes one of the last four propositions. For example, the first figure syllogism $all(n, x)\wedge most(b, n)\rightarrow most(b, x)$ is abbreviated as *AMM-1.* An example of this generalized syllogism is as follows:

Major premise: All dogs like to eat bones.

Minor premise: Most of the pets in our community are dogs.

Conclusion: Most of the pets in our community like to eat bones.

3. Generalized Syllogism System with the Quantifier 'most'

This system includes the following: primitive symbols, formation and deductive rules, and basic axioms, etc.

3.1 Primitive Symbols

(1) lexical variables: *b, n, x*

(2) quantifiers: *no, most*

(3) operators: $\neg$, $\rightarrow$

(4) brackets: (,)

3.2 Formation Rules

(1) If *Q* is a quantifier, *b* and *x* are lexical variables, then *Q(b, x)* is a wff.

(2) If ψ is a wff, then so is $\neg\psi$.

(3) If π and ψ are wffs, then so is $\pi\rightarrow\psi$.

(4) Only the formulas constructed based on the above three rules are wffs.

3.3 Basic Axioms

A1: If ψ is a valid formula in first-order logic, then $\vdash\psi$.

A2: $\vdash all(n, x)\wedge most(b, n)\rightarrow most(b, x)$ (that is, the syllogism *AMM-1*).

3.4 Rules of Deduction

Rule 1(subsequent weakening): From $\vdash(\pi\wedge\psi\rightarrow\delta)$ and $\vdash(\delta\rightarrow\gamma)$ infer $\vdash(\pi\wedge\psi\rightarrow\gamma)$.

Rule 2(anti-syllogism): From $\vdash(\pi\wedge\psi\rightarrow\delta)$ infer $\vdash(\neg\delta\wedge\pi\rightarrow\neg\psi)$.

Rule 3(anti-syllogism): From $\vdash(\pi\wedge\psi\rightarrow\delta)$ infer $\vdash(\neg\delta\wedge\psi\rightarrow\neg\pi)$.

3.5 Relevant Definitions

D1 (conjunction): $(\pi\wedge\psi)=_{\text{def}}\neg(\pi\rightarrow\neg\psi)$;

D2 (bicondition): $(\pi\leftrightarrow\psi) =_{\text{def}} (\pi\rightarrow\psi)\wedge(\psi\rightarrow\pi)$;

D3 (inner negation): $(Q\neg)(b, x)=_{\text{def}} Q(b, D-x)$;

D4 (outer negation): $(\neg Q)(b, x)=_{\text{def}}$ It is not that *Q(b, x);*

D5 (truth value): $all(b, x)=_{\text{def}} B\subseteq X$;

D6 (truth value): $some(b, x)=_{\text{def}} B\cap X\neq\varnothing$;

D8 (truth value): $no(b, x)=_{\text{def}} B\cap X=\varnothing$;

D9 (truth value): $not\ all(b, x)=_{\text{def}} B\not\subseteq X$;

D10 (truth value): *most*(*b, x)* is true iff $|B\cap X|>0.5|B|$ is true;

D11 (truth value): *at most half of the*(*b, x)* is true iff $|B\cap X|\leq 0.5|B|$;

D12 (truth value): *fewer than half of the*(*b, x)* is true iff $|B\cap X|<0.5|B|$ is true;

D13 (truth value): *at least half of the*(*b, x)* is true iff $|B\cap X|\geq 0.5|B|$ is true.

3.5 Relevant Facts

Fact 1 (inner negation):

(1.1) *all(b, x)=no¬(b, x)*;

(1.2) *no(b, x)=all¬(b, x)*;

(1.3) *some(b, x)=not all¬(b, x)*;

(1.4) *not all(b, x)=some¬(b, x)*;

(1.5) *most(b, x)=fewer than half of the¬(b, x)*;

(1.6) *fewer than half of the(b, x)=most¬(b, x)*;

(1.7) *at least half of the(b, x)=at most half of the(b, x)*;

(1.8) *at most half of the(b, x)=at least half of the(b, x).*

Fact 2 (outer negation):

(2.1) $\neg$*all(b, x)=not all(b, x)*;

(2.2) $\neg$*not all(b, x)=all(b, x)*;

(2.3) $\neg$*no(b, x)=some(b, x)*;

(2.4) $\neg$*some(b, x)=no(b, x)*;

(2.5) $\neg$*most(b, x)=at most half of the(b, x)*;

(2.6) $\neg$*at most half of the(b, x)=most(b, x)*.

(2.7) $\neg$*fewer than half of the(b, x)=at least half of the(b, x)*;

(2.8) $\neg$*at least half of the(b, x)=fewer than half of the(b, x)* ;

Fact 3 (symmetry):

(3.1) *some(b, x)*$\leftrightarrow$*some(x, b)*;

(3.2) *no(b, x)*$\leftrightarrow$*no(x, b)*.

Fact 4 (subordination) :

(4.1) $\vdash$*all(b, x)*$\rightarrow$*some(b, x)*;

(4.2) $\vdash$*no(b, x)*$\rightarrow$*not all(b, x)*;

(4.3) $\vdash$*all(b, x)*$\rightarrow$*most(b, x)*;

(4.4) $\vdash$*most(b, x)*$\rightarrow$*some(b, x)*;

(4.5) $\vdash$*at least half of the(b, x)*$\rightarrow$*some(b, x)*;

(4.6) $\vdash$*all(b, x)*$\rightarrow$*at least half of the(b, x)*;

(4.7) $\vdash$*at most half of the(b, x)*$\rightarrow$*not all(b, x)*;

(4.8) $\vdash$*fewer than half of the(b, x)*$\rightarrow$*not all(b, x)*.

Fact 1-4 are theoretical basis of first-order logic (Hamilton, 1978) and generalized quantifier theory (Peters & Westerståhl, 2006), so their proofs are omitted.

4. The Reducible Relationships between/among Generalized Syllogisms

If the validity of one syllogism can be deduced from that of another one, it is said that there is a reducible relationship between these two syllogisms. More specifically, the following Theorem 2 illustrates that the validity of generalized syllogisms after the implication symbol (i.e., $\rightarrow$) can be derived from that of the generalized syllogism *AMM-1*. In other words, there are reducible relationships between/among these valid syllogisms.

Theorem 1 (*AMM-1*): The generalized syllogism *all*(*n, x*)$\wedge$*most*(*b, n*)$\rightarrow$*most*(*b, x*) is valid.

Proof: Suppose that *all*(*n, x*) and *most*(*b, n*) are true, then $N\subseteq X$ and $|B\cap N|>0.5|B|$ are true according to Definition D5 and D10, respectively. Hence it can be concluded that $|B\cap X|>0.5|B|$ is true. Thus, *most(b, x)* is true by Definition D10, just as expected.

Theorem 2: There are at least the following 24 valid generalized syllogisms inferred from *AMM-1*:

(2.1) $\vdash$*AMM-1*$\rightarrow$*AMI-1*

(2.2) $\vdash$*AMM-1*$\rightarrow$*AMI-1*$\rightarrow$*MAI-4*

(2.3) $\vdash$*AMM-1*$\rightarrow$*AHH-2*

(2.4) $\vdash$*AMM-1*$\rightarrow$*AHH-2*$\rightarrow$*AHO-2*

(2.5) $\vdash$*AMM-1*$\rightarrow$*HMO-3*

(2.6) $\vdash$*AMM-1*$\rightarrow$*EMF-1*

(2.7) $\vdash$*AMM-1*$\rightarrow$*EMF-1*$\rightarrow$*EMO-1*

(2.8) $\vdash$*AMM-1*$\rightarrow$*EMF-1*$\rightarrow$*EMF-2*

(2.9) $\vdash$*AMM-1*$\rightarrow$*EMF-1*$\rightarrow$*EMF-2*$\rightarrow$*EMO-2*

(2.10) $\vdash$*AMM-1*$\rightarrow$*AMI-1*$\rightarrow$*AEH-2*

(2.11) $\vdash$*AMM-1*$\rightarrow$*AMI-1*$\rightarrow$*AEH-2*$\rightarrow$*AEH-4*

(2.12) $\vdash$*AMM-1*$\rightarrow$*AMI-1*$\rightarrow$*EMO-3*

(2.13) $\vdash$*AMM-1*$\rightarrow$*AHH-2*$\rightarrow$*ESH-2*

(2.14) $\vdash$*AMM-1*$\rightarrow$*AHH-2*$\rightarrow$*ESH-2*$\rightarrow$*ESO-2*

(2.15) $\vdash$*AMM-1*$\rightarrow$*AHH-2*$\rightarrow$*ESH-2*$\rightarrow$*ESH-1*

(2.16) $\vdash$*AMM-1*$\rightarrow$*AHH-2*$\rightarrow$*ESH-2*$\rightarrow$*ESO-2*$\rightarrow$*ESO-1*

(2.17) $\vdash$*AMM-1*$\rightarrow$*AHO-2*$\rightarrow$*HAO-3*

(2.18) $\vdash$*AMM-1*$\rightarrow$*AHO-2*$\rightarrow$*AAM-1*

(2.19) $\vdash$*AMM-1*$\rightarrow$*HMO-3*$\rightarrow$*SMI-3*

(2.20) $\vdash$*AMM-1*$\rightarrow$*HMO-3*$\rightarrow$*SMI-3*$\rightarrow$*MSI-3*

(2.21) $\vdash$*AMM-1*$\rightarrow$*EMF-1*$\rightarrow$*EMO-1*$\rightarrow$*EAH-2*

(2.22) $\vdash$*AMM-1*$\rightarrow$*EMF-1*$\rightarrow$*EMO-1*$\rightarrow$*AMI-3*

(2.23) $\vdash$*AMM-1*$\rightarrow$*EMF-1*$\rightarrow$*EMO-1*$\rightarrow$*EAH-2*$\rightarrow$*EAH-1*

(2.24) $\vdash$*AMM-1*$\rightarrow$*EMF-1*$\rightarrow$*EMO-1*$\rightarrow$*AMI-3*$\rightarrow$*MAI-3*

Proof:

[1] $\vdash$*all*(*n*, *x*)$\wedge$*most*(*b*, *n*)$\rightarrow$*most*(*b*, *x*) (i.e., *AMM-1*, Axiom A2)

[2] $\vdash$*all*(*n*, *x*)$\wedge$*most*(*b*, *n*)$\rightarrow$*some*(*b*, *x*) (i.e., *AMI-1*, by [1] and Fact (4.4))

[3] $\vdash$*all*(*n*, *x*)$\wedge$*most*(*b*, *n*)$\rightarrow$*some(x*, *b*) (i.e., *MAI-4*, by [2] and Fact (3.1))

[4] $\vdash\neg$*most*(*b*, *x*)$\wedge$*all*(*n*, *x*)$\rightarrow\neg$*most*(*b*, *n*) (by [1] and Rule 2)

[5] $\vdash$*at most half of the*(*b*, *x*)$\wedge$*all*(*n*, *x*)$\rightarrow$*at most half of the*(*b*, *n*) (i.e., *AHH-2*, by [4] and Fact (2.5))

[6] $\vdash$*at most half of the*(*b*, *x*)$\wedge$*all*(*n*, *x*)$\rightarrow$*not all*(*b*, *n*) (i.e., *AHO-2*, by [5], Rule 1 and Fact (4.7))

[7] $\vdash\neg$*most*(*b*, *x*)$\wedge$*most*(*b*, *n*)$\rightarrow\neg$*all*(*n*, *x*) (by [1] and Rule 3)

[8] $\vdash$*at most half of the*(*b*, *x*)$\wedge$*most*(*b*, *n*)$\rightarrow$*not all*(*n*, *x*) (i.e., *HMO-3*, by [7], Fact (2.1) and (2.5))

[9] $\vdash$*no*$\neg$(*n*, *x*)$\wedge$*most*(*b*, *n*)$\rightarrow$*fewer than half of the*$\neg$(*b*, *x*) (by [1], Fact (1.1) and Fact (1.5))

[10] $\vdash$*no(n*, *D−x*)$\wedge$*most*(*b*, *n*)$\rightarrow$*fewer than half of the*(*b*, *D−x*) (i.e., *EMF-1*, by [9] and Definition D3)

[11] $\vdash$*no(n*, *D−x*)$\wedge$*most*(*b*, *n*)$\rightarrow$*not all*(*b*, *D−x*) (i.e., *EMO-1*, by [10], Rule 1 and Fact (4.8))

[12] $\vdash$*no*(*D−x*, *n*)$\wedge$*most*(*b*, *n*)$\rightarrow$*fewer than half of the*(*b*, *D−x*) (i.e., *EMF-2*, by [10] and Fact (3.2))

[13] $\vdash$*no*(*D−x*, *n*)$\wedge$*most*(*b*, *n*)$\rightarrow$*not all*(*b*, *D−x*) (i.e., *EMO-2*, by [12]), Rule 1 and Fact (4.8))

[14] $\vdash\neg$*some*(*b*, *x*)$\wedge$*all*(*n*, *x*)$\rightarrow\neg$*most*(*b*, *n*) (by [2] and Rule 2)

[15] $\vdash no(b, x) \wedge all(n, x) \rightarrow at\ most\ half\ of\ the(b, n)$ (i.e., *AEH-2*, by [14], and Fact (2.4) and (2.5))

[16] $\vdash no(x, b) \wedge all(n, x) \rightarrow at\ most\ half\ of\ the(b, n)$ (i.e., *AEH-4*, by [15], and Fact (3.2))

[17] $\vdash \neg some(b, x) \wedge most(b, n) \rightarrow \neg all(n, x)$ (by [2] and Rule 3)

[18] $\vdash no(b, x) \wedge most(b, n) \rightarrow not\ all(n, x)$ (i.e., *EMO-3*, by [17], and Fact (2.4) and (2.1))

[19] $\vdash at\ least\ half\ of\ the\quad (b, x) \wedge no\neg(n, x) \rightarrow at\ most\ half\ of\ the(b, n)$ (by [5], and Fact (1.8) and (1.1))

[20] $\vdash at\ least\ half\ of\ the(b, D-x) \wedge no(n, D-x) \rightarrow at\ most\ half\ of\ the(b, n)$

(i.e., *ESH-2*, by [19] and Definition D3)

[21] $\vdash at\ least\ half\ of\ the(b, D-x) \wedge no(n, D-x) \rightarrow not\ all(b, n)$

(i.e., *ESO-2*, by [20], Rule 1 and Fact (4.7))

[22] $\vdash at\ least\ half\ of\ the(b, D-x) \wedge no(D-x, n) \rightarrow at\ most\ half\ of\ the(b, n)$

(i.e., *ESH-1*, by [20] and Fact (3.2))

[23] $\vdash at\ least\ half\ of\ the(b, D-x) \wedge no(D-x, n) \rightarrow not\ all(b, n)$ (i.e., *ESO-1*, by [21] and Fact (3.2))

[24] $\vdash \neg not\ all(b, n) \wedge at\ most\ half\ of\ the(b, x) \rightarrow \neg all(n, x)$ (by [6] and Rule 2)

[25] $\vdash all(b, n) \wedge at\ most\ half\ of\ the(b, x) \rightarrow not\ all(n, x)$ (i.e., *HAO-3*, by [24], and Fact (2.2) and (2.1))

[26] $\vdash \neg not\ all(b, n) \wedge all(n, x) \rightarrow \neg at\ most\ half\ of\ the(b, x)$ (by [6] and Rule 3)

[27] $\vdash all(b, n) \wedge all(n, x) \rightarrow most(b, x)$ (i.e., AAM-1, by [26], and Fact (2.2) and (2.6))

[28] $\vdash at\ least\ half\ of\ the\quad (b, x) \wedge most(b, n) \rightarrow some\neg(n, x)$ (by [8] and Fact (1.8) and (1.4))

[29] $\vdash at\ least\ half\ of\ the(b, D-x) \wedge most(b, n) \rightarrow some(n, D-x)$ (i.e., *SMI-3*, by [28] and Definition D3)

[30] $\vdash at\ least\ half\ of\ the(b, D-x) \wedge most(b, n) \rightarrow some(D-x, n)$ (i.e., *MSI-3*, by [29] and Fact (3.1))

[31] $\vdash \quad not\ all(b, D-x) \wedge no(n, D-x) \rightarrow \quad most(b, n)$ (by [11] and Rule 2)

[32] $\vdash all(b, D-x) \wedge no(n, D-x) \rightarrow at\ most\ half\ of\ the(b, n)$ (i.e., *EAH-2*, by [31], and Fact (2.2) and (2.5))

[33] $\vdash \quad not\ all(b, D-x) \wedge most(b, n) \rightarrow \quad no(n, D-x)$ (by [11] and Rule 3)

[34] $\vdash all(b, D-x) \wedge most(b, n) \rightarrow some(n, D-x)$ (i.e., *AMI-3*, by [33], and Fact (2.2) and (2.3))

[35] $\vdash all(b, D-x) \wedge no(D-x, n) \rightarrow at\ most\ half\ of\ the(b, n)$ (i.e., *EAH-1*, by [32] and Fact (3.2))

[36] $\vdash all(b, D-x) \wedge most(b, n) \rightarrow some(D-x, n)$ (i.e., *MAI-3*, by [34], and Fact (3.1))

So far, through the above 36 step reduction operations, the 24 valid generalized syllogisms in Theorem 2 have been deduced from the validity of the generalized syllogism *AMM-1*. Similarly, if making best of the above symmetry, subordination law, anti-syllogism rules, and inner/outer negation, etc., more valid generalized syllogisms can be derived when one continues to infer the steps after proving step [11] in Theorem 2. The validity of these syllogisms can be proven using the truth definitions in Definition 3.5 and Facts 1-4, just like Theorem 1.

5. Conclusion and Future Work

This paper mainly discusses the non-trivial generalized syllogisms reasoning with the quantifiers in Square{*no*} and Square{*most*}. To this end, this paper firstly formalizes generalized syllogisms, then proves the validity of the syllogism *AMM-1* with the generalized quantifier *most*, and further deduces the other 24 valid syllogisms. The reason why the valid generalized syllogisms studied in this paper can be mutually reduced is because: (1) any of the four Aristotelian quantifiers in Square{*no*} (that is, *no*, *all*, *some*, *not all*) can define the other three ones; (2) so can any of the four generalized quantifiers in Square{*most*} (that is, *most*, *fewer than half of the*, *at most half of the*, *at least half of the*).

There are (8×8×8×4–4×4×4×4=) 1972 non-trivial generalized syllogisms involving the 8 propositions mentioned earlier. How can we screen out all the valid ones among them? This question requires in-depth discussion.

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