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Original Paper

Blast Wave

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Abstract

The main results of the development of calculation methods and the creation of new antenna options based on the theoretical achievements of the middle of the last century are considered.

Keywords

Antenna theory, distribution of current, integral equation

1. Introduction

The development of antenna technology, like any other new technology, has a stepped nature. In the middle of the last century, it seemed that this development had stopped. The method of induced EMF confidently took upon oneself the calculation of the electrical characteristics of antennas. Unfortunately, it was not rigorous and precise enough to analyze complex radiators. In 1941, Stratton's famous book "The Theory of Electromagnetism" (Stratton, 1941) was published, which was later called gold. In this book, as a rigorous method for analyzing a straight linear antenna, it was recommended to use a method based on calculating the eigenfunctions of a spheroid (an elongated ellipsoid with an eccentricity slightly different from unity). This proposal was clearly pessimistic, because spheroidal functions are difficult to compute, and the method gives an approximate result and is practically applicable to only simplest antenna. Therefore, it might seem that the powerful blast wave that arose soon after the derivation and solution of integral equations for the currents in the antennas became a kind of reaction to this pessimistic forecast. Nevertheless, the calculation method using spheroidal eigenfunctions was brought to numerical results (Duncan & Hinchey, 1960), similar to the known ones, and after that this method was forever forgotten due to the limited possibilities of its use.

Antenna theory is based on the basic equations of electrodynamics - Maxwell's equations. A straight ideally conducting filament with an infinitely small radius or a tube with a finite radius a is used as a model of a cylindrical radiator. A conduction current $J(z)$ flows along the filament, which coincides, for example, with the z-axis. When the antenna is excited by one generator, the current is considered sinusoidal - based on the measurement results and the simple idea that when the wires of an open two-wire long line are moved apart in different directions, the current distribution along its wires changes weakly. The solution of the integral equations for the currents in the radiators confirms that the sinusoidal distribution is the first approximation to the true distribution.

2. Integral Equations

The integral equations of Hallen and Leontovich played a most important role in the development of antenna theory. They are based on the condition

$$E_z(J)|_{-L\leq z\leq L} + K(z) = 0,$$

according to which the resulting field on an ideally conducting metal surface, which is the sum of the field $E_z(J)$, created by the current $J(z)$ and the external field $K(z) = e\delta(z)$ is equal to zero. There is no current at the ends of the radiator (at points $z = \pm L$). The equation form is mainly determined by the choice of the function $E(J)$. In the case of a tubular antenna we obtain the Hallen equation with an exact kernel, in the case of an infinitely thin filament we obtain the Hallen equation for the filament. The widely used integral equation for a radiator of finite radius a is called the equation with an approximate kernel:

$$\int_{-L}^{L} J(\varsigma)Gd\varsigma = j\frac{1}{Z_0}\left(C\cos kz + \frac{e}{2}\sin|z|\right).$$

Here $G = exp(-jkR)/(4\pi R)$, $R = \sqrt{(z-\varsigma)^2 + a^2}$.

The solution to this equation was given by Hallen himself (1938), a detailed description of the solution is presented in the book of Aharoni (1946). During the solution, the method of successive approximations (iterative process) was applied. In this case, the function $J(z)$ is expanded into a series in inverse powers of the magnitude

$$\Omega = 2\,ln(2L/a).$$

The process, proposed by R. King and D. Middleton (1956) gives more accurate results. In it, the expansion parameter Ω is replaced by the magnitude Ψ. As Ψ, the value of the function $\Psi(z)$ is taken, which is proportional to the ratio of the vector potential to the total current at a point near the maximum, where the value of $\Psi(z)$ is almost constant. The first term in the expansion of the current in a series has the form

$$J_0 = j\frac{e}{60\Psi\cos kL}\sin k(L - |z|),$$

where k is the wave propagation constant along the antenna wire.

The integral equation of Leontovich and its solution are described in Leontovich and Levin (1944). This equation has the form

$$\frac{d^2J}{dz^2} + k^2J = j\frac{k}{30}\chi K(z) + \chi W(J).$$

Here $\chi = 1/\Omega$ is a small parameter,

$$\chi W(J) = \frac{d^2V}{dz^2} + k^2V$$

is the EMF, created by the antennas' currents. In this equation, the logarithmic singularity is excluded and there is no dependence on the azimuth φ, since the integration over φ has already been performed. Nevertheless, it is equivalent to the Hallen equation with an exact kernel. When solving it, the method of perturbations is used, i.e., the solution is sought in the form of a series in powers of the small parameter χ:

$$J(z) = \chi J_1(z) + \chi^2 J_2(z) + \chi^3 J_3(z) + \cdots$$

The noted features of the equation substantially simplify the calculations.

Comparison of the numerical results obtained by means of integral equations and by the method of induced EMF showed that for a straight metal antenna without losses these results are close to each other in most part of the frequency range. The similarity also exists in the same functions included in the obtained results. Finally, the question of the similarity of the both results was resolved when a new version of the Leontovich equation solution, based on the mathematical method of variation of constants, was developed. The new solution was published for the first time in Levin (1992) and described in Levin (2021). With its help, analytical expressions were obtained for the input impedance of a straight metal antenna without losses, which completely coincided with the results of the induced EMF method.

Further development of the new method was based on the fact that when calculating the next member of the series for the current, the field created by the previous member of the series, i.e., the current of a smaller order of smallness, was taken as the external EMF:

$$e_{n-1}(\varsigma) = -E_{\varsigma}(J_{n-1}).$$

This equality provides the freedom to choose constant coefficients for different members of the series. Choosing the simplest option $J_n = J_1$, it is easy to show that the members of the series form a geometric progression with a constant denominator $q = -\chi$, and that allows to determine the total sum of the series, and hence the total impedance of the antenna. The results show that the series sum is close to the sum of the first two or three terms. L. A. Vainshtein (1959) came to similar conclusions when solving the so-called key equation. This fundamental result is, to a certain extent, a distant consequence of the blast wave of the middle of the last century. On Figure 1 it is shown, as an example, the reactive component X_A of the dipole impedance with the ratio $L/a = 5$ and 50. The total reactance is given by a solid line, the second approximation - by a dotted line.

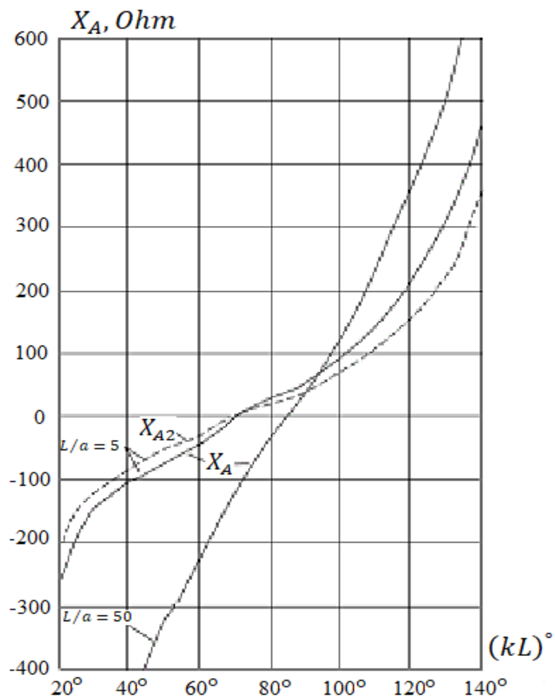

Figure 1. The Dipole Reactivity

3. Method of Induced EMF and Poynting Vector Method

The method of integral equations breathed new life into other calculation methods, in particular, it significantly changed the understanding of a method of induced EMF. As is known, it is based on equating the complex power given by the source (generator) and the complex power passing through the closed surface surrounding the antenna. Combining a closed cylindrical surface with the radiator surface and considering its radius to be small, it is easy to make sure that the complex power passing through the surface is determined as a result of integration over the cylinder side surface and is equal to

$$P_1 = -\int_{-L}^{L} E_z J^*(z)\, dz.$$

If the current is excited by a single generator located in the middle of the radiator, then the complex power creating by it is equal to

$$P_2 = |J(0)|^2 Z_A,$$

where Z_A is the input impedance of the radiator. Equating both powers, we find:

$$Z_A = -\frac{1}{|J(0)|^2}\int_{-L}^{L} E_z J^*(z)\, dz.$$

This expression is called the first formulation of the induced EMF method. Here the quantity $Im\{E_z J(z)\}$ has no clear physical meaning.

The second formulation is based on the theorem about an oscillating power (Vainshtein, 1957), passing through the surface, and proceeds from the equality of the instantaneous values of the powers on different sides of this surface. The instantaneous power value is equal to the product of instantaneous values of two quantities, in this case E_z and $J(z)$. As shown in Vainshtein (1957), the product $s_z(t)$ of two instantaneous values consists of a constant magnitude $\bar{S}_z$, equal to the average value over the oscillation period, and a variable magnitude $\tilde{S}_{z,}$ oscillating in time at a double frequency. At that

$\bar{S}_z = Re\{E_z J^*(z)\}$, $\tilde{S}_z = Re\{E_z J(z) e^{2j\omega t}\}$.

This implies the physical meaning of these quantities: $\bar{S}_z$ is the active part of the power flow, equal to its average value; $E_z J(z)$ is the amplitude of the power flux, oscillating at a double frequency. The oscillating power transmitted through the closed surface is

$$P_{O1} = -0.5 \int_{-L}^{L} E_z J(z) dz.$$

The oscillating power created by the generator with EMF e is equal to

$$P_{O2} = eJ(0) = J^2(0) Z_{A.}$$

Equating these values, we find:

$$Z_A = -\frac{1}{J^2(0)} \int_{-L}^{L} E_z J(z)\, dz.$$

This expression is called the second formulation of the induced EMF method. It was first obtained in Kontorovich (1951) in accordance with the principle of reciprocity. As shown here, it is easily deduced using ratios between the powers.

It should be emphasized that the derivation of the second formulation is strictly made, while in the derivation of the first formulation the concept of reactive power was used, which has no physical meaning. An important advantage of the new formulation is its applicability to lossy antennas and to antennas located in lossy environments. The contradiction between the induced EMF method and the reciprocity theorem was eliminated. The analysis also showed that the second formulation, in contrast to the first, has a stable character. Simultaneously with the received correct result, the radiation process became clear. The oscillating power theorem permitted to obtain, in particular, a rigorous solution to the problem of calculating the loss resistance in the ground for a vertical monopole.

An analytical solution to the problems about an antenna radiation was obtained for a comparatively small number of simple variants of radiators with limited dimensions located in free space. In this regard, numerical methods are widely used. This approach is implemented by means of the moments' method. Unknown currents are represented as the sum of elementary currents multiplied by linearly independent functions, which are called basis ones. After substitution of the basis functions, both parts of the integral equation are sequentially multiplied by linearly independent functions of the second system (weight functions) with integration over the antenna length. As a result, the problem is reduced to solving a system of linear algebraic equations for elementary currents with coefficients that are determined by numerical integration.

If the system of weight functions coincides with the system of basis functions, then the resulting version of the moments' method is called the Galerkin method. The variant of the such method chosen by Richmond (1966) uses piecewise sinusoidal functions as basis and weight functions of subdomains and provides fast convergence of the results. The coefficients of algebraic equations are closed expressions, which coincide with the mutual impedances between isolated short dipoles. Therefore, the Richmond method was called the generalized induced EMF method.

The generalized method of induced EMF permits to determine and, in many cases, to weaken the effect of metal structures located near the antenna on its properties. This influence is especially great on moving objects (on cars, ships), where it is necessary to deal with spatial restrictions. Replacing metal surfaces with objects of simple shape, convenient for calculation, gives unsuitable results. The method of analysis based on replacing a metal body with a system of thin wires is much more efficient (Richmond, 1966; Perini & Buchanan, 1982). In this case, there is no need to idealize, for example, the shape of a metal superstructure or consider its transverse dimensions to be small. The results of calculating the directional patterns of whip antennas located near superstructures of different heights and shapes at different frequencies of the HF range are given, in particular, in Levin (1998). In the same books the characteristics of antennas installed on one of the ship's decks with help of brackets also examined. As can be seen from the results, the placement conditions change all characteristics of the antennas, and the coincidence of the calculation and experiment indirectly confirms the correctness of selecting the calculated structure consisting of wires.

It is possible to weaken the influence of superstructures by using antennas with the required characteristics. If, for example, the superstructure sharply attenuates the signals of an antenna or antenna array located near it in a given direction, then this disadvantage can be eliminated by using radiators of a greater height. At that the necessary frequency range is provided in these antennas by means of a surface impedance or concentrated loads (Levin, 2021).

New results were also obtained using the method of Poynting vector. As is known, the electric field of a symmetric linear radiator with a sinusoidal current, located along the z-axis, with the center at the origin is equal to

$$E_\theta = j\frac{60J(0)}{\varepsilon_r \sin kL}\frac{exp(-jkR)}{R}\frac{cos(kLcos\theta) - coskL}{sin\,\theta}.$$

Here the last multiplier determines the directional pattern of the vertical radiator in the vertical plane. The radiated power of an antenna passing through the surface of the surrounding sphere is

$$P = \frac{R_0^2}{120\pi}\int_0^{2\pi} d\varphi \int_0^{\pi/2} \frac{E_\theta^2}{R_0^2} sin\theta d\theta.$$

Dividing the power by the square of the generator current, we find the resistance of antenna radiation:

$$R_\Sigma = \frac{P}{J^2(0)} = \frac{1}{30J^2(0)}\int_0^{\pi/2} E_\theta^2 sin\theta d\theta.$$

Unfortunately, the value E_θ has a complex character that complicates calculating the integral. If to accept that the field at an arbitrary angle θ coincides with the field in the horizontal direction, equal to

$$E_\theta = j30J(0)kl_e\frac{exp(-jkR)}{\varepsilon_r R},$$

where $kl_e = 2\,tan(kL/2)$, then the obviously wrong result is obtained.

The problem is solved, if we take into account an infinite radius of the surface, through which the transmitted power pass (Levin, 2021). For finite sizes of the radiator, one can consider that the distance from any radiator point to the surface of the infinite radius is the same, i.e., the field in the far zone has the character of a spherical wave, and the value of the signal passing through the surface depends only on the angle in the vertical plane between the antenna axis and the tangent to the surface in the given point of it. In such a situation, in accordance with the desire to simplify the calculations as much as possible it is expedient as the origin field to choose the field in the direction $\theta_0 = \pi/2$. In this case,

$$R_\Sigma = \frac{1}{30J^2(0)} \int_0^{\frac{\pi}{2}} |E_{\theta 0}|^2 sin^3\theta d\theta,$$

i.e.,

$$R_{\Sigma 0} = 20(kl_e)^2.$$

One may to verify this expression, using integration by parts, but this work is much more difficult. The described technique also was used to calculate the mutual active resistances of two parallel radiators located with and without a shift in height. The calculation results coincide with the characteristics obtained by other methods.

4. In-phase Current Distribution

The blast wave had numerous consequences. After the integral equations for metal antennas, an equation was written for a straight slot antenna (Pheld, 1948), and a rigorous method was proposed for the analysis of metal and slot antennas with constant and varying along the length surface impedance (Miller, 1954). The use of inductive impedance (of ferrite coating) has shown that it increases the wave propagation constant and the electrical length of the antenna, partially increases the radiation resistance at a constant geometric length of the radiator, and significantly reduces this resistance, if the electrical length of the antenna does not change.

By replacing the surface impedance with concentrated capacitive loads, it is possible, as shown by the experimental results (Rao, Ferris, & Zimmerman, 1969), to obtain a current distribution along the antenna that is different from the sinusoidal one. In accordance with Hallen hypothesis about the usefulness of capacitive loads, the inverse problem of the theory of radiators was solved - to determine the magnitude of loads that provide in-phase current distribution along the antenna wire in a wide frequency range (Levin, 1998). It was shown that each load should be a parallel connection of a resistor and a reactive element, and capacitors should be used as reactive elements (the use of more complex reactive two-terminal networks, which include inductances, narrows the operating range). Capacitors make it possible to create an electromagnetic wave along the antenna with a real propagation constant, i.e., obtain an exponential distribution of the current amplitude with a positive decrement (concave current curve). A convex current curve cannot be created with simple elements.

In a particular case, if it is necessary to obtain a law close to linear for the current envelope, the capacitance of the loads should decrease to the free end of the antenna in proportion to the distance from it. The resistance of resistors must increase: $C_n = C_N \frac{n-1}{N-1}$, $R_n = R_N \frac{N-1}{n-1}$. Here C_N and R_N are capacitance and resistance near the base of the asymmetric radiator. For the propagation constant to be real at a given frequency *f*, the capacitances of the capacitors must be less than

$$C_N \leq \frac{2.54 \cdot 10^5 \chi_n}{f^2 b}.$$

Here C_N is the capacitance in picofarads, *f* is frequency (in megahertz), χ_n is a small parameter on *n*th segment, *b* is segment length (in meters). The use of resistors reduces the efficiency, so the question of their use must be decided in each specific case.

At low frequencies, the propagation constant k is real along the entire antenna. As the frequency increases, it becomes purely imaginary (primarily near the generator), and the current becomes sinusoidal. This effect limits the frequency range of the antenna from above. To expand the range, the capacitances of the capacitors must decrease with increasing frequency in inverse proportion to its square. For this, it is necessary to change the capacitances, for example, to switch the capacitors. Calculations and experiments confirm that the installation of loads significantly expands the frequency range. With the same requirements for electrical characteristics and the same antenna sizes, the frequency overlap factor for a thin whip antenna is changes from 1.3 to 1.5, for an antenna with capacitive loads - from 1.5 to 3, with capacitive-resistive loads - from 3 to 4, with tunable capacitors - from 4 to 8 and higher.

5. Optimizing Antennas with Help of the Mathematical Programming

The described method of analyzing antennas with loads, based on the theory of an impedance radiator, makes it possible to determine the potential capabilities of the considered radiators. The obtained results were used to optimize antennas by the mathematical programming method (Himmelblau, 1972). This method makes it possible to create an antenna with characteristics that are as close as possible to the required ones. This reservation is due to the fact that the range of variation of the radiator parameters is limited, i.e., not every value of the electrical characteristic of a particular antenna can be practically realized. Different characteristics are optimal at different parameter values. The antenna must have the desired characteristics not at one fixed frequency, but over the entire frequency range. Therefore, the selected parameters are the result of a compromise, which is achieved by the method of mathematical programming.

The problem is formulated as follows: one must need to find a vector of parameters $\vec{x}$, that minimizes the objective function $\Phi(\vec{x})$ under the imposed constraints $\varphi_i(\vec{x}) \geq 0$. The objective function is the sum of the particular functionals $\Phi_j(\vec{x})$. Each such functional corresponds to one of the antenna characteristics f_j and is multiplied by the weight coefficient p_j, which takes into account the importance of this characteristic. When constructing a particular functional, the quasichebyshev criterion gives good results.

The choice of functions f_j depends on the task at hand. Of great practical importance is a particular case of this problem - the creation of a radiator that provides a high level of matching and a maximum radiation in the plane perpendicular to the axis of the radiator in a wide frequency band. In this case, the coefficient of the traveling wave in the cable and the coefficient of the radiation pattern, equal to the average level of radiation under angles close to the horizon, are used as functions f_j. Since there is no analytical expression for the objective function, the minimum of the function is determined by a numerical method, based on finding the gradient. The described process presupposes a repeated calculation of the radiator characteristics for various initial parameters. For this calculation, a program based on the method of moments is applied. As the initial values of the parameters, the values calculated by the method based on the theory of the impedance radiator are used. As a result, the process of finding the minimum of the objective function is accelerated, but most importantly, the probability of an error, caused by the fact that with an arbitrary choice of initial parameters this optimization process can lead not to a true, but to a local extremum, decreases.

In Figure 2 the result of optimization (minimal TWR) of 12-meter antenna with an isolator capacitance $C_i = 15$ pF in its base is presented for the different radii a and bandwidth ratios k_f depending on relative antenna length L/λ_m (λ_m is the maximal wavelength).

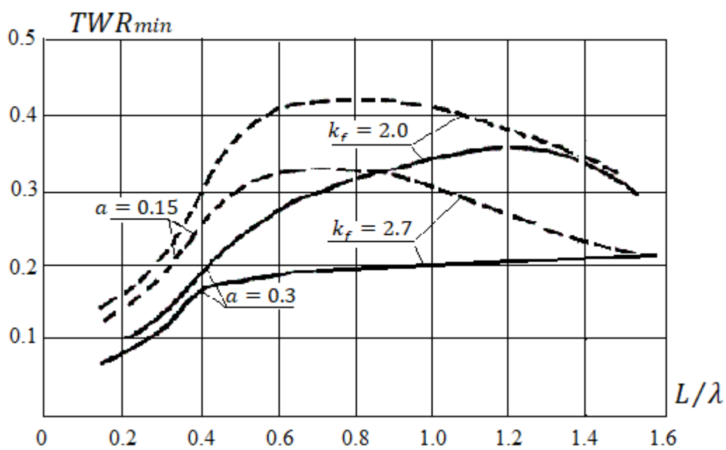

Figure 2. The Maximum Level of Matching

For the first time, the mathematical programming method for optimizing antenna devices was applied to the end-fire antenna array of Yagi-Uda in order to obtain maximum directivity (Chaplin, 1969). The solution of the problem was brought to practical results (Chaplin, Buchazky, & Mihailov, 1983). The similar methods allowed to solve the problems of creating the required current distribution in the linear radiator, optimizing the characteristics of V-antennas with loads (Levin, 1998), etc.

6. New Antennas and Methods

The widespread use of radiators in various areas of technology has led to the development of new antenna structures. The principle of duality based on the symmetry of Maxwell's equations with respect to the quantities $\vec{E}\varepsilon_0$ and $-\mu_0\vec{H}$ provided the calculating properties of magnetic (slot) radiators in accordance with the characteristics of electric (metal) ones. The Babinet's principle (Mushiake, 1996) substantiated the possibility of developing self-supplementary antennas. The input impedances of such antennas are related by the expression

$Zs = (60\pi)^2/Ze.$

Here Z_s is the input impedance of the slot radiator, equal to the input impedance of the adjacent metal radiator, Z_e is the input impedance of the metal radiator, the shape and dimensions of which coincide with the shape and dimensions of the slot. If these antennas have the same shape and dimensions, they are called self-complementary. In this case,

$$Z_s = Z_e = 60\pi,$$

i.e., the input impedance of each radiator does not depend on frequency, is purely active and is equal to 60π. The special properties of such structures make them unique. These properties are partially preserved if the dimensions of the metal sheet are finite.

As shown for the first time in Levin and Markov (1997), an antenna consisting of metal and slot radiators of the same shape and size can be three-dimensional. The such radiator can be placed on the surface of a circular cone, paraboloid or pyramid (Levin, 2019). In the case of a double-sided slot antenna located on a circular metal cone of infinite length and excited at its apex (see Figure 3), the input impedance of the structure is

$$Z_M = (120\pi)^2/Z_e,$$

where Z_M and Z_e are the input impedances of the magnetic and electric radiators, respectively.

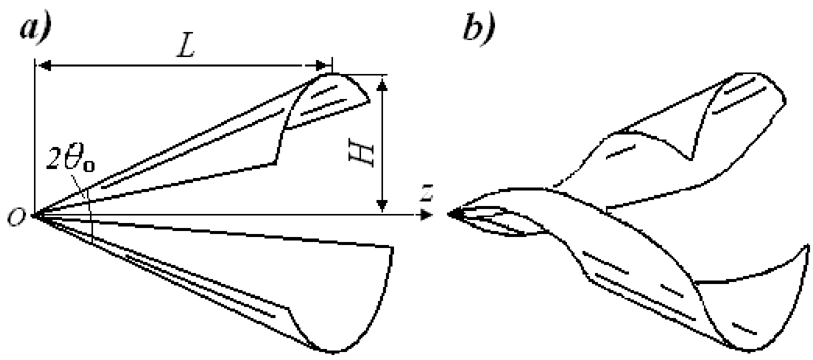

Figure 3. The Slot Antennas at the Cone with a Straight-line (a) and the Helical (b) Edges

Experts have considered and applied a wide variety of antenna structures, including loop antennas (round and square), folded and multi-folded (metal and with inductive surface impedance), transparent (with resistive surface impedance), multi-wire and multi-radiator, including antennas with an absorber

(Halpern & Mittra, 1985)). Microstrip antennas (simple and multiple-stack) characterized by simplicity and wide possibilities of their installation, have been proposed and applied on various moving objects. A variety of antenna options for cellular telephones allows to reduce irradiation of the user's head. In addition to a dipole log-periodic antenna providing operation in a wide frequency range, a monopole log-periodic antenna (Yakovlev & Pyatnenkov, 2007) and a multi-tiered antenna with a directional pattern pressed to the ground (Levin & Yakovlev, 1992) were developed.

Also new interesting methods of antenna analysis and synthesis recently have been proposed, including: method of electrostatic analogy, application of the complex potential method to microstrip antenna analysis, approximate integral equation for a non-straight antenna, reduction of three-dimensional problem to a plane one, reciprocity theorem for reflect array, expanding frequency band of microstrip antenna, methods of struggle with influence of moving external metallic objects. The blast wave still works.

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