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Original Paper

Method of Electrostatic Analogy

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Abstract

It is considered the analysis method for electromagnetic fields and directional characteristics of antenna arrays consisting of parallel linear radiators based on the calculation of the electrostatic fields of charges located on parallel wires. It is demonstrated the advantage of using radiators with in-phase current distribution. It is shown that the application of radiators with in-phase current distribution allows to decrease the length of antenna array and to provide the high directivity in a wide frequency range. It is stated the task of optimization of considered arrays by methods of mathematical programming.

Keywords

Antenna theory, directional antennas, in-phase current distribution, optimization

1. Introduction

The purpose of antenna synthesis is creating antennas with high electrical characteristics. In order to solve this problem engineers tried for a long time to find a law of current distribution, which provides required characteristics. The issue of creating the desired distribution of the current remained open. The task of choosing of antenna elements and their dimensions with the aim of optimizing antenna characteristics was first staged as the task of mathematical programming in relation to the Yagi-Uda antenna (Chaplin, 1969; Chen & Cheng, 1975). The task solution confirmed rightness of chosen approach.

The more general problem is a problem of creating a wideband radiator. It consists in determining a type and magnitudes of concentrated loads, which are placed along a linear radiator and provide in a given frequency band high electrical characteristics, including matching level, efficiency and required directional pattern (Levin, 1998). Its solution was based on understanding advantages of the in-phase current distribution and on the hypothesis of Hallen about usefulness of capacitive loads, whose magnitudes are changed along a radiator axis in accordance with linear or exponential law. Selected approach confirmed the hypothesis of Hallen, demonstrated the rightness of applying capacitive loads and gave numerical results.

The first of these was determination of loads magnitudes, which provide in a given frequency range the required current distribution. The second result is decreasing distortion of directional patterns by closely spaced superstructures with help of increasing height of antennas and using loads providing an operation in required frequensies band. Finally, the method was used for selecting loads in the wires of a symmetrical V-antenna in order to expand significantly the frequencies band, in which this antenna has high directivity in the direction of the bisector of its angular aperture. These tasks are consistently considered in (Levin, 2017).

Based on these results, it was decided to consider the possibility of using capacitive loads in directional antennas, in particular to use in-phase currents in a director-type antenna. During the work a reasonable sequence of solving each problem was defined. At the first stage an approximate method of solution must be proposed. Its results are used at the second stage as initial values for numerical solution of the problem by the method of mathematical programming. For analyzing the radiator with loads, the method of two-wire impedance long line and the method of two-wire metal long line with loads were used as approximate methods. Effectiveness of the such methodology is obvious compared with helpless method of trial and error. In the case of antennas systems consisting of parallel linear radiators, the method of electrostatic analogy can be used as an approximated method of this type.

The specified method and the procedure of its application are described in Section 2. In Section 3 the known variant of directional antenna is considered as an example. This antenna was developed as result of the first profound efforts devoted to the optimization of the such antennas. The method of electrostatic analogy allow us to consider the issues of expanding the operating range of directional antenna and to apply obtained results to a log-periodic antenna. The results of using the approximate method to the calculation of a log-periodic antenna are given in Section 4. Based on these results, the problem of applying the mathematical programming method to this antenna was stated.

2. Method of Electrostatic Analogy

Basic directional characteristics of antennas are the two characteristics: directivity D and pattern factor PF. The directivity is the ratio of radiation intensity in the direction of maximum to the average value of intensity in all directions (to the value of isotropic radiation). In accordance with this formulation, the directivity is the coefficient of increasing total radiated power P, if a maximum power is radiated in all directions. In this case, a total power is

$$DP = 4\pi S_m,$$

where S_m is a power flux in the direction of maximum radiation, i.e., a total power is equal to a product of a power in the direction of a maximum radiation by the area $S = 4\pi R^2$ of a sphere of an infinite large radius R. In a cylindrical coordinate system, the radiated power of a typical antenna with an axial structure, symmetrical with respect to two planes, passing through coordinates origin, is

$$P = 2\int_0^{2\pi} d\varphi \int_0^{\pi/2} S_m F^2(\theta,\varphi)d\theta = 4\pi \int_0^{\pi/2} S_m F^2(\theta,\varphi)d\theta.$$

According to these two expressions

$$1/D = P/(4\pi E_m^2) = \int_0^{\pi/2} \frac{S_m F^2(\theta)}{E_m^2} d\theta,$$

where E_m is the electric field in the direction of maximum radiation.

The pattern factor is equal to the average level of radiation in a predetermined range of angles, for example, in the range of vertical angles θ from 60 to 90 degrees:

$$PF = \frac{1}{K}\sum_{k=1}^{K} F(\theta_k),$$

where $F(\theta_k)$ is the magnitude of normalized vertical directional pattern at an angle θ within an angular sector from θ_1 to θ_K. The distance of radio communication depends on the field magnitude in the pointed range of angles. Therefore pattern factor is an important characteristic of any antenna. If a maximal directivity of an antenna is large, but is located outside the required angular sector, it is useless for radio communication.

As already said, for calculating electrical characteristics of systems consisting of parallel linear radiators we will use the method of electrostatic analogy as an approximate method. It is based on the similarity of electromagnetic fields created by high-frequency currents of linear radiators with electrostatic fields of charges placed on linear conductors. Both fields are directly proportional to the magnitude of the current or the magnitude of the charge, and in the far zone they are inversely proportional to the distance from the source. This similarity of their fields is a sign of the identical mathematical problem and the identical form of equations. This coincidence permits to use the principle of correspondence in the solution process.

When solving mathematical problems, an analogous form of equation often lends essential assistance. For example, the first two Maxwell's equations that became a basis for classical electromagnetic theory allowed substantiate the principle of duality (Pistolkors, 1948). Let's write these equations in the form:

$$curl\vec{H} = \vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}, \quad curl\vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t},$$

where $\vec{E}$ and $\vec{H}$ are strengths vectors of electric and magnetic field respectively, $\vec{j}$ is a conduction current, ε_0 and μ_0 are permittivity and permeability of a surrounding medium. By replacing variables in accordance with the equalities $\vec{E}_1 = \sqrt{\varepsilon_0}\vec{E}$, $\vec{H}_1 = \sqrt{-\mu_0}\vec{H}$ and introducing conventionally a magnetic conduction current, we obtain

$$curl\vec{H}_1 = \vec{j}_1 + \sqrt{-\varepsilon_0\mu_0}\frac{\partial \vec{E}_1}{\partial t}, curl\vec{E}_1 = +\sqrt{-\varepsilon_0\mu_0}\frac{\partial \vec{H}_1}{\partial t},$$

where $\vec{E}_1$ and $\vec{H}_1$ are also strength vectors of fields, and $\vec{j}_1$ is a conduction current, but in other units. The obtained equations are completely symmetric with respect to $\vec{E}_1$ and $\vec{H}_1$. An introduction of a magnetic conduction current makes theirs also symmetric with respect to electric and magnetic conduction currents, i.e., makes possible to treat theirs not only as equations for an electric radiator, but also as equations for a magnetic radiator with a magnetic conduction current $\vec{J}_{m1}$. Expressions for fields and characteristics of a magnetic radiator can be recorded by means of the expressions analogous to expressions for an electric radiator. For example, an input impedance of a magnetic radiator Z_m, whose shape and dimensions coincide with the shape and dimensions of an electric radiator, is equal to

$$Z_m = (120\pi)^2/Z_e.$$

Here Z_e is an input impedance of an electric radiator. A slot is a real embodiment of a magnetic radiator.

A similar method of solving problems is used in many other cases. For example, there is the known analogy between a picture of electrostatic field of charged conducting bodies located in a homogeneous and isotropic dielectric and a picture of constant currents in a homogeneous, weakly-conducting medium. In this case, bodies placed in the medium must have high conductivity and their shape and geometric dimensions must coincide with the shape and dimensions of the conducting bodies located in a dielectric.

If a picture of an electric field of linear charges is known, then, using the correspondence principle, one can construct a picture of a magnetic field of constant linear currents, provided the currents and charges are distributed in space identically. The difference between these images is only that lines of equal magnetic potential are located on places of lines of electric field strength, and lines of magnetic field strength are located on places of lines of equal electric potential (Neyman & Kalantarov, 1959).

Generalizing the principle of correspondence, it is advisable to compare electromagnetic fields created by high-frequency currents of linear radiators with electrostatic fields of charges, which are placed on linear conductors. This approach is based on an analogy between two structures consisting of high-frequency currents and constant charges. It is assumed that shapes and dimensions of radiators coincide with shapes and dimensions of conductors. In the case of several radiators a ratio of emf in their centers is equal to a ratio of charges placed on the conductors. The positive charge, equal to Q_0, is located on the conductor *0*, which corresponds to the active radiator. The negative charges (their number is equal to *N*) are located on the conductors *i*, corresponding to passive radiators. They are equal to $-Q_i$ and their sum is $\sum_{i=1}^{N}(-Q_i) = -Q_0$, i.e., the sum of all charges is zero and the conductors form an electrically neutral system. In this system

$$Q_i \Big/ Q_0 = C_{0i} \Big/ \sum_{(i)} C_{0i},$$

where C_{0i} is the partial capacitance between conductors *0* and *i*. As it follows from this equality, the charges of the conductors *i* are directly proportional to the partial capacitances C_{0i} between these conductors and the conductor *0*. See, for example, (Iossel, Kochanov, & Strunsky, 1981). Equivalent replacement of a complex structure of high-frequency radiators by a structure with constant charges placed on conductors sharply simplifies the problem, reducing it to the electrostatic problem. In accordance with what has been said, it is natural to call the proposed method by the method of electrostatic analogy.

The considered method allows analyzing the problem in a general view, for example, to study and to compare different laws of current distribution along the individual radiators. This is an undoubted advantage of the method. Characteristics of complex antennas are usually calculated using complex programs based on the moments method. For discussed problems, such a method is in essence a trial-and-error method. The moments method does not permit comparison of antennas with different distribution of currents in a common view. Therefore, this method is not applicable here. The approximate method does not give exact results. But if this method is correct, i.e. corresponds to the physical essence of the problem, its accuracy is the same for different distributions of the current, and it allows us to choose the best option.

3. Director Antenna

It is expedient to consider the procedure of applying the electrostatic analogy method on a concrete example. As an example, the director antenna (Yagi-Uda antenna) described in (Chaplin, Buchazky, & Mihailov, 1983). was adopted. This article was one of the first and most profound works devoted to optimization of directional antennas.

The antenna circuit is given in Figure 1. The antenna consists of four metal radiators, their numbers are given in parentheses: active radiator 0, reflector 1, directors 2 and 3. The antenna dimensions are shown in figure in metres. They were defined by solving the optimization problem in a rigorous formulation. Let's start with the capacitance between the wires. If the radii of the wires *i* and *0* are the same and equal to a = 0.001 m and the lengths l_0 and l_i of these wires are slightly different from each other, then the partial capacitance C_{0i} between these wires in the first approximation is equal to

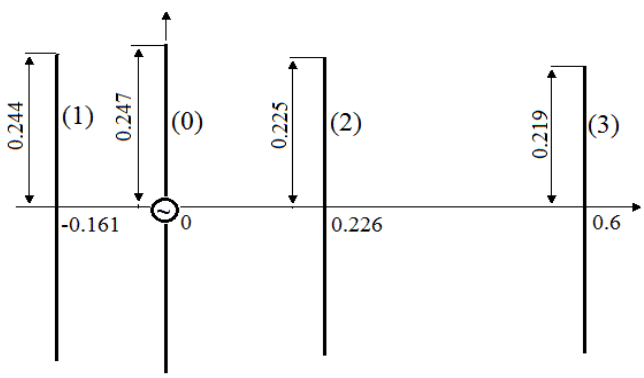

Figure 1. Dimensions of Optimized Directional Antenna from Metal Radiators

$$C_{0i} = \pi\varepsilon_0 l_i / ln(b_i/a),$$

where ε_0 is the medium permittivity and b_i is the distance between the wires.

If to divide the wire *0* of the antenna into three conductors and to denote these conductors by indices *0i*, then the circuit is divided into three circuits. Each circuit consists of two conductors: of conductor *i* and conductor *0i* (Figure 2). The generator is consists of three generators located in the centers of conductors *0i*. Their emfs are defined as $e_i = e\,Q_i/Q_0$, where *e* is the emf of the active radiator *0*.

As follows from the said, the calculation of the electromagnetic field of the presented antenna is reduced to a calculation of the fields created by the separate generators and to the summation of the obtained results. Since all elements of the system are known, calculations can be performed using a program based on the method of moments. As another variant of the calculation, the procedure based on the described in (King & Harrison, 1970) theory of folded antennas can be used. This procedure makes possible to receive results in the form of analytical expression. In this case the structure of the directional antenna is reduced to three circuits, one of them is shown in Figure 2. Using approximate expressions for the active and reactive components of the dipole input impedance and for the reactance of a long line, calculating the currents in the circuit elements and summing the currents in the wires, it is possible to determine the total field at the observation point, taking in the account distance from each wire to this point.

Electrostatic analogy between emfs in the radiator centers and the charges of conductors is approximate in nature. But if this approach is correct, i.e., corresponds to the physical essence of the problem, then its accuracy is the same for different laws of current distribution and allows us to choose the best option.

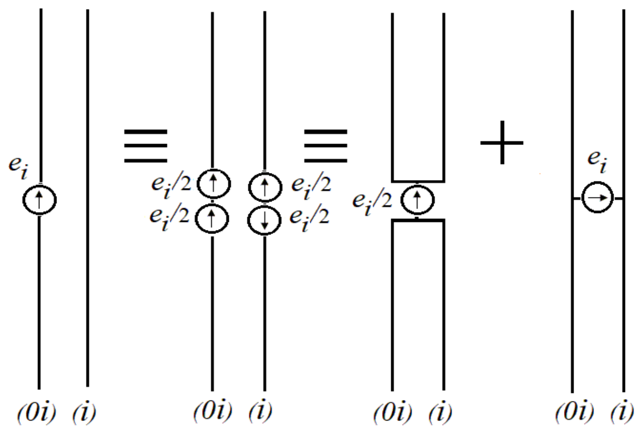

Figure 2. Circuit from Two Parallel Vertical Conductors

As is shown in the theory of folded antennas, the circuit of two parallel vertical wires with identical dimensions located at a distance b_i from each other can be divided into a dipole and a long line, open at both ends. The current at the center of each dipole is equal to

$$J_{id} = e_i/(4Z_{id}).$$

The reactive component of its input impedance is $X_{id} = -120\,ln(2L_0/a_{ei})\,cot\,kL_0$, where $L_0 = l_0/2$ is the arm length of the active radiator, $a_{ei} = \sqrt{ab_i}$ is its equivalent radius, k is the propagation constant. The current at the center of each wire of the long line is equal to

$$J_{il} = e_i/(2jX_{il}),$$

where $jX_{il} = -j120\,ln(b_i/a)\,cot\,k\,L_i$ is the reactive input impedance of the long line with length $L_i = l_i/2$. The currents of the active (*0i*) and passive (*i*) radiators are equal to the sum and difference of the dipole wire current J_{id} and the current J_{il} of the long line, i.e., the total current J_0 of the active radiator and the total current J_i of each passive radiator are

$$J_0 = \sum_{i=1}^{3}(J_{id} + J_{il}),\ \ J_i = J_{id} - J_{il}.$$

The amplitude and phase of the fields created by each radiator depend on its structure. The radiator structure determines the law of current distribution along it. If the radiator arm is a straight metallic conductor, its current is distributed according to the sinusoidal law $J(z) = J(0)\, sin\, k\,(L - |z|)$.

In this case the far field of the radiator is equal to

$$E_\theta = \frac{j60J(0)}{Sin\theta} \cdot \frac{exp(-jkR)}{\varepsilon_r R}[cos(kL\, cos\, \theta) - cos\, k\, L],$$

where R is the distance to the observation point. If the capacitive loads realize the in-phase current, distributed along the radiator in accordance with a linear law $J(z) = J(0)(1 - \frac{|z|}{L})$, then

$$E_\theta = j60J(0) \cdot \frac{exp(-jkR)}{\varepsilon_r R \sin\theta} \frac{1 - cos(kL\, cos\, \theta)}{kLcos^2\theta}.$$

From Figure 1 it is clear that the maximum radiation of the considered antenna is directed to the right, that is, towards the radiator *3*. Since the radiator *1* is located on the left from the active radiator *0* at a distance b_1, its field lags behind the field of the active radiator, firstly, per phase corresponding to a time of signal propagation from radiator *0* to radiator *1* and, secondly, per phase corresponding to the propagation time of the signal in the opposite direction - from the radiator *1* to the radiator *0* (the signal of radiator *1* must come to the radiator *0* an angle θ, i.e., the path length between the wires is $b_1/\, sin\, \theta$). The total phase difference is equal to $\psi_1 = -kb_1\,(1 + sin\, \theta)/sin\, \theta$. Similarly, in the case of radiators *2* and *3*, this phase difference is equal to $\psi_2 = kb_2\,(sin\, \theta - 1)/sin\, \theta$ and to $\psi_3 = -kb_3\,(sin\, \theta - 1)/sin\, \theta$, respectively.

The described procedure permits to determine the total field of the director antenna. In accordance with expression for e_i emfs of the different radiators are equal to $e_1 = 0.388e,\ e_2 = 0.335e,\ e_3 = 0.277e$. The total field of this antenna with radiators in the form of straight metal wires is

$$E_\theta = \frac{j60J(0)}{\sin\theta} \sum_{i=1}^{3} e_i \cdot \frac{exp(-jkR)}{\varepsilon_r R} \cdot$$

$$\left\{\left(\frac{1}{4Z_{id}} + \frac{1}{2Z_{il}}\right)[\cos(kL_0 \cos\theta) - \cos k\, L_0] + \left(\frac{1}{4Z_{id}} - \frac{1}{2Z_{il}}\right) exp(j\psi_i)[\cos(kL_i \cos\theta) - \cos kL_i]\right\}.$$

The inclusion of concentrated capacitive loads along the linear radiators, the magnitudes of which vary in accordance with the linear law, permits to create radiators with in-phase current. While retaining the dimensions of the radiators and the distances between them, we get the director antenna shown in Figure 3. The total field of such an antenna is calculated by the formula

$$E_\theta = \frac{j60J(0)}{kL\sin\theta\, cos^2\theta} \sum_{i=1}^{3} e_i \cdot \frac{exp(-jkR)}{\varepsilon_r R} \cdot$$

$$\left\{\left(\frac{1}{4Z_{id}}+\frac{1}{2Z_{il}}\right)[1-\cos(kL_0\cos\theta)]+\left(\frac{1}{4Z_{id}}-\frac{1}{2Z_{il}}\right)exp(j\psi_i)[1-\cos(kL_i\cos\theta)]\right\}.$$

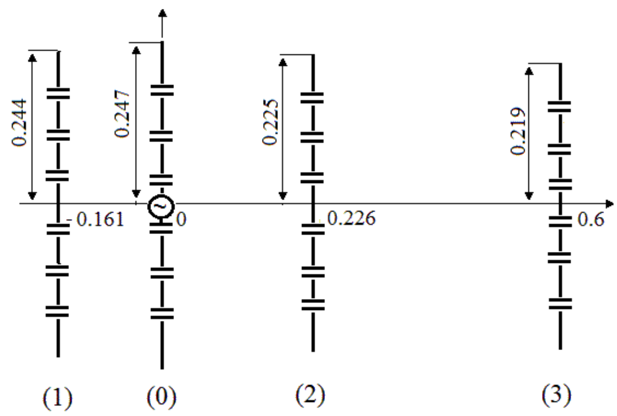

Figure 3. Directional Antenna with In-phase Straight Radiators

The results of calculating the directivity and the pattern factor of a director antenna with straight metal wires are given in Figure 4 (curves 1). Since here an approximate calculation procedure was used, these results are not identical to the results presented in (Chaplin, Buchazky, & Mihailov, 1983), but are similar to them. Practically the antenna operates at the same frequency. The results of calculating the same characteristics for the antenna with in-phase currents are also given in Figure 4 (curves 2). They speak for themselves. This antenna operates over a wide frequency range and its directivity steadily and smoothly increases with increasing frequency, that is, the quality factor of this antenna is small. Of course, we must bear in mind that these characteristics are valid only if the current is in-phase. But the frequency ratio of antennas with capacitive loads with a high level of matching is a value of the order 10.

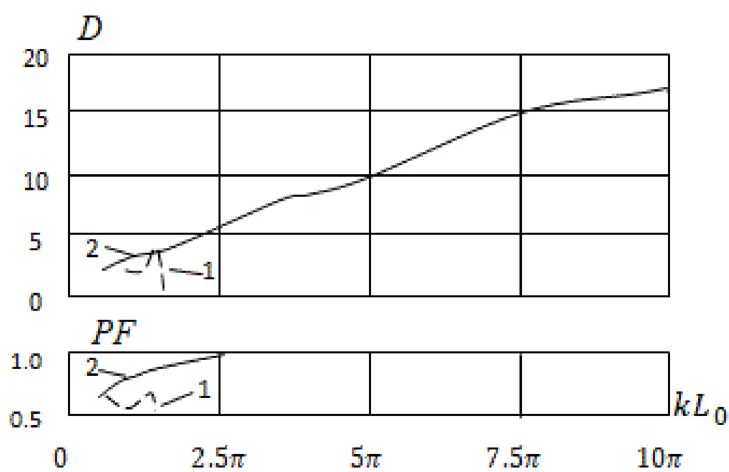

Figure 4. Directional Characteristics of Director Antennas with Sinusoidal (1) and In-phase (2) Currents

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As has already been said, the method of electrostatic analogy is based on the resemblance of a structure with high-frequency currents and a structure with constant charges. Comparison of electromagnetic fields created by high-frequency alternating currents of linear radiators with electrostatic fields of charges placed on linear conductors of electrically neutral system shows similarity of the mathematical structures of both fields.

This method allows us to propose a simple and effective procedure for calculating the directional characteristics of the director antennas consisting from linear radiators. The procedure uses knowledge about the basic antenna dimensions and current distributions along the radiators. The detailed information on the types and magnitudes of concentrated loads does not require. As calculations were shown, the director antennas consisting from the linear radiators with in-phase currents provide a high directivity and smooth variation of characteristics in a wide frequency range. The results obtained with help of this method can be used to solve the problem of optimizing various director antennas by mathematical programming methods.

The electrostatic analogy method was proposed in (Levin, 2017). But the need for such a method was understood long ago. A.F. Chaplin with his pupils stubbornly paved a way to its creation. A.F. Yakovlev persistently tried to improve the electrical characteristics of a log-periodic antenna (Yakovlev & Pyatnenkov, 2007). Their results will be discussed in Section 4.

4. Log-periodic Antenna

In Section 3 it is shown that using in-phase currents in director antennas provide high directivity in a wide frequency range. The result was obtained for the antenna array, consisting from the radiators with loads, by a method of electrostatic analogy. This method allows us to propose a new approach for solving the problem of reducing dimensions of the log-periodic antenna. Its structure does not withstand rough intervention, i.e., attempts to decrease longitudinal dimensions of an antenna by violating geometric progression relationships is causing a sharp deterioration of electrical characteristics and gives an insignificant decrease in overall dimensions.

In accordance with a well-known method of calculating log-periodic antenna (Carrel, 1961) we consider an active region of this antenna array, consisting of three radiators (Figure 5), and determine fields of these radiators when emf e is located in the center of a middle radiator. Let the arm length of the middle radiator is $L_0=\lambda/4$ (λ is the wavelength), the arm length of the left (longer) radiator is $L_1=\lambda/(4\tau)$, and the arm length of the right (shorter) radiator is $L_2=\lambda\tau/4$, where τ is a denominator of a geometric progression, according to which the radiators' dimensions are changed. A magnitude σ is another parameter, it is equal to $\sigma = 0.25(1-\tau)\cot\alpha$, where α is an angle between an array axis and a line passing through radiators ends. The value σ is the distance measured in the wavelengths between the half-wave radiator and the smaller neighboring radiator: $b_2 = \sigma\lambda$, and respectively the distance measured in the wavelengths between the half-wave radiator and the larger neighboring radiator is equal to $b_1 = \sigma\lambda/\tau$. As said in (Yakovlev & Pyatnenkov, 2007), the generalization of technic information leads to the conclusion that the minimal changes in the electrical characteristics of the log-periodic antenna with metal radiators within the frequency range from f to $f\tau$ occur when $\sigma/\tau = 0.19$. If to assume that this relation is also valid for the considered array and for definiteness to take that τ is equal to 0.9, then $\alpha = 0.146$, b_1=0.19 $\lambda, b_2 = 0.171\lambda$, $L_1 = L_0/0.9 = 0.278\lambda, L_2 = L_0 \cdot 0.9 = 0.225\lambda$,

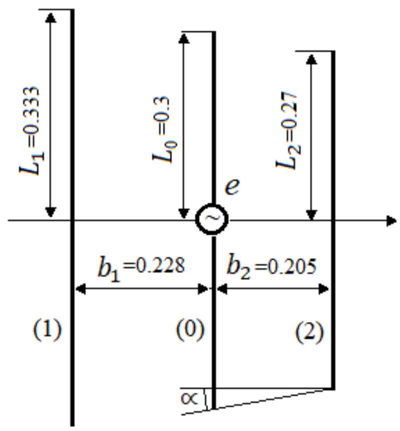

Figure 5. Directional Antenna of Three Radiators

As it was said in Section 3, since all elements of the system are known, calculations can be performed in accordance with the known methods. Let us apply the calculation method used in the analysis of the antenna array presented in Figure 1. At that we will consider that radii of all radiators are the same. The radiators' lengths, strictly speaking, are not the same, but they differ slightly. In the written below formulas, the difference between them is not fixed in order to simplify the written expressions. But the calculation with allowance for difference in their lengths shows that this difference has practically no effect on the result.

In order to use this procedure, we divide the active radiator (conductor *0*) into two parallel conductors (with numbers *01* and *02*), and afterwards the obtained system of the four conductors is divided into two circuits (conductors with numbers *01* and *1* in the first circuit, conductors with numbers *02* and *2* in the second circuit). In accordance with the method of the electrostatic analogy for two arrays, that is, accordingly to the physical content of the problem, one must assume that the ratio of emf in the centers of wires *01* and *02* is equal to the ratio of the partial capacitances C_{01} and C_{02}, i.e., $e_1 = 0.52e$, $e_2 = 0.48e$. A system of two parallel conductors located at the distance b_1 can be divided into a dipole and a long line, opened on both ends. The current in the center of each dipole is equal to $J_{id} = e_i/(4Z_{id})$; the current in the center of each conductor of a long line is equal to $J_{il} = e_i/(2Z_{il})$. Accordingly, the current in the active radiator center is the sum of the currents of the dipole and the long line, and the current in the passive radiator center is the difference between these magnitudes, i.e., $J_0 = \sum_{i=1}^{2}(J_{id} + J_{il})$, $J_i = J_{id} - J_{il}$. In the case of metal wires in the first approximation

$$Z_{id} = R_d + jX_{id}, \qquad R_d = 80\tan^2(kL_0/2),$$

$$X_{id} = 120ln(2L_0/a_{ei})\cot kL_0, \qquad jX_{il}(L_i, b_i) = -j120\, ln(b_i/a)\, cot\, k\, L_i,$$

where $a_{ei} = \sqrt{ab_i}$ is the equivalent radius of dipole.

The field amplitude and phase of each radiator depends on its structure and location in an antenna. Expressions for the currents distributions and the far fields of metal radiators and radiators with capacitive loads decreasing to antenna ends in accordance with the linear law are given in Section 3.

Consider an impact of the radiator location on the far field of the antenna by way of a specific example of the directional antenna, shown in Figure 5. The arm length of the middle (active) radiator is equal to 0.3 м, i.e., the wave length of the first (series) resonance is equal to 1.2 м. Radii of all conductors are the same and are equal to 0.001 m. The magnitude τ is 0.9. It is obvious that the maximal radiation of the antenna should be directed to the right, toward the radiator *2*. Since the radiator *1* is located from the left of the active radiator *0*, at a distance b_1 from it, its field lags behind the active radiator field, firstly by kb_1 in phase, i.e., on the propagation time of the signal from the active radiator to the passive radiator *1*, and, secondly, by $kb_1/\sin\theta$ in phase, i.e., on the time of the signal propagation in the opposite direction, from the radiator *1* to the active radiator (signal of the wire *1*, radiated at an angle θ, must come to the active radiator at the same angle θ, i.e., it travels the distance $b_1/\sin\theta$, and not the distance b_1. The total change in phase is equal to $\psi_1 = -kb_1 \cdot \frac{1+\sin\theta}{\sin\theta}$. Similarly, in the case of radiator 2, this phase change is equal to $\psi_2 = -kb_2 \cdot \frac{\sin\theta - 1}{\sin\theta}$.

The total field of the antenna structure shown in Figure 5 with in-phase current distribution at angle θ on the base of aforesaid may be written in the form

$$E = \frac{AJ(0)\,sin\,\theta}{cos^2\theta}\sum_{i=1}^{2} e_i\left\{\left(\frac{1}{4Z_{id}} + \frac{1}{2Z_{il}}\right)[1 - cos(kL_0\,cos\langle\theta - \theta_0\rangle)] + \right.$$

$$\left. + [1/(4Z_{id}) - 1/(2Z_{ii})]exp(j\psi_i)[1 - \cos(kL_l\cos\theta)] \right\}.$$

The directivity magnitude is determined by the expression

$$D = |E(\pi/2)|^2 / \sum_{n=1}^{N}[|E(\theta_n)|^2 \Delta\, sin\,\theta_n],$$

where Δ is the interval between neighboring values θ_n, N is the number of these intervals between $\theta = 0$ and $\theta = \pi/2$.

The results of calculating directivity of a structure, shown in Figure 5, are given in Figure 6 depending on an electrical length kL_0 of the active radiator arm. The curve 1 demonstrates the

directivity of the structure with in-phase currents in each element. The directivity of the structure with sinusoidal currents is given for comparison by the curve 2. Radiators with

concentrated capacitances distributed in accordance with linear or exponential law along each arm allow to ensure a high level of matching with a cable in the range with a frequency ratio of the order 10. Therefore, graphics are made so that to show the structure directivity in the range from $kL_0 = \pi$ to $kL_0 = 10\pi$.

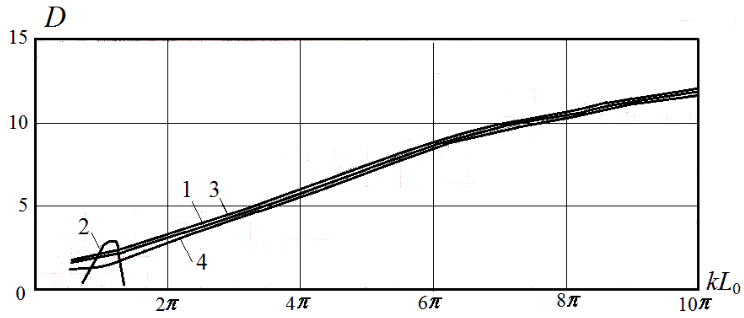

Figure 6. Directivity of Antennas of Three Radiators

Radiation of neighboring elements of log-periodic antenna with the arm length 0.27 m and 0.333 m respectively requires calculating fields in the structures presented in Figures 7a and 7b. Results of these calculations are given in Figure 6: curve 3 corresponds to Figure 7a, curve 4 - to Figure 7b. The directivity magnitudes in Figure 6 for specific values kL correspond to the same frequencies, i.e., the same values kL correspond to the elements of equal length in the all three circuits (for example, to the elements with the arm length 0.3 m). The figure shows that the directivity magnitudes at the same kL are close to each other. This is natural, since the directivity in each scheme increases slowly with increasing frequency. Small increasing the active radiator length causes at the same kL the small increase of the resonant wavelength, i.e., the small decrease of the resonance frequency and the directivity.

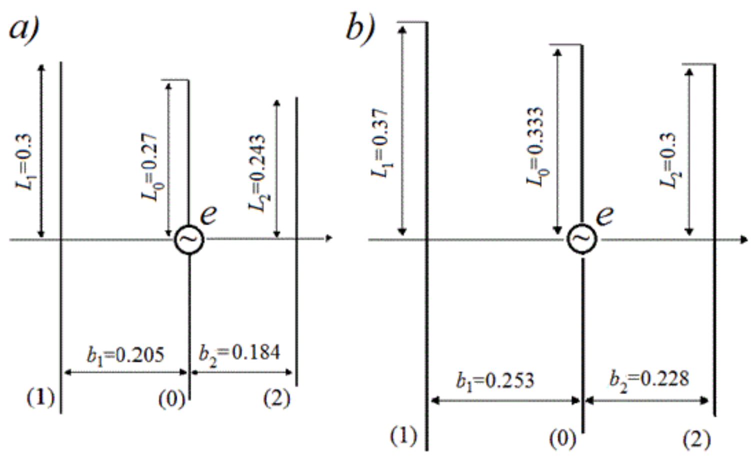

Figure 7. Structures of Smaller (*a*) and Greater (*b*) Neighboring Radiators

The obtained results show that each active radiator with in-phase current, included in the structure of the log-periodic antenna, provides high radiation directivity in a wide frequency range. Neighboring radiators have similar directivity magnitude. The direction of the radiation is the same. A signal propagates along the distribution line from short elements to the long dipoles (in Figures 5 and 7 to the left), the radiated signal propagates in the opposite direction. The total path difference is quite large.

For example, for the signal radiated by a half-wave neighboring radiator it is equal to 0.38λ, i.e., it is close to a half wavelength. Crossing wires of the distribution line in an interval between the elements permits to reduce dramatically this path difference.

Summarize the analysis of the log-periodic antenna. Known log-periodic antenna with sinusoidal current distribution along the radiators have a property of automatic currents "cut-off", i.e., a separate antenna segment (active region) radiates a signal in a narrow frequency band. Outside this band and outside the borders of the active region the signal decays rapidly. Wide frequency range is provided by a large antenna length, which is equal to the sum of the lengths of the active regions. Attempts to reduce the antenna length by disturbing a geometric progression and increasing a radiators number lead to a small decrease of dimensions and a sharp deterioration of electrical characteristics. The more effective methods are firstly a two-fold use of each active region by an application of linear-spiral radiators and secondly an employment of an asymmetrical log-periodic antenna with coaxial distribution line (Yakovlev & Pyatnenkov, 2007). This allows to decrease the antenna length by 25-30%.

Replacement of the straight metal radiators by the radiators with concentrated capacitive loads provides reusable active area and allows us to obtain a high directivity in a wide frequency range, using a simple structure with three radiators. Results obtained with the help of the method of electrostatic analogy may be used for solving optimization problem by methods of mathematical programming. Increasing the radiators number in the structure may allow dramatically increasing its directivity.

5. Conclusions

In this work the following results are obtained.

A simple and effective procedure, based on the method of electrostatic analogy, in accord with which it is assumed that the ratio of emf in the radiator centers is equal to the ratio of the charges placed on the conductors, is used for calculating directional characteristics of log-periodic antennas.

With the help of this procedure the directional characteristics of log-periodic antennas with sinusoidal and in-phase current distribution in linear radiators are compared. It is shown that replacement of the straight metal radiators by the radiators with concentrated capacitive loads allows to provide, when only three radiators are used, the high directivity in a wide frequency range, i.e., decrease sharply the antenna length. Increasing the number of radiators allows to increase substantially the antenna directivity.

Results obtained with the help of the method of electrostatic analogy may be used for solving optimization problem by methods of mathematical programming.

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