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Original Paper

About Microstrip Antennas

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Abstract

The electrical characteristics of a microstrip antenna are considered. Its design and electrical characteristics are compared with design and electrical characteristics of volume self-complementary antennas and an antenna located on the faces of a lying rectangular pyramid. The method of a complex potential is used for an analysis of the substrate influence on the properties of this antenna. In accordance with the results of comparing the characteristics of linear radiators with different current distributions, the design of a microstrip antenna with an in-phase distribution was proposed, which ensures operation in a wide frequency band.

Keywords

Antenna theory, current distribution, extension of frequency band, influence of substrate

1. Introduction

An explosive wave of theoretical work on antenna technology in the middle of the last century led to the creation of rigorous methods for analyzing and then synthesizing antennas. The widespread use of new antenna options, one of which was microstrip antenna, was a side effect of this wave. The reason for a widespread use of microstrip antennas was the fact that on aircraft, ships and other moving objects, where size, weight, price and ease of installation are especially important, low-profile antennas of the appropriate shape and design are needed. Such antennas can be mounted flush on metal surfaces, and it was need only feed lines, which are usually placed under the ground plane. The designs and electrical characteristics of microstrip antennas, as well as examples of their application, are described in books (Balanis, 2005; Johnson, 1993) and in articles included in the collection (Microstrip Antennas, 1995).

The microstrip antenna (Figure 1) presents a thin metal strip 1 (with thickness t) located at a small height h above the ground plate 2, at that $t, h \ll \lambda,$ where λ is the wavelength. The strip and ground plate are separated by dielectric plate 3 (substrate) as shown in the figure. The radiating elements and the feed lines are usually photoetched on the dielectric substrate. The metallic strip may be any configuration, but square, rectangular, and circular are the most common because simpler fabrication and analysis. The feed line is made or as a conducting strip 4, as a rule, of smaller width (Figure 1a) or as coaxial line 5 (Figure 1b), where the inner conductor of the coax is attached to the metal strip 1. With help of microstrip antennas one can create linear and circular polarization. Arrays of microstrip antennas allow to provide the high directivity.

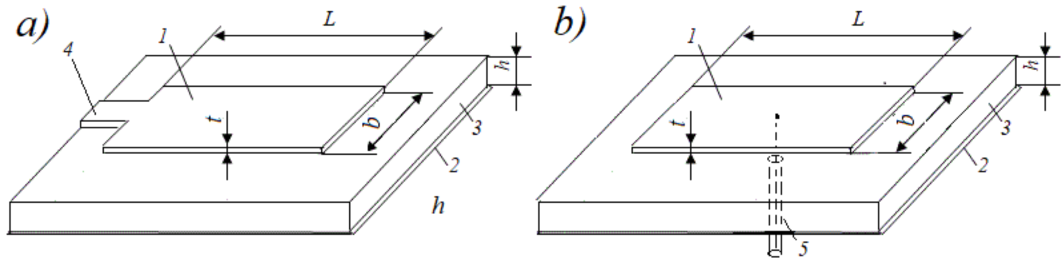

Figure 1. Microstrip Antennas with Feed Lines Made as a Conducting Strip (a) and as Coaxial Line (b)

Major operational disadvantages of this antenna are its small efficiency and very narrow frequency bandwidth, which is typically only a fraction of a percent or a few percent. The reason of this inefficiency is that its thickness is usually very small, and the waves generated within the dielectric substrate undergo considerable reflections when they go along the edge of the strip.

In order to analyze the electrical characteristics of microstrip antenna (patch), several models have been proposed. The greatest attention was paid to model in the form of a two-wire long line. This model treats the antenna as a segment of a transmission line with length L terminated at both ends by a radiation admittance Y_T. It is considered that the transmission line doesn't radiate because of reflections and acts as a transformer. Each radiation admittance is perpendicular to the feed line, and it is made in the form of a slot with length b and width h. If the length of the transmission line is approximately $\lambda/2$, where λ is the wavelength in the line, then the fields at the aperture of two slots have the same directions, and the slots form a two-element array with a distance $\lambda/2$ between the elements. The field components parallel to the antenna axis add up in phase and create a maximum radiation along the perpendicular to the ground plane.

As is convincingly stated in (Munson, 1974) the transmission-line model "has the advantage of yielding very simple expressions for both the real and imaginary part of Y_T, but it has three important shortcomings:

a) The expressions used for Y_T are inaccurate for narrow patches (i.e., for $b \leq \lambda_0$, where λ_0 is free-space wavelength).

b) The mutual coupling between the main radiating slots is neglected.

c) The influence of the side slots on the radiation conductance is neglected.

Here the slots with length b are called the main radiating slots and the slots with length L are called the side slots. Let's give the author the right to such terminology, but it is necessary to change the order of the listed deficiencies, since the main deficiency is the weak interest to the side slots.

It may seem surprising, but the closest relatives of microstrip antennas should be considered volume antennas located on a cone, a paraboloid and a pyramid shown in Figure 2. All of the listed antennas consist of two symmetrical radiators: metal and slotted, located under a right angle to each other. They are excited at the point located on the top of a cone, a paraboloid and a pyramid.

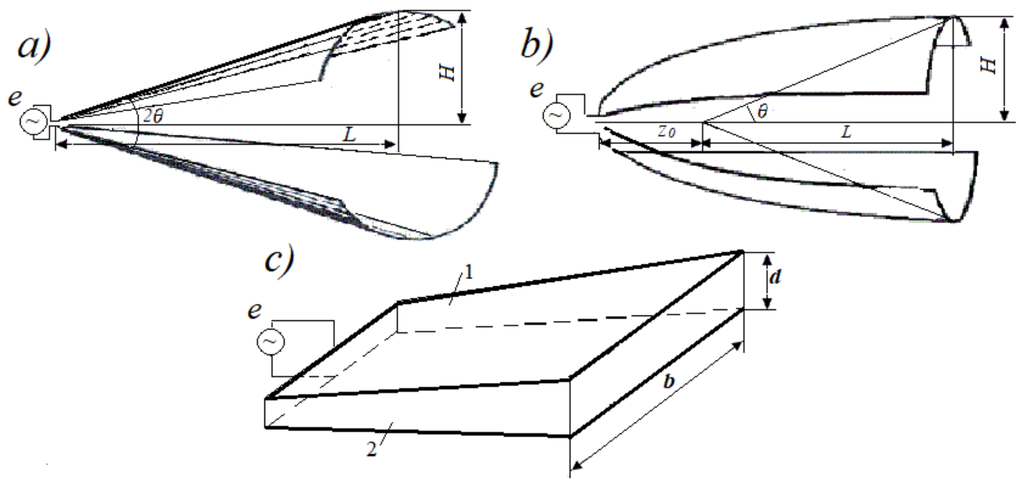

Figure 2. Antenna on a Cone (a), a Paraboloid (b) and a Pyramid (c)

Antennas located on a cone and a paraboloid are usually called complementary or self-complementary, but it would be more correct to call them self-complementary antennas of limited dimensions, in contrast to flat self- complementary antennas that occupy the entire infinite plane. The surface of an antenna on the pyramid is not a smooth surface of rotation with axial symmetry and strictly speaking such an antenna cannot be called complementary. But its characteristics are similar to those of complementary antennas. The upper and lower faces of the pyramid are made metallic and they form an electric dipole; the two side faces are slots and they form a magnetic dipole. The pyramid cross-section has a rectangular shape. Such structure is easier to realize, than for example conical, and on medium waves the lower arm of the antenna is created as a reflection of the upper arm in the ground.

As can be seen from Figure 2c, the pyramid structure is closest to the structure of the microstrip antenna. In the case of the microstrip antenna the upper and lower trapezoid faces of the pyramid are replaced by rectangular strips, and the metal strips of the antenna and the feed line are joined with each other. The slot radiator is also rectangular, although its width is substantially less than the width of the metal radiator. Basically, microstrip antenna is no different from the structure of the antenna located on the pyramid. The differences are minor.

From said, it follows that the all elements of the so-called transmission-line located along the length *L*, do not simply transfer energy from an excitation point to an aperture at an opposite end, but radiate the signal, i.e., all elements that make up a transmitting strip antenna create the field - just like all elements of a two-wire line radiate weak signals, if the distance between the wires is not zero. Ignoring this radiation is the main disadvantage of the so-called transmission-line model. Wave attenuation in the antenna substrate cannot serve as its justification.

2. Structure of Microstrip Antenna

Let us compare the microstrip antenna with the antenna on the pyramid, not taking into account the permittivity of the substrate. A cross-section of the microstrip antenna is given in Figure 3a. The dotted line in the figure shows a mirror image in the metal plane of grounding. The capacitance of the symmetric structure between a microstrip antenna and its mirror image is two times less than the capacitance of the asymmetric structure. Considering the microstrip antenna as a combination of two longitudinal radiators: metal and slot, we determine an input impedance of each one. The capacitance of a metal radiator per unit length can be calculated as the capacitance between two identical plates with a common plane of symmetry.

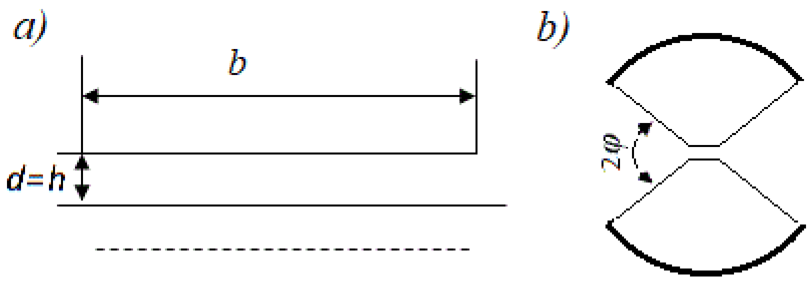

Figure 3. Cross-section of a Microstrip (a) and a Cylindrical (b) Antenna

A strict expression for its calculation contains complete and incomplete elliptic integrals of the first and second kind and requires the solution of an equation consisting of such functions. For calculations approximate functions can be used. Here and below, we extensive will use the formulas given in (Iossel, Kochanov, & Strunsky, 1981). If the width of the metal plate is equal to *b*, and the distance between the plates is d, then at $b/d > 27$ the capacitance between the plates per unit length is $C_l \approx \varepsilon\, b/d$. When

$$C_l \approx \varepsilon \frac{b}{d}\left\{1 + \frac{d}{\pi b}[1 + \ln(2\pi b/d)]\right\}.$$

If, for example, d=h=0.7874 mm, b=33.15 mm, then for the capacitance between the metal plates (according to the approximate first formula and the more accurate second one) we obtain

$$C_{l1} = 42.1\varepsilon, C_{l2} = 44.2\varepsilon.$$

The wave impedance of a long line formed by two metal strips of width *b*, located in a free space at a distance *d* from each other, is $W = k/(\omega C)$, where $k = 2\pi/\lambda$ is propagation constant of the wave in air, $\omega = 2\pi f$ is circular frequency, λ is wave length, f is frequency, $\lambda f = c = 3 \cdot 10^8$ is speed of light. Since $1/c = 120\pi\,\varepsilon_0$, where $\varepsilon_0 = 10^{-9}/(36\pi)$ is the absolute permittivity of the free space, we get that $W = 120\pi\varepsilon_0/C_{l2} \cong 2.7\pi$. This magnitude is equal to the wave impedance W_M of the metal radiator included in the structure of the complementary antenna.

To calculate the wave impedance W_S of a metal radiator identical to a slot in shape and size, it is need to determine the capacitance between the metal strips having dimensions of slots and located on their sites. Capacitance between plates with width *d*, located at a distance *b* from each other, if $d/b \leq 1$, is equal to $C_l \approx \pi\varepsilon_0/\ln(4\,b/d)$, i.e., $C_l = 0.613\varepsilon_{0,}$ and $W_S = 195.8\pi$. The product of asymmetric wave impedances is $W_E W_s \cong (23\pi)^2$. For a symmetrical antenna this product is 4 times larger, i.e., it is equal to $(46\pi)^2$. As is known, the product of the wave impedances of self-complementary antennas is $(60\pi)^2$. In the given case, this magnitude is significantly smaller, since the surface on which they are located, cannot be called smooth.

The symmetrical structure consisting of a microstrip antenna and its mirror image is similar to the structure of a complementary antenna consisting of two coaxial unclosed shells (Figure 3b) separated by angles 2φ. The capacitance between them per unit length of shells is equal to (see Iossel, Kochanov, & Strunsky, 1981) $C_l = \varepsilon\,K\left(\sqrt{1-k^2}\right)/K(k)$, where $K(k)$ is the complete elliptic integral of the first kind from the argument $k = tan^2\,\varphi/2$. Let's compare this structure with the structure of the microstrip antenna. For the given dimensions of this antenna, assuming that the structure radius is equal to $R = b/2$, and the angle φ is equal to $\varphi = h/R$, we find: $\varphi/2 = h/b = 0.024$. In accordance with expression for C_l we get: $k = 0.00056$, $\sqrt{1-k^2}$=0.9999998, $K(k) = 1.5708$, $K\left(\sqrt{1-k^2}\right) = 8.87$, that is $C_{l1} = 5.7\varepsilon$. If replace the angle width 2φ of the slot with the angular width of the shell (it is equal to $2\propto = \pi - 2\varphi$), we get: $\propto/2 = 0.7616$, $k = 0.909$, $K(k) = 2.3217$, $\sqrt{1-k^2} = 0.416, K\left(\sqrt{1-k^2}\right) = 1.6464$, that is the capacitance between the slots is $C_{l2} = 0.71\varepsilon$. Wave impedances are respectively $W_E = 21.05\pi$, and $W_s = 169\pi$. The product of the wave impedances is $W_E W_s = (60\pi)^2$ - in full agreement with the properties of the self-complementary structures. The difference between the results for the structures shown in Figure 3a and Figure 3b is, of course, caused with the difference in the distances between the arms of the metal radiators.

As mentioned above, each of the radiators presented in Figure 1 can be simultaneously viewed as a long line and as an antenna. This approach is no different from the usual comparison of a linear antenna with an equivalent long line. That greatly simplifies calculating the wave impedance and other characteristics of an antenna. The structure of the microstrip antenna and the dimensions of its cross-section are constant along its length L, i.e., the equivalent long line is uniform, and its wave impedance is constant.

However, it is necessary to note a significant difference between a microstrip antenna and a long line (and antennas associated with it). Known antennas are often divided into symmetrical (e.g., the dipole) and asymmetric (e.g., the monopole). In this case, a structure is called asymmetric, if it consists of a real antenna and its mirror image in the ground or grounding. Asymmetry essentially means a change in the sign or direction of the current. The asymmetry of the metal plate of a microstrip antenna is caused by the difference in the media adjacent to it from above and below and the large width of the plate compared to the thickness of the substrate. This results in a different magnitude of currents along the top and bottom surfaces of the plate. In this case, the EMF on both surfaces at the point of connection of the generator is the same. The voltages are also the same on both sides of the plate along all its length.

Thus, the structure of a microstrip antenna is more complex than that of a common long line. To simplify this structure, let's replace the metal plate with width b by a metal cylinder 1 with a radius $a = b/\pi$ (with the same surface area). The cross-section of such a structure is shown in Figure 4.

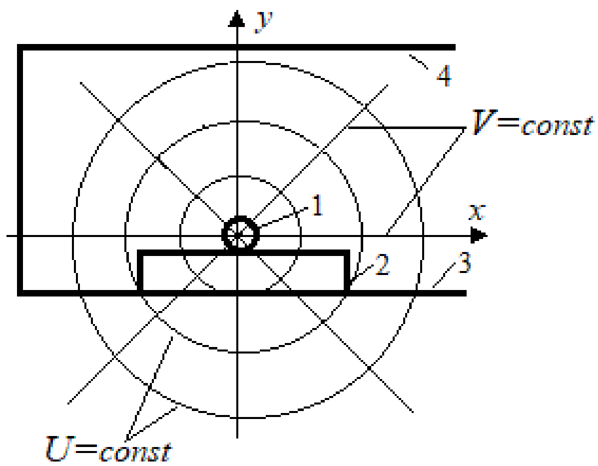

Figure 4. Approximate Equivalent Structure of the Microstrip Antenna: 1 - radiator, 2 - substrate, 3 – grounding, 4 – surface of zero potential

As it can be seen from the figure, in the vicinity of the radiator there are two surfaces with the same (zero) potential. This is, firstly, the metal surface 3, on which the antenna is located (grounding). This is, secondly, the surface of zero potential 4, which by definition is located at a distance from the radiator equal to the total length $2L$ of the antenna. The substrate is noted by the number 2.

The wave impedance of a long line located between the lower surface of the plate and the surface 3 of zero potential is equal to $W_1 = k_1/(\omega C_1)$, where $k_1 = kn$ is a wave propagation constant along the line (here $n = \sqrt{\varepsilon_1}$ is a wave slowing, ε_1 is a relative substrate permittivity). Capacitance C_1 in the first approximation, as previously shown, is equal to $C_1 = \varepsilon_1 \varepsilon_0 b/h$. As a result, we get

$$W_1 = 120 \frac{\pi h}{b\sqrt{\varepsilon_1}} \ . \tag{1}$$

This value is usually specified in microstrip antenna manuals.

If the structure used in this calculation is replaced by the new structure shown in Figure 4, then the result gives some error. But the new structure permits to take into account the influence of the zero potential surface, indicated by the number 4. This surface is usually not taken into account, and that leads to a significant error. From the solution of the integral equation for the current in a symmetrical linear radiator with a length $2L$ it follows that its wave impedance is approximately equal to $W_A = 120(\ln 2L/a - 1)$, where a is the wire radius. The capacitance of a single wire per unit length is twice as large as the capacitance of the long line, and the wave impedance of a single wire is half as much as the wave impedance of the long line, i.e., it is equal to

$$W_2 = 60(\ln 2L/a - 1). \tag{2}$$

As follows from the above, the replacement of the calculated structure by the structure shown in Figure 4 leads to an approximate choice of the value a in the expression for the antenna wave impedance. But this is a standard substitution used in the derivation and solution of integral equations for current, and the error caused by it is minimal.

Hence, the total wave impedance of the antenna shown in Figure 1a is

$$W = \frac{W_1 W_2}{W_1 + W_2}. \tag{3}$$

To determine the reactive component of the input impedance, it is necessary to clarify the value of the propagation constant.

As can be seen from Figure 1, microstrip antennas are made in two versions: asymmetric and symmetrical. In the first version, the antenna equivalent is a segment of a long line, excited at one end and open at the other end (analogous to a monopole). Wave impedance W was obtained for the first option. The symmetrical radiator (dipole) is the equivalent of the second option. It consists of two identical line segments, excited in the center of the structure and open at the ends remote from each other. If the dimensions of the arms coincide with the dimensions of the first option, then the wave impedance of the second option is twice as large.

3. Substrate

One of the main elements of a microstrip antenna is the substrate. A substrate changes all electric characteristics of antenna: capacitance per unit length, wave length, propagation constant, electric field and complicates antenna calculation. If the antenna is located in a homogeneous medium with $\varepsilon_r \neq 1$, the capacitance C_1 per unit length of the antenna increases ε_r times. At the same time in an antenna with a substrate, cut from a material with the same dielectric constant, the capacitance increases much less, only ε_{re} times, and this increase depends on the antenna design and substrate dimensions. In this case the Leontovich's equation for the current in a mode of own oscillations has the form

$$\frac{d^2 J}{dz^2} + k_1^2 J = 0,$$

where the wave propagation constant $k_1 = k\sqrt{\varepsilon_{re}}$ is increased by $\sqrt{\varepsilon_{re}}$ times. Assuming that the radius a of the antenna conductor is small compared to the antenna length $2L$, i.e., the surface of zero potential is located at a distance $2L$ from the antenna surface, we obtain for the self-capacitance of a conductor per unit its length $C_{10} = 2\pi\varepsilon_{re}/(\ln 2L/a - 1)$, and for the capacitance per unit length between the antenna arms $C_1 = C_{10}/2$. An increase of the propagation constant leads to a corresponding increase of the antenna wave impedance and the reactive component of its input impedance.

From the said it follows that in order to determine the electrical characteristics of a radiator located on a substrate, it is necessary to find out to what extent the substrate changes a capacitance per unit antenna length. This means that the analysis of the substrate effect is reduced to an electrostatic problem. The electrostatic problem considers a system of charged conducting bodies located in a dielectric medium, in which volume charges are absent. If the charge of each body and the charge distribution over the surface of the body are known, then in a homogeneous medium with a dielectric constant ε the potential *U*, determined by the entire set of charges located in the volume *V*, can be found using the expression. Here ρ_v is the charge density of the conducting body, *r* is the distance from the point of charge placement (integration point) to the observation point. The presented integral is a solution of the Poisson equation.

If the distribution of charges is unknown, then in accordance with the uniqueness theorem, one should to establish the necessary and sufficient conditions, which uniquely determine the field. With allowance for these conditions, the field on the surface of the conducting body must satisfy the Laplace's equation, and these surfaces must be surfaces of equal potential, i.e., on each of them $U = U_m = \text{const}$. The total charge q_m of each body *m* should be equal $q_m = -\int_{(S_m)} \varepsilon \frac{\partial U}{\partial n} dS$. Here S_m is the surface of a body *m*, *n* is the normal to its surface. The structure of an electrostatic field is determined by curves of equal potential, along which *U* = const, and by curves of field strength (by force lines) *V* = const. Each of these curves is a collection of points, at which this or that equality is true. The curves corresponding to these two equalities are two families of curves intersecting at right angles.

The problem of calculating the electric field of charged bodies is greatly simplified, if the field depends on only two coordinates. Such a field is called by a plane-parallel field. This, for example, is the field of a system consisting of several infinitely long cylindrical wires parallel to each other and to z axis of a rectangular coordinate system. In this case the wires charges are uniformly distributed along their length, the field structure is the same in all transverse planes *z* = const, the lines of field strength and equal potential in these planes depend only on two coordinates, *x* and *y*.

Let *dl* is an element of the line of field strength, and *ds* is an element of the line of equal potential. As already mentioned, they are mutually perpendicular. The potential *U* increases in the direction opposite to the vector $\vec{E}$, i.e., in the direction of decreasing *l*. It is generally accepted that the flux Ψ_E of the electric field vector grows in the direction of increasing *s*. In accordance with these conditions, the electric field strength may be related with magnitudes U and V by dependences $E = -\partial U/\partial l = \partial V/\partial s$. In a rectangular coordinate system, we obtain $E_x = -\partial U/\partial x = \partial V/\partial y$, $E_y = -\partial U/\partial y = -\partial V/\partial x$. Repeated differentiation shows that both functions (U and *V*) satisfy

Laplace's equation, which in the rectangular coordinate system has the form (e.g., for *U*) $\frac{\partial^2 U}{dx^2} + \frac{\partial^2 U}{dy^2} + \frac{\partial^2 U}{dz^2} = 0,$ and consequently, they satisfy the first requirement of the uniqueness theorem.

If to consider the plane of the cross-section of a plane-parallel field as the plane of a complex variable $z = x + jy$, then for the regular analytic function $\varsigma(z) = \xi(x, y) + j\eta(x, y)$ of the variable *z* we can write in accordance with the Cauchy-Riemann conditions $\partial\eta/\partial x = -\partial\xi/\partial y,\ \partial\eta/\partial y = -\partial\xi/\partial x.$ It is easy to be convinced that these equalities coincide with the expressions for E_x and E_y, i.e., we can adopt that $\xi = V$, $\eta = U$ and use the regular analytic function

$$\zeta(z) = V(x, y) + j\,U(x, y),$$

which is called by the complex potential of the field. In each specific case, the problem of calculating the field will be completely solved if the function ζ (z) is found that satisfies the boundary conditions on the surfaces of conducting bodies.

The method of complex potential and examples of its use for solving problems devoted to conducting bodies located in a homogeneous medium are described in the monograph (Mirolubov, Kostenko, Levinstein, & Tichodeev, 1963). In particular, there it is shown that for a solitary circular wire with a charge τ per unit length, located in a homogeneous medium with a dielectric constant ε, the complex potential has the form

$$\zeta(z) = -j\frac{\tau}{2\pi\varepsilon} lnz + C. \quad (4)$$

Here $C = C_1 + jC_2$ is a constant, and factors C_1 and C_2 depend on the choice of the initial line of field strength and the line of zero potential. Introducing the notation $z = \rho e^{j\varphi}$, we find

$$V(z) = \frac{\tau\varphi}{2\pi\varepsilon} + C_1, U(z) = -\frac{\tau}{2\pi\varepsilon} ln\rho + C_2, \quad (5)$$

whence for the line of field strength and the line of equal potential we obtain, respectively

$$\varphi = const,\ \rho = const. \quad (6)$$

The constant coefficients included in expression (5) are determined based on the condition that when going around a closed contour around the wire section, the angle φ increases by 2π, and the function $V(z)$ receives an increment equal to Ψ_E/l, where Ψ_E is the flux of the electric field vector $\vec{E}$ through a cylindrical surface covering the wire segment with a length *l*.

The method of complex potential allows us to determine all the electrical characteristics of a metal radiator located in the homogeneous medium, in particular, the self-capacitance of a conductor per unit of its length and the capacitance between the antenna arms.

If the medium is inhomogeneous, for example, piecewise homogeneous, the problem is significantly complicated. Such a problem, in particular, arises if the wire is located at the boundary of two media, air and dielectric. In this case it is advisable to go from the flux Ψ_E of the vector of electric field $\vec{E}$ to the flux Ψ_D of the electric displacement vector $\vec{D}$, taking into account the different density of this flux in different media.

The corresponding generalization of the complex potential method was accomplished in the article (Levin, 1997). In this case the increment of the function *V*(z) in going along closed contour is $\Delta V = \frac{1}{l}\sum_{(m)}\Psi_{Dm}/\varepsilon_m$ (*m* is a number of a medium). Integrating with respect to the volume *v* of the conductor the left and right parts of the Maxwell equation $div\vec{D} = \rho_v$ (ρ_v is the charge density on the conductor):

$$\int_v div\vec{D}dv = \sum_{(m)}\oint_{S_m}\vec{D}\,d\vec{S}\ ,\ \int_v \rho_v\,dv = \sum_m\int_{S_m}\sigma_m dS$$

(here S is the conductor surface, S_m is the part of this surface located beside to the medium *m*, σ_m is the charge density of this surface part) and equating these results, we find that $\sum_{(m)}\Psi_{Dm} = \sum_{(m)}q_m$, i.e., the flux Ψ_{Dm} of the displacement vector through the conductor surface to medium *m* is equal to the charge q_m of the surface part located beside to this medium. Respectively

$$\Delta V = \frac{1}{l}\sum_{(m)}q_m/\varepsilon_m\,. \tag{7}$$

Let's introduce the magnitude $\varepsilon_{e,}$ satisfying the equality $\sum_{(m)}q_m/\varepsilon_m = q/\varepsilon_{e\,,}$ where q is the total charge of the conductor. It can be seen that this magnitude has the meaning of the equivalent permittivity of an inhomogeneous medium. If $\Delta\varphi_m$ is the circumference arc beside to medium *m*, and $\sum_{(m)}q_m = q,$ then it is easy to show that

$$\varepsilon_{\mathrm{e}} = \frac{1}{2\pi}\sum_{(m)}\varepsilon_m\Delta\varphi_m. \tag{8}$$

If *N* media with the same angular width are located beside to the wire, then $\varepsilon_{\mathrm{e}} = \frac{1}{N}\sum_{m=1}^{N}\varepsilon_m$.

It is important to mark that in plane-parallel field the potential on the surface of each antenna wire (for example, on each dipole arm) is constant. The potential at an infinitely distant point is not equal to zero, otherwise at finite distances from the radiator it will turn out to be infinitely large. At the same time, it is obvious that when moving in the plane $z = c\mathrm{onst}$ away from the antenna axis, the potential is the same in all directions. Therefore, it should be assumed that lines of equal potential are circumferences with centers at the origin, and surfaces of equal potential are circular cylinder.

As it follows from expression (7), in the case of a homogeneous medium, the magnitude of the electric field flux between two wires depends on the angle φ between the lines of equal potential passing through the axes of the wires. As shown in Figure 5a, in the case of a symmetrical two-wire long line located in free space, the lines 1 of equal potential are arcs of a circumference, and the angle between them near each wire 2 is close to π. This means that half of the field flux passes through the volume bounded by the dotted line. Half of the energy between the wires of the long line passes through the same volume. In the case of a single-wire long line located above a perfectly conducting ground 3, contrary to popular opinion, only half of the energy passes through the space bounded by a semicircle. The second half passes through the surrounding space. These simple facts are given in (Levin, 2021). If the wire is located at the boundary of two media with the same form and volume (Figure 5c), then half of the energy passes through each medium, $\Delta\varphi_1 = \Delta\varphi_2 = \pi,$ and as can be seen from (8), $\varepsilon_{\mathrm{e}} = 0.5(\varepsilon_0 + \varepsilon_1)$.

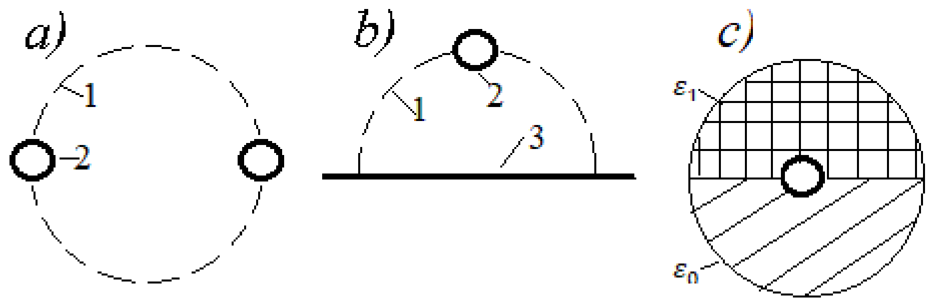

Figure 5. Cross-sections of Two-wire (a) and Single-wire (b) Long Lines and a Wire between Two Media (c)

In the case of a microstrip antenna, an approximate equivalent of its simpler version (monopole) is shown in Figure 4. In this case, the field structure is plane-parallel, since it is formed by two parallel horizontal surfaces of zero potential. As can be seen from the figure, the flux of the electric displacement is divided into two parts by a horizontal line coinciding with the axis of the antenna metal strip. This means that in this case, as in the case of Figure 5c, $\Delta\varphi_1 = \Delta\varphi_2 = \pi$, i.e., the fluxes through the air and the substrate are the same and the equivalent permittivity ε_e of the medium surrounding the radiator is equal to half sum of the permittivity of air and a substrate. Respectively the propagation constant of the current wave along the upper and lower plate surfaces is the same and equal to $k_e = k\sqrt{\varepsilon_e}$, i.e., reactive and active components of the input impedance of the microstrip antenna shown in Figure 1a, are equal to

$$X_{A1} = -W \cot k_e L, \; R_A = 80\left(\frac{k}{k_e}\tan\frac{k_e L}{2}\right)^2. \tag{9}$$

In order to experimentally verify the validity of the proposed method of calculating the equivalent permittivity of a medium next to a microstrip antenna, we will use the results of the development of a microstrip antenna with circular polarization (Barbakadze, Tabatadze, Petoev, & Zaridze, 2018). As known, a circular polarization compared to linear polarization has a number of advantages in the transmission and reception of radio signals. In this paper it was considered a square microstrip antenna excited by a coaxial cable (Figure 6). It has become significant step forward in expanding the employment of microstrip antennas.

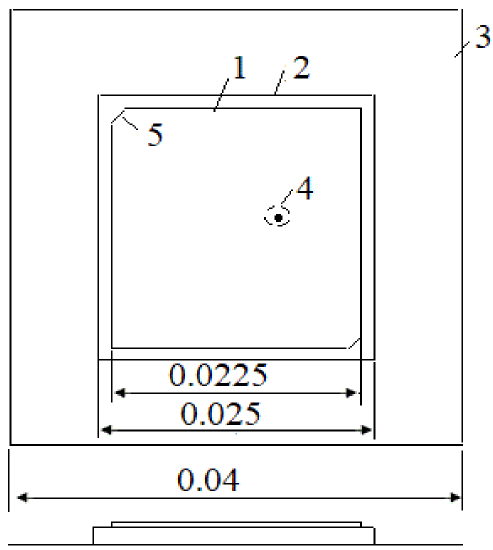

Figure 6. General View of the Antenna: 1 - Metal Plate, 2 - Substrate, 3 - Grounding, 4 –Cable, 5 - Cut Corners

The main option of the antenna was designed to operate at the frequency 2.49 GHz. The dimensions of the antenna model (in meters) are given in the figure. The thickness of the metal plate (strip) is approximately 0.01 mm. Two corners of the conductive strip were cut off in order to obtain circular polarization. The circular polarization is created by the rotating current and depends on the depth of the cut off. This depth is selected experimentally and is equal to 0.0028 m., The thickness of the substrate is 3.48 mm, its relative permittivity is $\varepsilon_r = 6.8$. To optimize the model performance at the specified frequency, the cable connection point is shifted from the center and located at a distance of 0.0242 and 0.2095 m from the left and bottom edges of the grounding. The cable radius is 1.6 mm.

The paper confirms the possibility of creating a circularly polarized antenna with sufficiently high electrical characteristics: the antenna SWR at the fundamental frequency is 1.5, the 6 dB bandwidth is equal to 0.13 GHz, and the axial ratio is 1.27 db.

Let us apply to this antenna the described method of calculating characteristics. The equivalent permittivity of the antenna substrate is equal to $\varepsilon_{\mathrm{e}} = (6.8 + 1)/2$=3.9. Since frequency is $f = 2{,}49 \cdot 10^9$, the wave length is $\lambda = c/f = 0.12$ m, and slowing is $n = \sqrt{\varepsilon_{\mathrm{e}}}$ =1.97, then the resonant length of the antenna is $L = \lambda/\left(2\sqrt{\varepsilon_e}\right) = 0.03$ m. Its geometric length, i.e., the diagonal of the square, taking into account the cuts, is somewhat less - 0.026 m. Therefore, it had to slightly increase this length by shifting the excitation point. This result serves as a clear confirmation of the validity of the described technique and shows that the substrate even of a small thickness significantly increases the equivalent permittivity of the environment.

4. Dependance of Antenna Characteristics on Current Distribution

In the article it was already said about the importance of expanding the frequency range and improving the characteristics of microstrip antennas. The solution of these problems primarily depends on the nature of the current distribution along a particular antenna. This Section is devoted to comparing the results of applying known distributions.

We will calculate the radiation resistance of the antennas by means of the method of the Poynting vector (Poynting, 1884). In accordance with this method the radiation resistance R_Σ is determined as the result of dividing the radiated power P by the square of the antenna input current $J_A(0)$: $R_\Sigma = P/J_A^2(0)$. At that the power is defined as the sum of fields fluxes through the surface of a sphere of infinitely large radius R_0, the center of which coincides with the antenna center. Such a radius allows us to assume that the field passing through the sphere is flat, i.e., the ratio of the electric and magnetic fields is equal to $E/H = Z_0$, where $Z_0 = 120\pi$ is the wave impedance of free space. This means that the power flux P through the unit area of the sphere S is equal to $P/S = E_\theta^2/Z_0$. Here E_θ is the projection of the electric field strength onto the sphere surface.

In order to determine the power flux through the surface of the sphere, it is necessary to calculate the integral of the value E_θ^2 over the angles θ and φ. Integration over φ is reduced to a multiplication by 2π, since E_θ does not depend on the angle φ. Integration over the angle θ is a laborious problem, which is solved, as a result of multiple integration by parts.

The used further method of calculation is based on two simplifications. Firstly, the expression for the electric field of a vertical radiator at an arbitrary angle θ is complex, while the analogous formula for the field in a direction perpendicular to the radiator is much simpler. Secondly, the antenna dimensions are small compared to the infinite radius of the spherical surface, i.e., the distance from any point of the antenna to any point of this surface is the same. Therefore, the field created by the antenna on this surface at an angle θ is equal to $E_\theta = E_\theta(\pi/2)\sin\theta$, and the power flux in the indicated direction is proportional to E_θ^2. That's why the integral of E_θ^2 over the entire spherical surface in the spherical coordinate system is

$$\int_0^{\pi/2} E_\theta^2 \sin\theta \, d\,\theta = \int_0^{\pi/2} E_\theta^2\,(\pi/2)\sin^3\theta d\theta.$$

This result is easy to verify and confirm by a well-known example. In accordance with the above expressions, the radiation resistance of the antenna is

$$R_\Sigma = 4\pi R_0^2 \int_0^{\pi/2} \frac{E_\theta^2}{Z_0 J_A^2(0)} \sin\theta d\theta = R_0^2 \int_0^{\pi/2} \frac{E_\theta^2(\pi/2)}{30 J_A^2(0)} \sin^3\theta d\theta. \quad (10)$$

The effective length of a dipole with an arm length $L = \lambda/4$ is equal to $l_e = 2/k$, i.e., in the horizontal direction $E_\theta(\pi/2)R_0 = 60$, and we obtain the well-known value of R_Σ:

$$R_\Sigma = 120 \int_0^{\pi/2} \sin^3\theta d\theta = 120 \cdot \frac{2}{3} = 80 \text{ 0hm}.$$

In accordance with this technique, we will consider different options of current distribution along the radiator. Let's start with the usual sinusoidal distribution. In this case the electric field of a symmetrical linear radiator is

$$E_\theta = j\frac{60J_A(0)}{\sin kL}\frac{\cos(kL\cos\theta)-coskL}{\sin\theta}\frac{exp(-jkR_0)}{\varepsilon_r R_0}. \quad (11)$$

In the horizontal direction, the field is equal to

$$E_\theta(\pi/2) = j60J_A(0)\frac{1-coskL}{\sin kL}\frac{exp(-jkR_0)}{\varepsilon_r R_0} = \frac{j30J_A(0)kl_e}{\varepsilon_r R_0}.$$

In this case the value l_e, which proceeding from the analogy with the value $2L$ in expression for the field of an elementary dipole is called the equivalent effective length, is $l_e = \frac{2}{k}\tan\frac{kL}{2}$. And in accordance with (10) and (11) we get

$$R_\Sigma = 20k^2l_e^2. \quad (12)$$

When $L > \lambda/4$, the value R_Σ grows with increasing L, but the expression (12) becomes very approximate. Along with the growth of R_Σ the reactive component of the input impedance increases, and the input current of the antenna decreases. At $L > \lambda/2$ R_Σ begins to decrease, since segments with anti-phase current appear along the antenna. At $L > \frac{\lambda}{4}$ the directivity *D* starts to grow. But an increase of *D* can lead to a decrease in a communication range, if the signal in a horizontal direction decreases. The pattern factor *PF* was introduced as an indicator of such a result. It is equal to the average level of radiation under angles θ from 60° to 90°. When $L < \lambda/4$, $PF_1 = \int_{\pi/3}^{\pi/2}\sin\theta d\theta = 0.5$. With growth of *L* this factor begins to decrease, sharply limiting the operation range of frequencies.

In a linear antenna with a constant surface impedance of an inductive nature, the current distribution remains sinusoidal, but the wave slows down, and the propagation constant k_1 increases. Calculations show that for the same geometric length, the radiation resistance of an antenna with a constant inductive impedance is somewhat greater than the radiation resistance of a metal antenna, and for the same electrical length, it is much less.

In-phase current distributions created by capacitive loads connected at regular intervals along the entire length of the antenna are of particular interest. In the case of a linear distribution, the current amplitude decreases uniformly from the generator to the free ends of the radiator. The electric field of a vertical antenna with allowance for the mutual compensation of components whose signs are opposite in different arms of a symmetrical radiator is calculated as

$$E_\theta = j30k\,J_A(0)\sin\theta\,exp(-jkR_0)/(\varepsilon_r R_0)\cdot\int_{-L}^{L} f_1(|z|)exp(jkz\cos\theta)dz.$$

Here $f_1(|z|) = 1 - |z|/L$ is a symmetric function of current distribution along the radiator. Respectively

$$E_\theta = 2A\int_0^L (1 - z/L)\cos(kz\cos\theta)dz,$$

where $A = j30k\,J_A(0)\sin\theta\,exp(-jkR_0)/(\varepsilon_r R_0)$. As a result of integration,

$$E_\theta = \frac{2A}{k^2Lcos^2\theta}[1 - \cos(kL\cos\theta)].$$

If θ→0, and $\frac{1-\cos(kL\cos\theta)}{cos^2\theta} \to 0.5k^2L^2$, we get $E_\theta(\pi/2) = j30kLJ_A(0)\,exp(-jkR_0)/(\varepsilon_r R_0)$, i.e., $R_\Sigma = 20k^2L^2$. In this case, in contrast to (12), the value R_Σ grows continuously with increasing L. At that the values D and PF do not change, i.e., with an increase in the radiator length the field and the radiation resistance grow within the frequency band, in which the in-phase distribution is preserved.

In the case of an exponential in-phase current distribution along the radiator, when the distribution function is equal to $f_2(|z|) = \frac{e^{-\propto z} - e^{-\propto L}}{1-e^{-\propto L}}$, the field is

$$E_\theta = \frac{2A}{1-e^{-\propto L}} \int_0^L [e^{-\propto z} - e^{-\propto L}] \cos(kz\cos\theta) dz.$$

As a result of integration, we find

$$E_\theta = \frac{2A \propto k\cos\theta}{(1-e^{-\propto L})(\propto^2 + k^2cos^2\theta)k\cos\theta} \cdot \{1 - e^{-\propto L}[\cos(kL\cos\theta) + \propto \sin(kL\cos\theta)/(k\cos\theta)]\}.$$

If $\propto$ is small, $E_\theta = [2A/(k^2L)cos^2\theta][1-\cos(kL\cos\theta)]$.

As θ→0, $E_\theta(\pi/2) \to j30kLJ_A(0)\frac{exp(-jkR_0)}{\varepsilon_r R_0}$, i.e., as expected, the value R_Σ coincides with (11). If $\propto$ is not small, then, taking into account that $\cos\left(kL\cos\frac{\pi}{2}\right) = 1$, we obtain:

$$E_\theta(\pi/2) = j60\frac{k}{\propto}J_A(0)\frac{exp(-jkR_0)}{\varepsilon_r R_0},$$

and therefore

$$R_\Sigma = 80\,k^2/\propto^2. \tag{13}$$

Figures 7 and 8 show the results of characteristics calculating for asymmetrical linear metal radiators (monopoles) with the same dimensions depending on the current distributions: sinusoidal (1), in-phase with a linear distribution (2), in-phase with an exponential distribution at $\propto = 1/L$ (3) and $\propto = 4/L$ (4). The radiators with an arm length 1 m and a radius 0.01 m are excited by the cable with a wave impedance 60 Ohm. The fields and the radiation resistances of radiators are given correspondingly on Figures 7a and 7b, the reactive components of input impedances for different current distributions are shown on Figure 8a. As is known, an operation frequency band depends on a level of travelling wave ratio (TWR). The magnitudes of TWR are presented in Figure 8b.

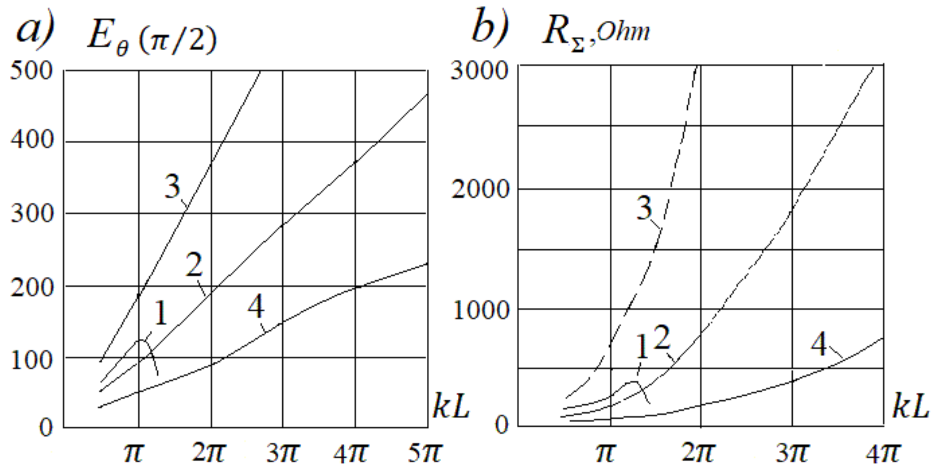

Figure 7. Fields (a) and Radiation Resistances (b) of Antennas with Different Current Distributions

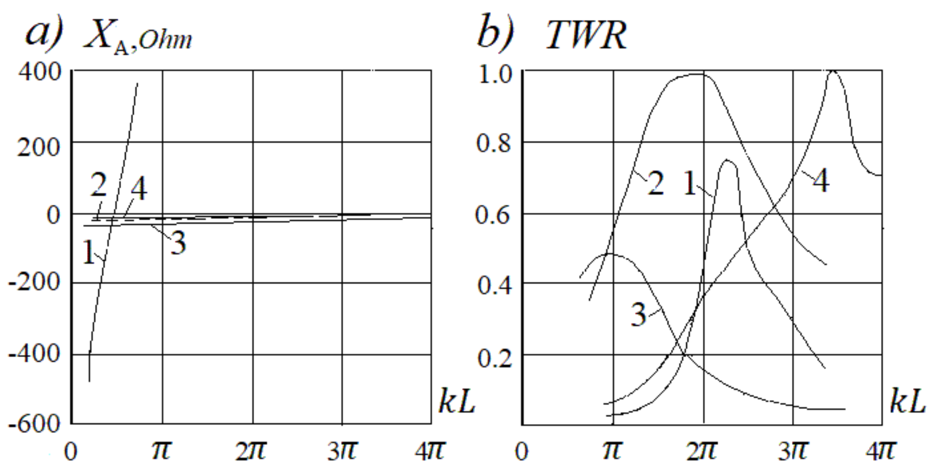

Figure 8. Input Reactance (a) and TWR (b) of Antennas with Different Current Distributions

The wave impedance of this radiator with sinusoidal current distribution is $W_1 = 120\,(\ln 2L/a - 1) = 516$ Ohm.

A reactive component of its input impedance is equal to X_{A1} =- $W_1 \cot kL$. The inclusion of capacitive loads leads to a sharp decrease in the inductance per unit length of the wire and the propagation constant k_1 along this wire. As a result, the wave impedance W_2 of an antenna with capacitive loads is decreased by $\sqrt{k_1/k}$ times. In the author's book (Levin, 2019) it is erroneously stated that this impedance coincides with W_1. Therefore, the reactive component of an input impedance of a radiator with a linear in-phase current distribution is X_{A2} =- $W_2/(kL)$.

In the case of an exponential in-phase distribution the reactive component of the input impedance is X_A =- $W_{3,4}/(kL)$, where the wave impedance $W_{3,4}$ is given depending on $\propto$ by the expression

$$W_{3,4} = 0.5 \propto LW_2[1 + \cot(\propto L/2)].$$

The formulas for calculating the radiation resistance of the compared radiators and their electric fields along the perpendicular to the antenna axis are given above.

The obtained results show that antennas with in-phase current distribution permit in the transmit mode to provide the high electrical characteristics in the wide frequency range, including the range, which used for operation with sinusoidal distribution. They confirm advantages of applicating the in-phase current distribution. Other options of loads allow to operate in different ranges using short waves, which length is much less than the length of an antenna arm, i.e., at frequencies where a sinusoidal distribution is not applicable.

At the same time, with increasing frequency, the signal level of the vertical radiator at angles close to the earth's surface, grows with the general growth of the field. This circumstance is very important for ensuring reliable radio communications over long distances. These properties are equally valid for both electric and magnetic radiators.

Antennas with switchable loads make it possible to operate in all need ranges.

The self-complementary antennas consisting of magnetic and electric radiators located side by side on a common surface and filling this entire surface deserve special consideration. In this case, the input impedance of each radiator is purely active, does not depend on frequency in the operating range and is equal to 60π. These characteristics, as is usually stated, are partially preserved at smaller sizes of the structure. Attentive analysis shows that this situation is valid at high frequencies, more precisely, when the wavelength of the radiated signal is half the dimension of the occupied surface.

Typical input characteristics of symmetrical flat complementary antennas with an arm length L depending on the wavelength λ, taken from the book (Levin, 2019) are shown in Figure 9. Curves 1 are given for an antenna consisting of one metal and one slot radiator with the identical angular width $\pi/2$, curves 2 are given for two antennas with the same angular width, which are connected to the generator poles in parallel to each other. It can be seen from the figure that as the frequency increases (wavelength decreases), the curves slowly converge to known limit values: the active resistance of each radiator converges to 60π, the reactive impedance - to zero. This limit is realized when $\lambda/L \approx 1$, i.e., at a wavelength equal to half the length of the radiator.

As follows from the foregoing, radiators with in-phase current distribution included in the self-complementary structure differs from single radiators with a similar current distribution by a purely active input impedance.

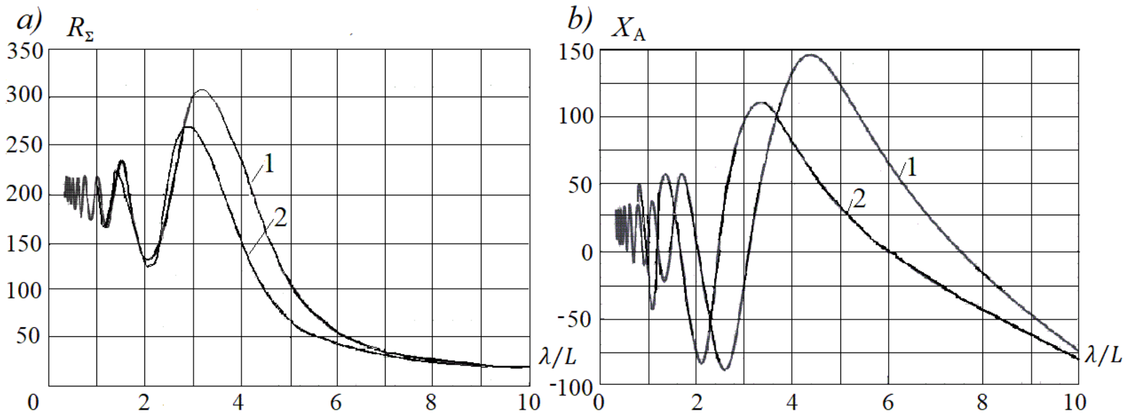

Figure 9. Active (a) and Reactive (b) Components of the Input Impedances of Flat Self-complementary Antennas

5. Expansion of Frequency Range

Obtained results allow us to analyze the performances of microstrip antennas and suggest methods for improving their characteristics. Their main disadvantage is low efficiency and narrow frequency range. The small thickness of the substrate leads to the losses in the substrate and reduces efficiency.

Despite the individual structural elements that distinguish microstrip antennas from other radiators, and features of electrical characteristics caused by these elements, a sober look at the operation of these antennas shows that in principle they little differ from ordinary radiators. And therefore, to improve their performance, one must use the usual methods. The current in a microstrip antenna is distributed along a sinusoid, as in a conventional linear radiator.

A linear antenna with a sinusoidal current distribution is equivalent to a resonant circuit operating in a narrow frequency range. To expand the frequency range of such a structure, it is necessary in particular to lower its Q factor, i.e., increase the resistance of radiation and losses. In the case of a microstrip antenna, this method can be implemented by using a two-story structure, in which the upper antenna (second floor) connected to the main (first floor), provides additional radiation and partially extends the operating range (see, for example, Croq & Pozar, 1991). But a significant expansion of the operating range is possible only by creating in a linear radiator the in-phase current distribution with the help of capacitive loads.

Capacitive loads, located along the axis of a linear electrical radiator, make it possible to provide over the entire its length a current close to the in-phase in a wide frequency range. Amplitude of in-phase current decrease toward antenna ends in a linear or exponential law. As a result, constant loads allow several times to expand a frequency range with a required matching level. Changing of loads by means of their switching greatly improve this result. In accordance with the duality principle the inductive loads connected along the slot provide similar result in a magnetic radiator.

This design can be applied in a microstrip antenna. Figure 10 shows three options of such an antenna with capacitive and inductive loads located along its axis. The capacitive loads are installed on a metal plate 1 and are made in the form of transverse slots 5 with metal selvages 6 (see Figure 10a) or with capacitors 7 (Figure 10b). The inductive loads are made in the form of metal tapes 8 and are placed on the substrates' sides 4. The third option (Figure 10c) makes it possible to switch capacitive loads and change operating ranges.

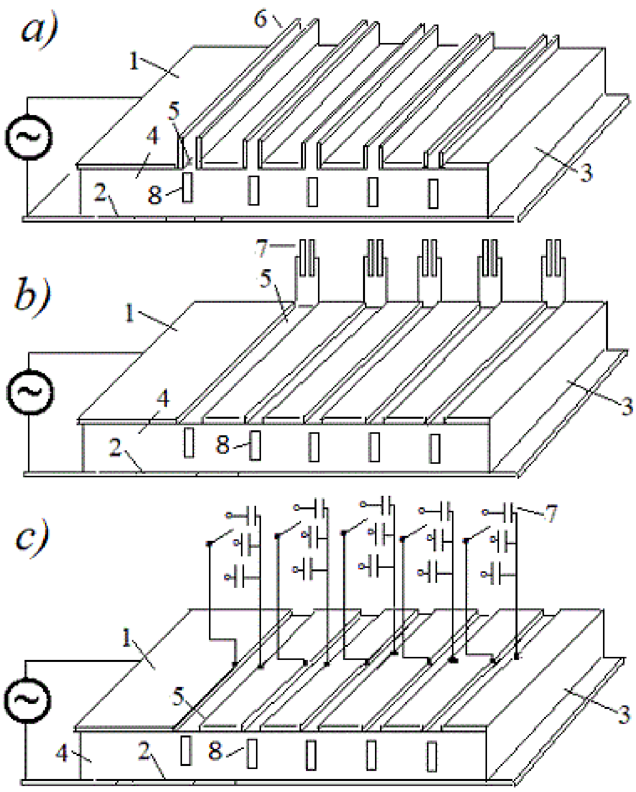

Figure 10. Designs of Microstrip Antenna: First Option (a), Second Option (b), Third Option (c) 1 – metal strip, 2 – ground plane, 3 – substrate, 4 – side of substrate, 5 – transverse slot, 6 – metal selvage, 7 – capacitor, 8 – transverse strip

The use of capacitive loads in an electric radiator of the microstrip antenna makes it possible to expand the operating range and significantly improve the characteristics of the antenna. Calculating the wave impedance of a metal radiator, identical to the slot in shape and size, performed in Section 2, shows that magnetic radiator of the microstrip antenna at the currently used sizes has little effect on the magnitude of the radiated signal. To improve the characteristics of the magnetic radiator, it is necessary to increase the thickness of the microstrip antenna.

6. Conclusion

Microstrip antennas are widely used in various objects. Sufficiently simple changes in their design can significantly improve their characteristics and expand the possibilities of application.

References

Balanis, C. A. (2005). *Antenna Theory: Analysis and Design*. Wiley & Sons: New York.

Barbakadze, V., Tabatadze, V., Petoev, I., & Zaridze, R. (2018). Directed circular polarized antenna, *Proc. 23th Seminar DIPED-2018* (Tbilisi, Georgia), 177-180. https://doi.org/10.1109/DIPED.2018.8543324

Croq, F., & Pozar, D. M. (1991). Millimeter-wave design of wade-band aperture coupled stacked microstrip antennas. *IEEE Trans. Antennas and Propa*gation, *39*(12), 1770-1776. https://doi.org/10.1109/8.121599

Iossel, Yu. Ya., Kochanov, E. S., & Strunsky, M. G. (1981). *Calculation of Electrical Capacitance*. Energoisdat: Leningrad. (In Russian).

Johnson, R. C. (1993). *Antenna Engineering Handbook*. McGrow-Hill: New York.

Levin, B. M. (1997). Calculation of electrostatic fields in inhomogeneous media. *Radiotechnics and Electronics*, *42*(8), 916-920.

Levin, B. M. (2019). *Wide-Range and Multi-Frequency Antennas*. CRC Press: London, New York.

Levin, B. M. (2021). *Antennas: Rigorous Methods of Analysis and Synthesis*. CRC Press: London, New York.

Microstrip Antennas, edited by Pozar, D. M., & Schaubert, D. H. (1995). IEEE Press: New York.

Mirolubov, N. N., Kostenko, M. V., Levinstein, M. L., & Tichodeev, N. N. (1963). *Methods of Electrostatic Field Calculation*. Visshaya Shkola: Leningrad. (In Russian).

Munson, R. E. (1974). Conformal microstrip antennas and microstrip phased arrays. *IEEE Trans. Antennas Propagation*, *22*(1), 74-78. https://doi.org/10.1109/TAP.1974.1140723

Poynting, J. H. (1884). On the transfer of energy in the electromagnetic field. *Phil. Trans. Roy. Soc. (London)*, *2*, 343. https://doi.org/10.1098/rstl.1884.0016