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Mathematics Teacher's Content Knowledge and Pedagogical Content Knowledge in Learner-centred Approaches in Secondary Schools

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Abstract

This paper describes how mathematic teacher's content knowledge informs teachers' pedagogical content knowledge in the use learner-centred approaches and decision when teaching mathematics. Two trained mathematics teachers in the use of learner-centred approaches who were teaching mathematics at ordinary secondary school level in Tanzania were involved. Data was collected through classroom observation, video recording of classroom events and teacher's self-reflections. Thematic analysis procedure both at conceptual and manifest level was employed. The findings indicated that, teacher's weak knowledge of contents as a component of pedagogical content knowledge in the use of learner-centred approaches led to teacher's inability to help students' construct knowledge of the subject matter that teachers taught. The trained teachers did not teach lessons that were learners' focused and were unable to help students discover the relationship between contents they taught with other contents in the syllabus. Teachers' lack of content knowledge led to teachers' communication of their misunderstanding to students through teacher-centred teaching approaches. In addition, these experienced teachers were unable to help students describe the rationale for learning contents. It is recommended that cementing on leaner-centred approaches during teachers' professional development programs should go hand in hand with capitalizing on teacher's knowledge of contents.

Keywords

teachers' PCK, learner-centred, teaching approaches, content knowledge

1. Introduction

Pedagogical Content Knowledge (PCK) was introduced by Shulman (1986) as a specialized professional knowledge for teachers that distinguish the teaching profession from other professions. Apart from being a specialized knowledge for teachers, researchers in different subjects has agreed on its contribution in the improvement of students' learning outcomes in a subject that students are learning (Lange, Kleickmann, & Moller, 2012). Teacher's PCK is described as a teacher's personal construct and implicit (Kind, 2009; Garritz, 2015) and is revealed in different forms ranging from topic and discipline specifics (Abell, 2007; Park & Oliver, 2008; Mavhunga & Rollnick, 2013) to subject specifics nature (Andrews, 2001; Kitta, 2004; Magnusson, Krajacik, & Borko, 1999). High level of teacher's PCK helps a teacher to deliver contents to learners in the most effective, appropriate and organized way that positively influence learners' achievement in the subject matter (Shulman, 1986; Shulman, 1987).

Teacher's PCK help teachers to refine their explanations, make decisions during teaching and learning process, formulate questions, ask questions and deal with students' learning aspects. Teacher's PCK as a personal construct grows stronger with external experiences such as professional training (Loughran, Milroy, Berry, Gunston, & Mulhall, 2001). A strong and well developed teacher's PCK is indicated by teacher's strong knowledge of subject contents, general pedagogy, curriculum, learners and their characteristics, educational contexts, education ends, purposes, values and learners' psychological historical background (Shulman, 1986). When teachers are implementing curriculum through teaching subject contents, teacher's PCK informs both their general performance of their day-to-day functions and when helping students to build their own knowledge on contents.

Mathematic teacher's PCK for teaching mathematics is reported to be divided into two categories; Subject Matter Knowledge (SMK) and PCK categories (Hill, Ball, & Schilling, 2008). The SMK is composed of three sub-elements; Common Content knowledge (CCK), Specialized Content Knowledge (SCK) and Horizontal Content Knowledge (HCK) in mathematics subject. The CCK is recognized when a person is using mathematics in everyday life, SCK is recognized through teacher's knowledge of the subject matter when teaching, while HCK helps teachers to recognize and helps students to construct relationship between concepts in the field of mathematics. In fact, during teaching and learning process, SMK helps a teacher to identify important ideas in the subject matter that students are supposed to acquire (Hauk, Jackson, Toney, Nair, & Tsay, 2014). In mathematics, the teacher's PCK category is composed of three sub-elements; teacher's Knowledge of Contents and Students (KCS) which helps teachers to understand how learner's learn contents, Knowledge of Content and Teaching (KCT) which helps teachers to represent contents and help learners' to learn and the third one is, Knowledge of Content and Curriculum (KCC) which help teacher's to organize content in the curriculum so that students can best learn. Therefore, a mathematics teacher with well-developed PCK in the use of Learner-Centred Approaches (LCA) is likely to have strong knowledge of both SMK and have developed PCK categories in the use of LCA. With the focus on both SMK and PCK, three research questions were answered by the researchers; How does mathematics teacher's content knowledge as component of PCK in the use of LCA inform teacher's decision on the use of LCA when teaching mathematics? How does mathematics teacher's content knowledge as a component of PCK contributes to teacher's growth of PCK in the use of LCA? In what ways do mathematic teacher's content knowledge as a component of PCK in the use of LCA contributes to students' knowledge of contents?

1.1 Conceptualizing and Assessing Content Knowledge as a Component of PCK in LCA

The conceptual framework for assessing teacher's PCK in the use of LCA in this qualitative study are five as were adapted from Magnusson et al. (1999). These components are; the teacher's orientation to teaching mathematics, teacher's knowledge of mathematics contents, teacher's knowledge of students' understanding of mathematics concepts, teacher's knowledge of instructional strategies and teacher's knowledge of assessing students' learning when employing LCA. Magnusson et al. (1999) had earlier proposed five components of PCK; teacher's orientation towards teaching science, teacher's knowledge and beliefs about science curriculum, teacher's knowledge and beliefs about students' understanding science topics, teacher's knowledge and beliefs about assessment in science, and teacher's knowledge and beliefs about instructional strategies for teaching science. Since teacher's PCK is recognized as an algamation of both pedagogical and content knowledge (Magnusson et al., 1999), these construct in mind helped the researchers to assess content knowledge as a component of teacher's PCK in the use of LCA as a teacher's personal construct.

The focus on teaching approaches based on Shulman (1987) argumentation that, teacher's PCK is a knowledge base which includes teacher's decision on which approach fits to the teaching of contents. Therefore, the study assessed ways in which a trained mathematics teacher make decision on when to use LCA for specific learners, group of learners and organize contents while putting learners at the centre of teaching and learning process. In addition, Hill, Ball, and Schilling (2008) knowledge base for teaching mathematics guided the assessment of content knowledge as a component of PCK in LCA using both the SMK and PCK categories (Figure 1). Measuring PCK as a knowledge base, a PCK rubric was developed to aid assessing teachers while teaching. The PCK rubric was adapted from the work of Hume and Berry (2010), Gardener and Gess-Newsome (2011) and that of Anney and Hume (2014). In terms of credibility, the PCK rubrics have been documented to well represent teacher's PCK (Loughran, Berry, & Mulhall, 2006). KCT and KCS was captured through teacher's evidence of knowledge of LCA and teacher's practices such as ability to ask higher order thinking questions, involving students and other actions that revealed teacher's intention to put learners at the focal point while teaching. Cochran and Jones (1998) identified four elements of Content Knowledge (CT); knowledge of majour facts and concept of a discipline, knowledge of basic principles, knowledge of how validity and invalidity are established within a discipline and knowledge of connecting topic to other entities in the discipline. SCK was captured through evidence of teacher's knowledge of mathematics concepts, knowledge of variety of correct explanations of a concept, knowledge of how errors are established and knowledge of connecting concepts with other mathematics concepts. For documentation purposes, each teacher's content knowledge data was finally presented in a continuum with weak content knowledge attributes on one end and strong content knowledge on the other end.

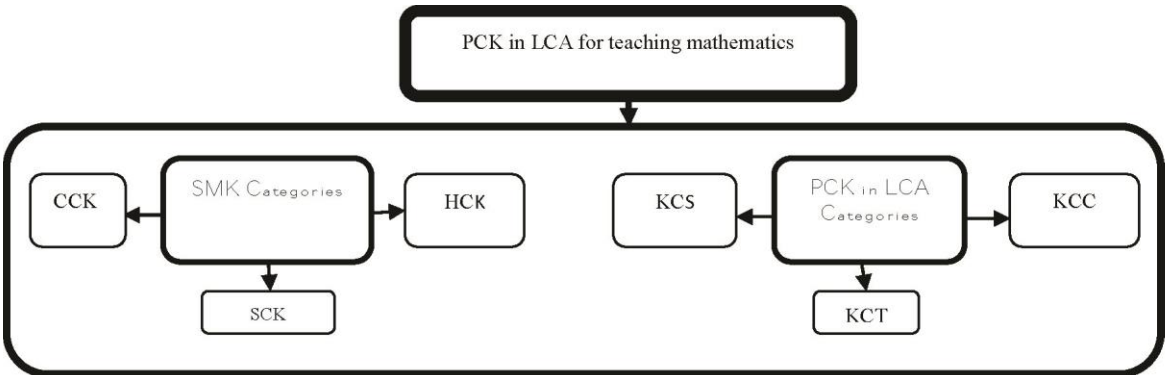

Figure 1. PCK in LCA for Teaching Mathematics as Adapted from Hill, Ball, and Schilling (2008)

2. Literature Review

Previous studies have shown that, science teachers with strong PCK are successful when teaching specific science topics (Mulhall, Berry, & Loughran, 2003). These studies have also confirmed that, for effective teaching of a topic, teacher's knowledge about common students' misconceptions in that topic is necessary. It was further concluded that, teacher's development of topic specific PCK is embedded in teacher's classroom practice on the same topic. This implies that, successful teachers who have taught a particular topic not only promote students' learning but also have well developed PCK in that specific content area (Gess-Newsome & Lederman, 1999; Hill, Ball, & Schilling, 2008; Kitta, 2004; Magnusson et al., 1999; Van Driel, Verloop, & De Vos, 1998). Being recognized through a teacher's classroom practice and use of approaches to teach contents is a common view that researchers agree upon the nature of teacher's PCK. In American teacher's education programmes, teacher's content knowledge and PCK have been identified to be important components of teacher's competence (NCTM, 2000; Mewborn, 2003). In addition, there is an agreement that, all American teachers' training programmes must seek to achieve a balance between CK and PCK.

Similarly, all teachers' training programmes in Tanzania are approved by education authorities after they verify that there is a balance between CK and PCK in the programme seeking for approval. Since 2005 when the competence based curriculum was innovated, teacher's competence in the use of LCA in teachers' preparation and other education programmes gained emphasis (MoEVT, 2005). The government through the Ministry of Education and Vocational Training (MoEVT)'s innovation focused on a paradigm shift from the then content-based curriculum to a Competency-Based

Curriculum (CBC) that could enhance learners' competency. Since then, the emphasis in the education provision is more on the improvement of students' learning, improvement in the teaching and learning process and quality of learning outcomes (TIE, 2005) rather than the teaching, content delivery and coverage (Richards & Rodgers, 2001). The CBC is in place to address more on what the learners can do rather than what they should cover during the learning process. In addition, secondary school teachers, and tutors in teachers training colleges are encouraged to the use of LCA (MoEVT, 2005).

In mathematics teaching and learning process, LCA are recommended for the purpose of raising students' learning and participation in the learning process (Philemon, 2010; TIE, 2010a). The use of LCA is expected to develop students' high level of competencies in different education levels (TIE, 2010b). Since its introduction in Tanzania, the government has been emphasizing on changing roles of teachers and tutors from that of teaching by imparting knowledge to facilitating students' learning (Vavrus, 2009). For about two to three years of CBC implementation (from 2005 to 2008), the education system in Tanzania began recruiting teachers who had already been trained in the CBC curriculum. Since then a number of studies have been conducted with the focus of generating knowledge on the use of LCA in the classrooms in Tanzania. Research reports are indicating that LCA is not fully implemented by teachers in classrooms (Kitta & Tilya, 2010; Mosha, 2012; Msonde, 2011; Philemon, 2010).

Challenges to the use of LCA have included class size, curriculum being overloaded with contents, and teachers with inadequate PCK (Anney, 2013; Kitta & Tilya, 2010). Teachers' orientation to teaching has remained to be a teacher-centred pedagogy. Top-down dissemination of emphasis on pedagogy has neglected teaching environment and therefore discouraged to bring expected changes in instructional methods (Mosha, 2012; Thornberg, 2010). Curriculum overloaded with contents has not encouraged the development of students' competence through the use of learner-centred approaches (Tilya & Mafumiko, 2010). Teachers are reported to be concerned mostly with coverage of all contents in the syllabus with a belief that they have all knowledge to transfer to learners (Mosha, 2012). Researchers have also reported that, the reasons for dominant teacher-centred orientation in teaching is attributed to the fact that during the paradigm shift in education, not all teachers were educated with emphasis in CBC. This implies that, teachers who are implementing the CBC at the secondary school level in Tanzania are those who had specifically followed CBC and who did not go through a CBC during their training.

Since PCK grows with experience there are teachers in Tanzania whose PCK is underdeveloped due to lack of experience. The untrained teachers in CBC as compared to experienced teachers are therefore unlikely to effectively enhance students' learning outcomes in the context of LCA. Available findings in Tanzania on the use of LCA show that, pre-service teachers have underdeveloped PCK (Anney, 2013). A study by Anney (2013) in Tanzania did not assess PCK as a whole rather on licensed and in-service teachers' effectiveness in the implementation of learner-centred pedagogy. An action research study by Kitta (2004) focused on enhancing teacher's topic specific PCK skills. Secondly, although there is an emphasis on use of LCA in Tanzania, teacher-centred teaching approaches in classroom are still dominant which suggested a need to assess teachers' PCK in the use of LCA. Thirdly, some teachers who are implementing the CBC are untrained in the use of LCA, hence assessing trained teachers' PCK in the use of LCA was necessary.

3. Method

The nature of PCK as a tacit knowledge demanded to obtain deep insights into what is in the mind of the participants' teachers through a qualitative approach. The participants in this study were two trained mathematic teachers in the use of LCA. The two teachers were identified by this study as MTR1 and MTU2. The first two letters M and T were used to identify mathematics teachers while R1 and U2 are identify a teacher in a rural and urban area respectively. MTR1 was purposively selected from a group of low performing schools in rural area while MTU2 was purposively selected from a group of low performing schools in an urban area. MTR1 had a degree in mathematics with education qualification while MTU2 had had a diploma in mathematic with education qualification. Both teachers were trained in the use of LCA and had completed their studies in 2010. Both teachers had been recruited as mathematics teachers by the government and worked for at least five years in one of the political administrative districts in Tanzania. The fact that PCK is a personal construct, interpretivism paradigm guided data collection and analysis of the findings. A multiple case study design was employed whereby each individual teacher was treated as a case study. A multiple case study design ensured transferability of the findings. Data was collected through classroom observation, video recording of at least three lessons of each teacher's classroom practices and teacher's self-reflection after the classroom observation. Since teacher's PCK is a personal knowledge, the researchers agreed with the teachers on the lesson that best represented teacher's PCK before documentation. Therefore, this study presents one of the best lessons that each teacher thought it well represented their PCK. The choice of these teachers began with discussion on their teaching plans, their syllabus coverage and finally the topic that they were ready to be observed and videotaped by the researcher while teaching was later on agreed. Since the purpose of the study was to measure a personal construct, teacher's plans and teacher's choice of topic was a priority. The purpose of the observation schedule was to formulate each teacher's content representation (CoRes) and finally represent each teacher's PCK. Loughran et al. (2006) and Hume and Berry (2010) guided the identification of themes. Each teacher's self-reflection on classroom practices as guided by Gardner and Gess-Newsome (2011) helped to capture and represent each teacher's professional development Repertoires (Pap-eRs).

Due to the implicit nature of PCK and this study being purely qualitative, Braun and Clarke's (2006) and Creswell's (2009) six steps thematic analysis procedures was adapted. The analysis process in this study was data driven and allowed capturing the themes they emerged from participant teachers. Thematic analysis allowed data analysis both at manifest and theoretical level as the data collection procedures was going on and from different sources. The themes that emerged during classroom observation and during interview were used to locate evidence of teacher's use of LCA and the level of teacher's PCK in the use of LCA. The analysis process began with transcription and describing each case with the data in order to become familiar with the data. This was followed by coding and organizing data from both video record and interview sessions into themes. FitzGerald (2012) stipulates that analysis of events in video records should be guided by research questions. The events from video records that was gauged from each teacher's classroom practices was then matched with each teacher's PCK in the use of LCA and the teacher's self-reflection during a min interview with the researcher. This stage helped the researcher to obtain general meaning of the data and extracting supporting evidence from data. This stage was followed by description and interpretation of the themes.

After individual case study analysis, crosscase analysis followed whereby, similarities and differences in the teacher's classroom practices in the use of LCA were done. Triangulation of the data from classroom observation, teaching plans, and teacher's self-reflection helped to formulate each teacher's profile of PCK in the use of LCA. A summary of the themes that emerged from both cases was formulated before documenting each teacher's PCK as contributed by their knowledge of contents that they taught. The themes as emerged during classroom practices (lessons) are represented in italic form in each teacher's lesson as teacher's content representations (CoRes).

4. Results and Discussion

We begin by presenting each teacher's content representation as were captured through video recording of each teacher's classroom practices (Box 1 and Box 2). Each teacher's classroom practice is followed by each teacher's self-reflection to document each teacher's Pap-eRs.

Box 1: Lesson 1 by MTR1: Topic "Graphs of a Relation"

Lesson beginning: … "we want to see how we can make graphs of different relations". MTR1's wrote an example from the textbook. *Example 1:* "Draw the graph of a relation $R = \{(x, y) : x + y \leq 2 \text{ and } y \leq 2\}$". MTR1 read the example, then continued … "Now, listen to me" ... "When you see this symbol or have a symbol, < or > in a relation, what you do is to draw a dotted line. But when you have a symbol $\leq or \geq$ in a relation, you draw a full line" "Do you understand?". MTR1 asked. "Yes". Students yield in a chorus. MTR1 *continued*: The inequality $y \leq 2$ is very much known by everyone while the inequality $x + y \leq 2$ is the same as $x + y = 2$. So, what you do here is to make a table of values. You assume that, the symbol > is equal to = because you cannot draw a table of values with inequalities". MTR1 prepared a table of values with x and y intercepts $(0,2)$ *and* $(2,0)$. "Having $y \leq 2$ you regard this as $y = 2$, this is a full line". MTR1 made a sketch of this relation on the chalk board (see Figure 1). Figure 1 showed MTR1's knowledge of concept of a line segment and a line (*knowledge of content, orientation to teacher-centred approaches (TCA*)).

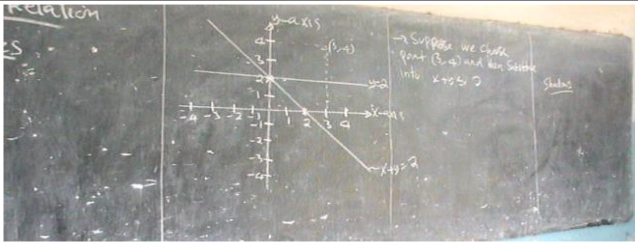

Figure 1. MTR1's Representing Steps when Sketching Graph of a Relation

MTR1 continued ... "Look at this graph. Each graph divides the plane into two parts; the down and upper part" (see Figure 1 and the part referred). "You choose any part, either the up or down part that will satisfy the relation. This will be the part that will be shaded. Now, suppose I choose a point $(3,4)$ and then substitute into $x+y \leq 2$. This means now $x=3, y=4$ then $3+4 \leq 2,\ then\ 7 \leq 2$, this is not true. What you do is to shade the other part that agrees. For example, choose $(-2,1)$ this gives $x+y \leq 2,\ then -2+1 \leq 2$ which gives $-1 \leq 2$. Is this true?". MTR1 asked. "Yes". Student replied in a chorus.

Lesson continued: "After that you go to the other inequality $y \leq 2$. "Would you shade up or below?". MTR1 asked. "I am asking you … where do we shade? Is it below or up?". MTR1 asked again after a long silence. "Below" The students responded in a chorus (*orientation to TCA).* "Our interest is to find the part of intersection. The shading now will be as shown" (See Figure 2). After that, MTR1 pointed the intersection that was earlier mentioned by MTR1 (*orientation to teaching*). This was then followed by MTR1's oral questions. "Any question?" MTR1's asked. However, students were silent (*knowledge of assessing students*).

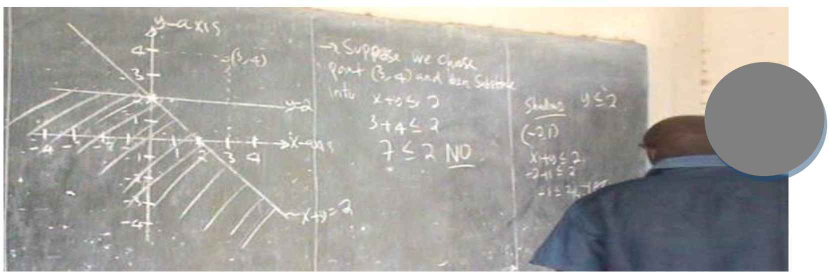

Figure 2. ASRL's Representation of a Graph of a Relation $R=\{(x,y): x+y \leq 2\ and\ \ y \leq 2\}$

"Suppose the words; find domain and range are added in the question, what will be the domain and range?" (*Knowledge about leaching and students learning*). MTR1 added these words in the example. "Domain are values of x that satisfy the equation while range are all values of y that satisfy the equation". MTR1 provided the definition of domain and range (*Knowledge of contents*). "This means if you proceed with the shading, all values of x will be included. The shading takes all values of x and that of y. Therefore, Domain $=\{all\ real\,number\,x\}$ while range $=\{all\ real\,number\ y\}$". MTR1 pointed on the shaded part while giving explanation about domain and range of relation. "On the second inequality $y \leq 2$ the shading stops at $y=2$ and don't go above. So, range $=\{y: y \leq 2\}$". MTR1 explained *(TCA orientation)*. "Any question?". MTR1 asked, but students remained silent *(Knowledge of assessing students).* "Let us have another example number 2". MTR1 wrote example 2 on the chalkboard.

Example 2: "Draw the graph of the relation $R = \{(x, y) : y < x\}$". MTR1 allowed students to copy the question in their notebooks after had already written the example on the chalkboard. Later on, MTR1 sketched the graph on the chalkboard (See figure 3). (MTR1's *knowledge of the concepts of a line and line segments)*. This indicated MTR1's inadequate knowledge of the concepts. In mathematics, a line has no end and is indicated by arrows on both ends while a segment has end points on both ends.

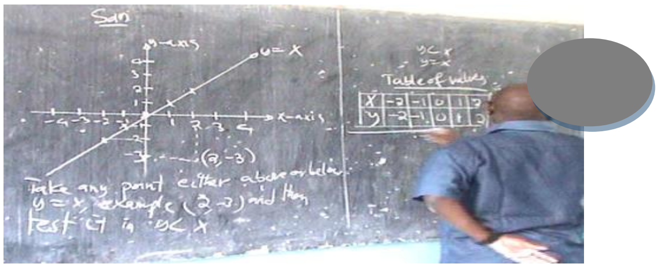

Figure 3. MTR1's Procedure when Drawing a Graph of a Relation $R = \{(x, y) : y < x\}$

Lesson continued: "You construct a table of values. What will be the value of y when x is -2, -1, 0, 1, and 2?". MTR1 continued on by asking a question. After had asked the same question to the students for several times, one student at a time, MTR1 finally filled the table of values (See figure 3). At this time, MTR1's students responded to MTR1 by raising their hands up. However, similar students participated (*Knowledge of LCA).* "You can now see the line that divides the plane into two equal parts. Taking any point above or below $y = x$ for example, take $(2,-3)$ *(TCA orientation)*, then $-3 < 2$ is true. "Is this true?". MTR1 asked (*Knowledge of assessing students*). "Yes". Students yielded in a chorus. *Lesson continued:* "This part agrees. Automatically, the other part will not agree. For example, $(-2,1)$, then $1 < -2$, is also true. Is this true?". MTR1 asked. "No". Students' responded in a chorus. "This means that we are still in the right truck". MTR1 commented that, the answer was correct.

Lesson continued: "What is a domain?". MTR1 asked. "All real numbers" The students shouted in a chorus "Why?" MTR1 asked a follow up question but students remained silent. *Lesson continued:* "I agree with you but why" (*Knowledge about students and learning*). There was a silence moment. "Don't claim". MTR1 insisted. After such a silence moment, MTR1 wrote: "$D = \{All\ real\ number\ x\}$". "Look here, the shading takes the negative and positive values". MTR1 explained (*TCA orientation*). "What is the range?" MTR1 asked. However, students remained silence. "Range $= \{all\ real\ numbers\ y\}$" MTR1 wrote on the chalkboard "Why?" You see here, if this line goes infinitely the shading will take all positive and negative numbers. This means that, all real numbers" MTR1's question and explanations. "Is there any question before I give you an exercise?". MTR1 asked, but there was silence (*Knowledge of assessment*).

Lesson continued: "Organize yourself in groups of five students then do the following" (*Knowledge of instructional strategies*). After that moment, MTR1 wrote a question on the chalkboard which read; "Draw a graph of the relation $R = \{(x, y) : y \geq x\}$ *and find domain and range*". At this time, MTR1 gave an opportunity for each student to work in small groups. MTR1 was passing through different groups as students worked (*Knowledge of strategies*). "Who can help us to do the work". After about 5 minutes, MTR1 asked for a student who can show how to solve the problem (*Knowledge of group accountability*). A student's response was read as*:* "We assume that $y \geq x$ is equal to $y = x$ then make a table of values" (*Students' knowledge of contents*).

Using the chalkboard, the student who had been appointed by MTR1 showed how to solve the problem. The student started by making a table of values, then drew a graph of the equation but did not shade. "Our line must be full line and we will find the domain and range". The student explained (*Students' knowledge of content*). "What about shading?". MTR1 interrupted. "Now, we shade". The student incorporated the idea. "Take $(2,-2)$ so you get $-2 \geq 2$. Is it correct? The student asked. "No". Some students replied in a chorus. "So, the upward side is not correct. We shade the downward part". The student shaded the downward part (See Figure 4). Figure 4 show how teacher's knowledge of contents is transferred to students and then reflected through students' knowledge of contents.

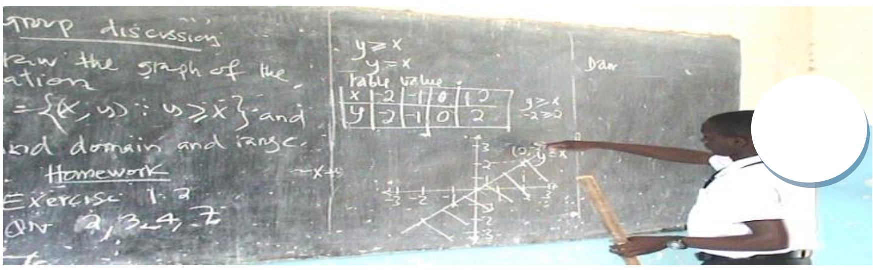

Figure 4. MTR1's Student Presenting a Graph of a Relation $R = \{(x, y) : y \geq x\}$

Lesson continued: "Is he correct? What is a mistake? Make sure you take a point which is above or below". MTR1 interrupted after a time. "Don't take a point within the line so that you cannot confuse. After that, do a test" (MTR1 continued with explanations). "What is the mistake here? MTR1 added. In your case take a coordinate point $(2,3)$ then test whether $3 \geq 2$ is true then shade". MTR1 added (*TCA orientation*). "Is there anybody who has not understood? MTR1 asked (*Knowledge of assessment*). Students remained silent. "So, you don't have any question?" MTR1 added after noticing that students were silent. "Note the use of dotted and full line. When you are given either a symbol < or > then, use a dotted line where the point within are not included. When you are given either a symbol $\leq$ *or* $\geq$ then, use a full line, whereby the points within the line are included". MTR1's commented before ending the lesson (*TCA orientation*).

Ending the lesson: Before ending the lesson, MTR1 introduced the next lesson and finally ended the lesson by providing an exercise.

MTR1's self-reflection and the components of PCK in LCA

The following extract from MTR1's self-reflection as was interviewed by the researcher revealed that, MTR1 had adequate HCK but this knowledge did not adequately inform both KCS and KCT;

> Researcher: What is the relationship between the subtopic you were teaching and other Mathematics topics or subtopics that you teach at this level?
>
> MTR1: *In drawing graphs of relation, you need to know about intercepts or the point where the lines cuts the Y-axis and where the lines cuts X-axis.* There is a relationship between the topic called Algebra whereby you will need to find the X and Y intercepts when constructing the table of values for an equation.
>
> Researcher: How useful was the relationship in the lesson that you taught?
>
> MTR1: *The use of x and y intercepts was useful when drawing graphs.*

Box 2: Lesson 2 by MTU2: Topic "Graphs of Relations"

Lesson beginning: MTU2 wrote the subtopic on the chalkboard "Graphs of Relations", later on, announced orally "today you will learn about the process of making graphs of Relations". "How do we make graphs of different Relations?" MTU2 asked. At this time, students remained silent. "These are examples of Relations. At this time, ESSUL began demonstrating ways to make graphs. "Let us see how we can make graphs of the relations". While demonstrating the nature of the graphs and how the graph are drawn, MTU2 continued on by writing different examples of Relations on the chalk board (*Orientation to teaching*). Example 1: "(i) Draw the graph of a Relation $R = \{x, y\}: x + y \leq 2 \; and \; y \leq 2$ and determine the Domain and Range". MTU2 wrote this first example on the chalkboard (see Figure 5).

Figure 5. MTU2 Demonstrating on how to Draw Graphs of Relations

Lesson proceeding: "Note that; when drawing graphs, you show these two symbols; $<$ $>$ by a dotted lines and these two; $\leq$ $\geq$ by a full lines. Is it understood?" MTU2 asked. "Yes". The class yielded in a chorus (*Orientation to teaching*). Before proceeding on with the first example, MTU2 decided to demonstrate on how inequality signs are represented on graphs of Relations. "We shall see this through examples. So, what you do, is first to construct a table of values using the equation $x + y = 2$. A simple way to do this is to use x-intercept and y-intercept. Who knows? what is an x-intercept and what is a y-intercept?" "Class, what is an x-intercept?" "What is a y-intercept?" MTU2 asked again (*Knowledge of students and orientation to teaching*). Two students raised their hands up at this moment. MTU2 picked one of the two students. “The x-intercept is 2 and the y-intercept is 2” A student answered. “Can you show us how you got 2?” MTU2 asked after the student had answered (*Knowledge of teaching and students’ learning*). “You plus two (2) and zero (0)” The student replied. It meant to the researcher that, you add 2 and 0. “Who can show the points (0, 2) and (2, 0) on a number plane?” MTU2 asked then picked two students one after the other. The two students went to the front of the class in order to indicate the points on the number line (See Figure 6) (*Knowledge of teaching and students*).

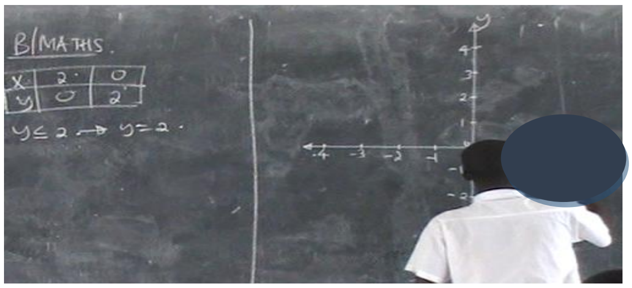

Figure 6. MTU2’s Student Locating a Coordinate Point on a Number Plane

After that session, MTU2 constructed a table of values, a number plane and a sketch of the graph (Figure 7). “These two lines divide the plane into a number of regions”. MTU2 explained. “How many regions are there?” MTU2 asked. The class remained silent while gazing at MTU2. After about 3 minutes of such a silence moment, there was an attempt from some students who answered orally. “9, 6, 5, 8,”. The students answered. The researcher noted that, the students’ answers ranged between a number 5 and 9. However, MTU2 did not comment on any of the students’ answers (*Knowledge of teaching and students*). “There are four regions”. MTU2 gave an answer after the students had answered (*orientation to teaching*). While pointing on the graph with a ruler and without exactly showing the named regions, MTU2 gave an answer which was quite different from the students’ answers (See Figure 7). (*Knowledge of content and students*)

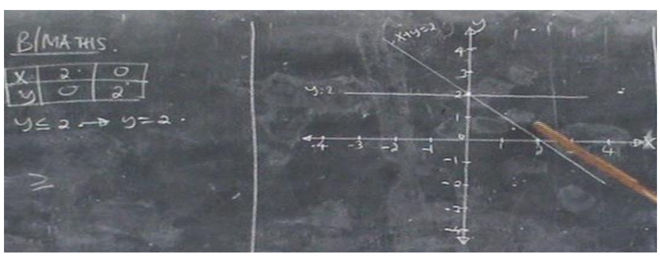

Figure 7. MTU2 Demostratng on the Four Regions in a Graph of a Relation on a Plane

Lesson continued: “Now what follows is shading. Where do we shade?” ESUL asked. “You shade any part; down or up, then you test” MTU2 answered the question that had earlier asked (*Knowledge of content and teaching, Orientation to teaching*) “Which part of the regions should we choose to shade now?” MTU2 asked for the second time. The whole class was silent; nobody attempted to answer (*Knowledge of content and teaching, Knowledge of instructional strategies*). In addition, MTU2 did not show the part on the graph that was above and the part which was below on the number plane. “Can you choose any point below this point? $(-1,1)$ or $(2,1)$?” MTU2 asked while pointing the graph on the chalkboard. “Now, I choose a point $(3,4)$”. MTU2 chose a coordinate point $(3,4)$. “We take the point $(3,4)$ then substitute in the inequality. We shall have, $3+4 \leq 2,\ \ 7 \leq 2$”. “Is this $7 \leq 2$ true?” MTU2 asked. However, students remained silent. Having student remaining silent, MTU2 commented; “No, the part does not agree, so you take the opposite. MTU2 added. (*Orientation to teaching, Knowledge of content and teaching*) “The opposite is the point $(-3,2)$. MTU2 indicated the point $(-3,2)$ on a graph (See Figure 8).

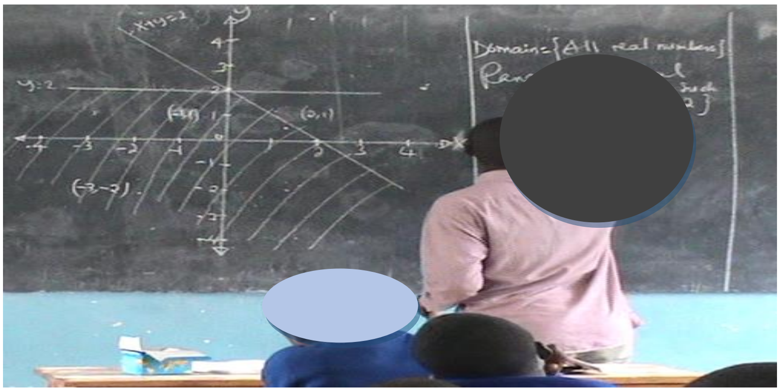

Figure 8. MTU2’s identifying a Domain and Rrange of a Relation on a Graph

“After that, substitute the point in the equation. You get $x+y \leq 2,\ \ -2+1 \leq 2,\ -1 \leq 2$. Which is now true” MTU2 continued, “We now shade the part which is true (Figure 8).” After knowing this, our next interest is to see the point of intersection” (*Orientations to teaching, Knowledge of content and teaching*). *“*We then go back to the next part $y \leq 2$ and find the domain and range. Any question? Do you have any question?” MTU2 asked (*Knowledge of content and teaching, Orientation to teaching*). The whole class was silent. “Domain will be all values of x”. MTU2’s demonstrated that the domain is all values of x. “Of course, if you proceed along the line, then all values of x will be included. So, domain is $D:\{all\ real\, number\, x\}$ and Range R is $R:\{y: y \leq 2\}$”. “Is there any question?” MTU2 asked. The class was silent. “I hope there is no question”. MTU2’s hoped that there were no questions (*Knowledge of instructional strategies*). “I give you two (2) minutes to finish this example 2” MTU2 wrote another example which was named example 2 on the chalkboard. Example 2; “Draw a graph of the relation $R=\{(x,y): y<2\}$ then find the domain and range”. None of MTU2’s students managed to attempt this task. The question was written with some mathematical mistakes. However, both MTU2 and the students did not identify the mistakes. After about 5 minutes had elapsed, MTU2 noticed the mistake, made some correction on example 2 which had written then asked students to work in small groups.

Group activity: After had improved the example, MTU2 asked student to work on the new and improved example 2 in their small groups. The new example 2 was written as; “$R=\{(x,y): y<x\}$”. MTU2 changed the question from $R=\{(x,y): y<2\}$ to $R=\{(x,y): y<x\}$). “Let us work together”. MTU2 decided to continue working with the whole class after had corrected the mistakes.

"Note that, in constructing graphs of this inequality, we assume that $y = x$ when preparing a table of values. Remember this will be a dotted line" MTU2 proceeded with construction of a table of values and a graph (*Orientation to teaching, Knowledge of content and teaching*). "Here is the table of values and the graph we are looking for" MTU2 prepared a table of values and sketched a graph without the dotted lines (see Figure 9). MTU2 had forgotten the initial statement about dotted and full lines. Also, MTU2 had forgotten labeling and differentiating graphs of lines and segments (*Knowledge of contents, Knowledge of content and teaching*).

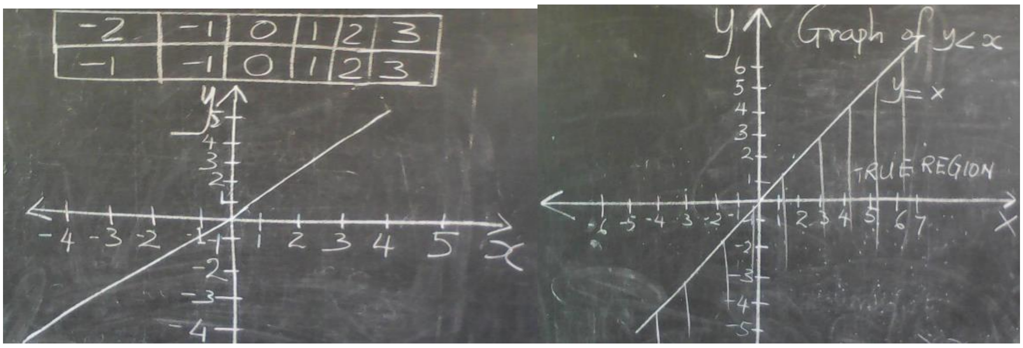

Figure 9. MTU2's representation of Graphs of a Relation $R = \{(x, y) : y < x\}$

Lesson proceeding: "Suppose, I choose/take any point above or below the line $y = x$ then test for the inequality $y < x$, say, if I take a point $(2,-3)$, we shall have $-3 < 2$ (*Knowledge of content and teaching*). The inequality shows that, 2 is greater than -3. Class, is the inequality true? or is it false?" After MTU2 had asked wrote the words; "Yes or no?" on the chalkboard. Class, is it "Yes or no?" MTU2 asked again (*Knowledge of content and teaching*). The class was silence. "Now, what is the domain and what is the range?" MTU2 asked again. "All real numbers x" A class replied in a chorus. "Why?" MTU2 asked after the class had responded. None of the students tried to answer this question (*Knowledge of content and teaching*). "You will agree with me that the domain is all real numbers x". MTU2 commented after such a moment of silence (*Orientation to teaching, knowledge of students*). "What is the range?" Silence continued. It appeared to the researcher that, students had no clear knowledge of the two concepts; "domain and range" (*knowledge of contents and students*). "The range is all real number y" MTU2 answered. However, it also appeared to the researcher that MTU2 had forgotten that the Relation that was referred involved an inequality sign (*Knowledge of contents and students*). This part of the lesson was followed by students' work in small groups.

Small group activity session: After such initial instruction, MTU2 wrote a problem on the chalkboard then asked students to continue working in small groups: "Draw the graph of the relation $R = \{(x, y) : y \geq x\}$ then find the domain and range". As students were working, MTU2 passed through different groups to see how students worked (*Knowledge of content and teaching*). After about 5 minutes, MTU2 discovered that some groups had failed to follow the steps or procedures. After such observation, MTU2 decided to provide an outline of all the steps on the chalkboard (*Orientation to teaching, Knowledge of contents and teaching*). "The first step is to make a table of values, second is graphing either with a dotted or full line and finally testing points" (*Orientation to teaching, Knowledge of content and teaching*). After about 15 minutes had elapsed, MTU2 chose randomly a student from one of the groups so as to present their solution to the problem. The student began with a table of values, plotted points on a number plane then chose a point $(2,-2)$ in order to test for a region which satisfied the inequality. However, the student got stuck and could not proceed. "Use this point $(2,3)$". MTU2 advised the student to use a point $(2,3)$ after had observed that the student used a point $(2,-2)$ (*Knowledge of content and teaching*). The student's answer was read as: "The domain D is, $\{all\ real number\, x\}$ while the range is, $\{all\ real number\, y\}$" (See Figure 10). The student finished after some assistance from MTU2 (*Knowledge of students, Knowledge of content and teaching*).

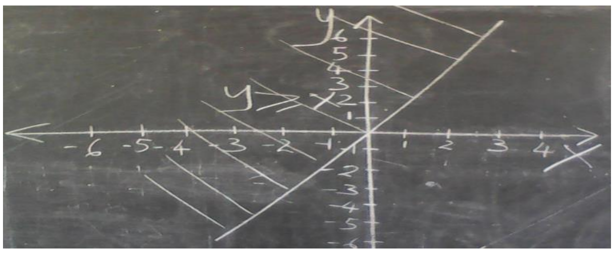

Figure 10. MTU2's Student's Graph of a Relation $R = \{(x, y) : y \geq x\}$

Later on, MTU2 continued on with a third example. Example 3 read as: "Draw a graph of $R = \{(x, y) : y = x^2\}$ then find the domain and range". MTU2 asked students to continue working in groups. However, there was no chance for small group discussion due to a shortage of instructional time as per school timetable. Having this situation, MTU2 decided to go on solving the problem (O*rientation to teaching*). (see 11).

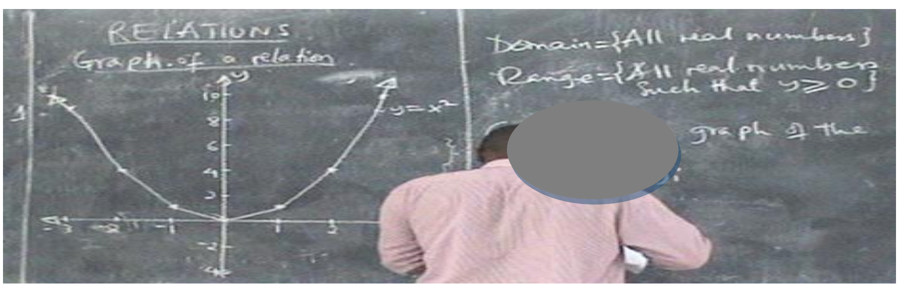

Figure 11. MTU2 Sketching a Graph of a Relation $R = \{(x, y) : y = x^2\}$

Ending the lesson: To wind up the lesson, MTU2 decided to provide a home work which was found in the students' textbook. The homework marked the end of MTU2's lesson (see Figure 12).

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Figure 12. MTU2 Writing Homework for the Students

MTU2's self-reflection and the components of PCK in LCA

Data from MTU2's self-reflection revealed that, MTU2 had adequate HCK but weak SCK which did not inform KCT. The extract from the MTU2's self-reflection session with the researcher revealed;

Researcher: What is the relationship between the graphs of relations and other mathematics topics or subtopics within the topic of relations?

MTU2: *There is no any subtopic that relate with relations. The topic relates with other mathematics topic like linear programming and coordinate geometry. This is because, in order to draw the graph of relation, you must first of all have the knowledge of coordinate geometry on how to locate points in the XY plane. In linear programming this knowledge is used to show the feasible region by shading* (*Horizontal content knowledge*).

Researcher: If the topic you mentioned is in Form Four then how useful was that relationship in the lesson that you taught?

MTU2: *I think shading and drawing are applied here*.

Experienced teachers' knowledge of content as a component of PCK in the use of LCA

The participant experienced teachers in this study lacked knowledge of concepts such as; mathematical relations, domain and range which led to communicating their misconceptions to the students. For example, during a mathematics lesson, students had a misconception that domain means; "values of "x" while range "values of y". During individual activities, the students copied the same in their exercise books. MTR1 was unable to differentiate between "a line" and "a line segment", the mathematics concepts which have both similarities and differences in the study of Geometry. This finding is contrary to the expectation that when teaching mathematics, the experienced teachers are able to help students identify important ideas in the subject and lesson (Hill, Ball, & Schilling, 2008). In different occasions, there were evidences of the experienced teachers use of wrong concepts and in particular when drawing graphs of equations. Instead of employing LCA, the experienced teachers directly communicated errors and misunderstanding to their students. This implies that the experienced teachers lacked the PCK categories in the use of LCA.

The curriculum developer recommended raising students' participation in the teaching and learning process (TIE, 2010a), however this expected outcome will not be achieved as the findings shows that, the experienced teachers lacked PCK categories in the use of LCA. Both experienced teachers in the study lacked questioning skills that hindered their ability to ask provoking questions so as to stimulate students' thinking. As a result, the teachers were unable to manage instructional time, provide constructive feedback to students and involve students in their lessons. Since teachers' lacked content knowledge for teaching, the teachers were forced to the use of demonstration method in order to finish what they planned to teach. Demonstration method misguided students' learning which led to students' misconception. In addition, lack of content knowledge lowered teacher's confidence which indirectly diminished teacher's growth of PCK in the use of LCA.

Experienced teacher's content knowledge and their decision on the use of LCA

The literature review is indicating that the inexperienced teachers in the use of LCA are unlikely to effectively enhance students learning outcomes in the use of LCA (Anney, 2013). However, the findings from this study show that, the participant experienced teachers had inadequate horizontal content knowledge and were oriented to TCA. These participant teachers who were oriented to teacher centered approaches were unable to decide and make use of LCA. If this situation continues on, it will discourage the government effort to have LCA implemented in classrooms. The teacher's lessons will continue to be dominated by telling facts about concepts which will finally not improve students' performance in mathematics. The experienced mathematics teachers' classroom practices which were more teachers' centred weakened their ability to help students to construct knowledge of contents that they taught. The participants' teachers made few attempts to assist students to construct, identify and understand concepts or ideas on their own. These suggested that, the participant teachers had underdeveloped PCK categories in LCA and SMK categories.

5. Conclusion and Recommendation

Based on the findings, it is concluded that, the experienced mathematics teacher's lack of content knowledge led to the teacher's difficulties in decision, planning and implementing learning activities that are students' focused. In addition, teacher's growth of PCK is limited by their weakness in guiding learners at the centre of their lessons. This will un address both teacher's growth of PCK in LCA and students' learning of contents. Students' misunderstanding of content will leave students' learning needs and problems unaddressed. Furthermore, this teaching and learning situation will not improve students' learning outcomes and achievements in the subject. Since teachers underdeveloped PCK in LCA hinder their decision and use of LCA when teaching of mathematics, it is recommended that, teachers' professional development programmes should cement on both teacher's content knowledge and PCK categories in the use of LCA.

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