

Original Paper

The Unpleasant Arithmetic of the Taylor Rule

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Abstract

This paper analyzes the effect of a monetary policy that raises the reference interest rate in order to reduce inflation in a situation where the fiscal policy parameters remain constant. In an overlapping generation's model and in the presence of an accelerationist Phillips curve and a Taylor rule of interest rates, it is observed that increasing the independent component of said rule leads to a solution that at least in a large number of cases is unstable. In the case where the elasticity of substitution is greater than one, inflation falls temporarily, but then it can increase in an unstable manner. One way to achieve stability is to establish an interest rate rule where Taylor's principle is not met. However, in this case many times the increase in the independent component of this rule will generate greater long-term inflation.

Keywords

monetary policy, Taylor rule, unpleasant arithmetic

1. Introduction

In 1981, Thomas Sargent and Neil Wallace published an article of great relevance in monetary theory. This work shows that not always what is understood as a restrictive monetary policy reduces inflation. In particular, when the fiscal policy parameters remain constant, a reduction in the rate of growth of the nominal amount of money sometimes reduces inflation temporarily, but then the price growth increases to levels higher than what it had before the implementation of the described policy. On other occasions, the aforementioned policy results in an immediate increase in inflation. These results are known as unpleasant arithmetic (Note 1).

The work of Sargent and Wallace (1981) clearly shows that in order to reduce inflation in the long term there must be coordination between monetary policy and fiscal policy. A lower price growth requires, on a large number of occasions, a fiscal adjustment in which the inflationary tax is replaced by another type of taxes, or with a lower public expenditure. Only in this way can inflation reduction be permanent (Note 2).

Since the 1980s, in many countries monetary policy began to use a reference interest rate as a primary instrument to reduce inflation. The assumption to use this instrument is that the demand for consumption and investment depends negatively on the real interest rate, so, in the presence of inertial inflation, an increase in the nominal rate leads to an increase in the real rate that reduces aggregate demand, causing inflation to fall.

At the theoretical level, a problem that had been raised at the end of the 19th century by Knut Wicksell (1898) was discussed, whereby the use of the nominal interest rate to reduce inflation can generate a situation in which the policy is successful, but it generates an unstable solution, where a single increase

in the nominal rate causes an unlimited fall in inflation (the problem of cumulative causation or Wicksell instability problem) (Note 3).

In 1993, John Taylor proposed a mechanism that solves the problem posed a century ago by Wicksell (1898). A rule where the nominal interest rate is over-indexed to inflation, so that if the latter rises (falls) one point, the nominal rate rises (falls) more than one point and, thus, the real rate increases (falls). The proposed mechanism eliminates Wicksell's instability problem. The over indexation of the nominal interest rate on inflation is known as the Taylor principle.

In the last twenty years the problem of unpleasant arithmetic has not been much analyzed. Most of the macroeconomic models of the new Keynesian approach, which already consider Taylor's rule as a fundamental instrument of monetary policy, are based on the representative agent assumption, which has an infinite time horizon and where the government is committed to a Ricardian policy, with which it can be financed with bonds, but eventually it will have to make a fiscal adjustment to prevent public debt from growing unsustainably (Note 4).

The seminal article by Sargent and Wallace (1981) shows the need for coordination of fiscal and monetary policy to reduce inflation permanently. The analysis of the new Keynesians indicates that, when the primary instrument of monetary policy is some reference interest rate, there must be a rule that complies with the Taylor principle so that a reduction in inflation allows the stability of the macroeconomic system.

To the best of our knowledge, there are no studies on the macroeconomic effects of implementing a monetary policy that follows Taylor's principle when, at the same time, fiscal policy parameters remain constant. Therefore, the objective of this work is to propose a situation of this type, where the monetary authority carries out a restrictive monetary policy following an interest rate rule that contains the Taylor principle, but, at least for a certain period of time, the fiscal authority maintains its policy parameters without movement.

The main result of the aforementioned exercise is that a situation like the one described above eventually produces higher inflation. In fact, this could arise from the moment of the implementation of the policy. Moreover, the macroeconomic model enters an unstable situation. Paradoxically, instability is caused by compliance with the Taylor principle. Eliminating this principle would not reduce inflation in relation to its original point, but it would give stability to the macroeconomic system (Note 5).

This work is divided into five sections:

The first section develops a model of overlapping generations, in which the only financial asset is government bonds. In a second section an IS function is generated and it is analyzed under what conditions an effective demand such as that of the simple Keynesian model would arise. In this same section, the model is extended to include an accelerationist Phillips curve and an interest rate rule for monetary policy that may or may not comply with Taylor's principle.

The third section assumes that the elasticity of substitution in consumption is unitary and analyzes the effect of monetary policy on the expanded macroeconomic model. The exercise consists of an increase in the independent component of the interest rate rule, which aims to reduce inflation. The analysis shows the dynamics of inflation, output and the real interest rate. We consider both an interest rate rule that complies with the Taylor principle and another that does not comply.

The fourth section carries out the same exercise as the third, but when the elasticity of substitution in consumption is greater than unity.

Finally, the fifth and last section analyzes potentially unstable cases produced by monetary policy and the way in which fiscal policy can stabilize the system.

2. The Overlapping Generation's Model and the Maximization of the Utility

We start with the development of an overlapping generation's model. This type of theoretical framework was first used by Maurice Allais in 1947 (see Malinvaud, 1987). Eleven years later Paul Samuelson (1958) popularized the use of the model (Note 6).

In the development of this work, generations are supposed to live two periods. There is only one financial asset, government bonds. In the first period people generate income, pay taxes and can make their purchases on credit. In the second period people do not generate income and can only make their consumption with the value of the capitalized bonds that they saved in the first period.

For even generations (superscript a): those born in periods $t, t+2, t+4, t+6 \dots$ the budget constraints of the two periods they live are:

$$Y_t - T_t = C_t^a + b_t \quad (1)$$

$$(1 + r_t)b_t = C_{t+1}^a \quad (2)$$

Where Y is the total gross income, T represents taxes, C_t^a is consumption of the even generation in period t , b_t are government bonds saved in period t , r is the real interest rate of public bonds. C_{t+1}^a is consumption of the even generation that was born in period t .

In turn, for nons (superscript b): those born in periods $t-1, t+1, t+3, t+5 \dots$ the budgetary restrictions in their corresponding periods are:

$$Y_{t-1} - T_{t-1} = C_{t-1}^b + b_{t-1} \quad (3)$$

$$(1 + r_{t-1})b_{t-1} = C_t^b \quad (4)$$

Where the variables are the same as for the even generation but out of date one period.

The national accounts identity indicates that

$$Y_t = C_t^a + C_t^b + g_t \quad (5)$$

The income is consumed entirely by young people of the even generation that was born in the period t (C_t^a), by old people of the non-generation who was born in $t-1$ (C_t^b) and by government consumption (g_t).

The public sector budget constraint is a linear combination of the above equations: equation (4) is replaced in the national accounts identity (5) and the $Y_t - C_t^a$ value is solved in (1) and replaced in (5), which results in:

$$g_t - T_t + (1 + r_{t-1})b_{t-1} = b_t \quad (6)$$

Equation (6) represents the budget constraint of the public sector. The public deficit is $g_t - T_t + r_{t-1}b_{t-1}$. The primary deficit is $g_t - T_t$. This result is relevant, as it indicates that if the private sector reaches equilibrium, the public sector will also be in equilibrium.

Each of the generations maximizes an intertemporal utility function. For the even generation that is born in t , this function is established as:

$$U = \frac{C_t^{a \frac{1-\frac{1}{\rho}}}}{(1-\frac{1}{\rho})} + \frac{C_{t+1}^{a \frac{1-\frac{1}{\rho}}}}{(1-\frac{1}{\rho})(1+\theta)} \quad (7)$$

(7) is an iso elastic utility function, being ρ the elasticity of substitution in intertemporal consumption. On the other hand, θ is the subjective discount rate.

The combination between equations (1) and (2) results in the intertemporal budget constraint:

$$Y_t - T_t = C_t^a + \frac{C_{t+1}^a}{(1+r_t)} \quad (8)$$

The maximization of the utility function (7) subject to the intertemporal budget constraint (8) gives rise to the following first-order conditions

$$C_t^{a-\frac{1}{\rho}} - \lambda = 0 \quad (9)$$

$$\frac{C_{t+1}^{a-\frac{1}{\rho}}}{(1+\theta)} - \frac{\lambda}{(1+r_t)} = 0 \quad (10)$$

$$Y_t - T_t - C_t^a - \frac{C_{t+1}^a}{(1+r_t)} \quad (11)$$

Where λ is the Lagrange multiplier associated with utility maximization.

Combining equations (9) and (10) we get

$$C_t^{a-\frac{1}{\rho}} = \frac{C_{t+1}^{a-\frac{1}{\rho}}(1+r_t)}{(1+\theta)} \quad (12)$$

(12) is Euler's equation in consumption: the marginal utility of consumption in period t is equal to the marginal utility of consumption in period t+1 multiplied by the gross real interest rate (1+r) and divided by subjective discount rate (1+θ).

Substituting Euler's equation (12) in the intertemporal budget constraint (8) or (11):

$$C_t^a = \frac{(1+\theta)^\rho(1+r_t)^{(1-\rho)}(Y_t-T_t)}{(1+(1+\theta)^\rho(1+r_t)^{(1-\rho)})} \quad (13)$$

$$C_{t+1}^a = \frac{(1+r_t)(Y_t-T_t)}{(1+(1+\theta)^\rho(1+r_t)^{(1-\rho)})} \quad (14)$$

The optimal levels of consumption of the generation that was born in period t are found.

The response of present and future consumption to changes in the gross real interest rate (1+r) is:

$$\frac{dC_t^a}{d(1+r_t)} = \frac{(1-\rho)(1+\theta)^\rho(1+r_t)^{-\rho}(Y_t-T_t)}{(1+(1+\theta)^\rho(1+r_t)^{(1-\rho)})^2} \geq \leq 0 \quad (15)$$

$$\frac{dC_{t+1}^a}{d(1+r_t)} = \frac{(1+\rho)(1+\theta)^\rho(1+r_t)^{1-\rho}(Y_t-T_t)}{(1+(1+\theta)^\rho(1+r_t)^{(1-\rho)})^2} > 0 \quad (16)$$

In the face of an increase in the real interest rate, present consumption may rise, remain constant or decrease. Instead, future consumption always increases.

The response of present consumption to an increase in the real interest rate depends on the elasticity of substitution in intertemporal consumption (ρ). If this is less than the unit, both present and future consumption increase. If it is equal to one, present consumption remains constant and future consumption increases. If this elasticity is greater than unity, present consumption falls and the future rises.

The increase in future consumption also depends on the elasticity of substitution. The greater this elasticity, the more future consumption increases. This is logical because it is also in this case when the present consumption falls the most, so, given the same budget constraint, individuals who have a greater elasticity of substitution will have a greater variation in their consumption over time.

The maximization of the utility of the non-generation that was born in t-1 is symmetric to that of the even generation that was born in t, so that the consumption in t of such generation has a form analogous to the consumption in t+1 of the even generation:

$$C_t^b = \frac{(1+r_{t-1})(Y_{t-1}-T_{t-1})}{(1+(1+\theta)^\rho(1+r_{t-1})^{(1-\rho)})} \quad (17)$$

3. The Macroeconomic Model

Macroeconomic models in which the central bank uses some reference interest rate rule as a primary monetary policy instrument are relatively new. In this context stand out the works of Clarida, Galí and Gertler (1999), McCallum and Nelson (1999), King (2000), Romer (2000), Taylor (2000), Woodford (2001, 2003), Blanchard (2008), Koenig (2008), among others, which use the rule proposed by Taylor (1993).

In the macroeconomic model proposed in this work, it is assumed that the only tax that exists is that of income, whereby $T_t = \tau Y_t$, where τ is the income tax.

In this way, the IS function is made up of the sum of youth consumption (13), the consumption of old people (17) and public consumption expenditure g . Therefore, that equation is expressed as

$$Y_t = \frac{(1+\theta)^\rho (1+r_t)^{(1-\rho)} Y_t (1-\tau)}{(1+(1+\theta)^\rho (1+r_t)^{1-\rho})} + \frac{(1+r_{t-1}) Y_{t-1} (1-\tau)}{(1+(1+\theta)^\rho (1+r_{t-1})^{1-\rho})} + g_t \quad (18) \text{ IS function}$$

3.1 The Simple Keynesian Model

The expression (18) is an equation in first order differences, which, solving for current output Y_t can be expressed as

$$Y_t = \frac{(1+(1+\theta)^\rho (1+r_t)^{1-\rho}) g_t}{(1+(1+\theta)^\rho (1+r_t)^{1-\rho})} + \frac{(1+r_{t-1})(1+(1+\theta)^\rho (1+r_t)^{1-\rho})(1-\tau) Y_{t-1}}{(1+(1+\theta)^\rho (1+r_{t-1})^{1-\rho})(1+\tau(1+\theta)^\rho (1+r_t)^{1-\rho})} \quad (19)$$

This equation has too many parameters. If the real interest rate were a constant, (19) would be transformed into:

$$Y_t = \frac{(1+(1+\theta)^\rho (1+r_t)^{1-\rho}) g_t}{(1+(1+\theta)^\rho (1+r_t)^{1-\rho})} + \frac{(1+r_t)(1-\tau) Y_{t-1}}{(1+\tau(1+\theta)^\rho (1+r_t)^{1-\rho})} \quad (20)$$

The equation is stable if the value of the compound factor that multiplies Y_{t-1} is less than one, which occurs when

$$\tau > \frac{r_t}{(1+r_t) + (1+\theta)^\rho (1+r_t)^{1-\rho}} \quad (21)$$

In this case, the tax rate has to be higher than the interest rate divided by a parameter that, in general, is greater than unity, which seems very feasible to be met in modern economies. If fulfilled (21) we find an effective Keynesian demand.

Blanchard and Kiyotaki (1987) point out that an effective demand of this kind, which breaks with Say's Law, can be generated when individuals use income not only for consumption, but for other activities: saving, having leisure or paying taxes. In this case, young individuals consume, save and pay taxes, which under condition (21) generates an effective demand independent of possible supply conditions.

Equation (20) shows that in the short term the multiplier of public spending is greater than one, since the denominator of the first term on the right side of that equation is less than the numerator. The same equation also shows that the response of output to the tax rate is negative, since such variable is in positive terms in the denominators of the two terms on the right side of the equation and with a negative sign in the numerator of the second term on the right side of the same equation. These are compatible results with the simple Keynesian income determination model.

If condition (21) is met, there is a long-term solution for income, which is obtained by matching Y_t to Y_{t-1} in (20), which results

$$Y_t = \frac{(1+(1+\theta)^\rho (1+r_t)^{1-\rho}) g_t}{(\tau((1+r_t) + (1+\theta)^\rho (1+r_t)^{1-\rho}) - r_t)} \quad (22)$$

Equation (22) is the long-term IS, which depends positively on public spending and inversely on the income tax rate. It is possible to prove that the long-term multiplier of the government expenditure is greater than one, since if we make the conjecture that the numerator of (22) is greater than the denominator, the solution is reached:

$$1 - \tau + (1 + \theta)^\rho (1 + r_t)^{1-\rho} (1 - \tau) > -r_t (1 - \tau) \quad (23)$$

This is always true if $r > 0$, but it would certainly also be true if r is negative but greater than -1 , which has to happen so that the saving of the population is not completely confiscated.

To prove the effect of the real interest rate on long-term output, the derivative of the ratio y/g in (22) is taken in relation to $(1+r)$. This exercise has a long algebraic process. The condition for long-term

output to increase when the real interest rate increases $\frac{d(\frac{y}{g})}{d(1+r)} > 0$ is

$$1 + r\rho + \frac{(1+r)^\rho}{(1+\theta)^\rho} > 0 \quad (24)$$

This condition is met as long as the real interest rate is positive. It is also true that the elasticity of substitution in consumption is less than or equal to one, since the real interest rate cannot be less than -1 .

However, there are cases in which the condition is not met, which can happen when the real interest rate is negative and the elasticity of substitution in consumption is very high. In such a situation, a marginal increase in the real interest rate at levels that remain negative would lead to a reduction in long-term output.

The simple Keynesian model described here suggests that, in a large number of cases, the increase in the real interest rate in the long term has a positive effect on demand, indicating that such increase can be inflationary at least in the long term.

3.2 A More Complete Model

The simple Keynesian model is not very realistic. For that reason, we add two more functions to the equation IS (18): an accelerationist Phillips curve and an interest rate rule.

The accelerationist Phillips curve is an equation that shows that the difference between current inflation and inflation in the previous period-the acceleration of inflation-depends on the difference between the current and the natural output. This function is posed as

$$\pi_t = \pi_{t-1} + \gamma(Y_t - Y^*) \quad (25) \text{ Phillips curve}$$

Being π inflation and Y^* the natural output.

This version of the Phillips curve, which some call the old curve, has been analyzed theoretically and empirically by many authors. In theory, the works of Phelps (1967), Friedman (1968), Tobin (1972), Blanchard (1990), Romer (2000) and Taylor (2000) stand out. Empirical estimates of this function, or a derivation thereof, have been carried out by Fuhrer (1995, 1997), Gordon (1997, 2013), Lindé (2005), Rudd and Whelan (2005), Roberts (2001), Ball and Mazumder (2011), Blanchard, Cerutti and Summers (2015), Blanchard (2016), among many others.

The third equation of the macroeconomic model is the interest rate rule, which may or may not comply with the Taylor principle (Taylor, 1993). In this way, the rule mentioned for the real interest rate, or the so-called MP function by Romer (2000), is established in a generic way, such as:

$$r_t = f(\pi_t) \quad (26) \text{ MP function}$$

In a rule that follows the Taylor principle, the nominal interest rate is over-indexed to inflation, so the real rate depends positively on inflation. Other rules may not obey Taylor's principle. A fixation of the nominal interest rate, as proposed by Wicksell (1898) over a hundred years ago, results in an MP function where the real interest rate depends negatively on inflation.

The objective of the model comprised by equations (18), (25) and (26) is to analyze how its parameters affect the trajectories of the endogenous variables: output, inflation and the real interest rate. In particular, we want to analyze what type of interest rate rule stabilizes the model and how the variables react to a change in the independent parameter of said rate rule.

If the interest rate rule (20) is linear in inflation, the proposed macroeconomic model is not linear, since in the IS equation present and past inflation would be multiplying both the current output and the lagged. This is a problem, because in these cases it may not be possible to find the dynamic characteristics of the model conclusively, so that, in many cases, all that remains to be done is to carry out simulations with numerical methods.

We will analyze two cases: in the first we will assume that the elasticity of substitution in consumption is one. In the second case the elasticity of substitution in consumption will be greater than one. In both cases, the use of a Taylor-type rule and that of another rule that does not follow the Taylor principle will be analyzed.

4. Monetary Policy When the Elasticity of Substitution in Consumption Is Equal to One

If the elasticity of substitution is one ($\rho=1$) and the MP function follows the Taylor principle, the linear equation is

$$r_t = z_t + \Omega\pi_t \quad (27)$$

In that case, the aggregate demand (the AD function in the Romer-Taylor model (Romer, 2000; Taylor, 2000)), which is the equation that arises when replacing the Taylor type rule (27) in the IS function (18), takes the form

$$Y_t = \frac{(1+\theta)}{(2+\theta)}(1-\tau)Y_t + \frac{(1+z_{t-1}+\Omega\pi_{t-1})}{(2+\theta)}(1-\tau)Y_{t-1} + g_t \quad (28)$$

We will intuitively analyze the seemingly restrictive monetary policy, where there is an increase in the z_t parameter of Taylor's rule (27).

The increase described in z in t leaves the IS curve unchanged in that period, as observed in equation (28). If at that time $Y_t=Y^*$, the system remains in equilibrium. However, in the following period the increase in z_t will have a positive effect on income. This can be observed because in the second term on the right side of equation (28) z_{t-1} has a positive effect on Y in the numerator.

Following the above, the increase in Y will lead to inflation and that will raise interest rates, but these rates have, eventually, a positive net effect on output. Therefore, inflation will continue to rise, the real interest rate as well and consumption, income and public debt will do so in an unstable and explosive process.

When any other parameter increases, the proposed Taylor rule will also cause instability. For example, if public spending (g) rises, there will be a positive effect on output that will generate an acceleration of inflation, with the Taylor rule established an increase in the real interest rate will be generated that will eventually lead to an explosive cycle of increases in consumption, income, debt and inflation.

An interest rate rule that complies with the Taylor principle does not stabilize the model when ρ is equal to one, because the income effect of an interest rate increase dominates the substitution effect almost from the short term.

One way to achieve stability is to propose a rule in which the real interest rate falls when inflation rises, so that:

$$r_t = z_t - \Omega \pi_t \quad (29) \text{ (Note 7)}$$

In this context, an increase in spending will lead to an increase in output and inflation. Instead of raising the real interest rate, it will go down and that will have a negative net effect on consumption, which will compensate, at least in part, for the initial inflationary effect and stabilize the system (Note 8).

When the interest rate rule is that established in (29) and $\rho=1$, aggregate demand is expressed as:

$$Y_t = \frac{(1+z_{t-1}-\Omega\pi_{t-1})Y_{t-1}}{(1+\tau(1+\theta))} + \frac{(2+\theta)g_t}{(1+\tau(1+\theta))} \quad (30) \text{ (Note 9)}$$

The interaction between demand (30) and the Phillips curve (25) solves in a stable way for inflation and income, since equation (30) depends negatively on past inflation. In this case, the model is very similar to the semi-reduced form of the so-called Romer-Taylor model (Romer, 2000; Taylor, 2000).

It is possible to calculate the value of long-term inflation, which arises from solving for $Y_t=Y_{t-1}=Y^*$ when $z_t=z_{t-1}=z$ $\pi_t=\pi_{t-1}$ in (30) and then for inflation, so that

$$Y_t = Y^* = \frac{(2+\theta)g_t}{\tau(2+\theta+z_t+\Omega\pi_t)-(z_t+\Omega\pi_t)} \quad (31)$$

Solving for π en (31)

$$\pi_t = \pi^* = \frac{(2+\theta)(g_t-\tau Y^*)+z(1-\tau)Y^*}{\Omega Y^*} \quad (32)$$

Long-term inflation π^* depends on four elements: the fiscal determined by the primary deficit $g-\tau Y^*$ and the income tax rate τ ; the monetary determined by the parameters of the monetary rule (29) z and Ω ; and the structural factors that determine the long-term output Y^* .

The primary deficit and the independent value of the interest rate rule have a positive effect on long-term inflation. The income tax rate will have a negative effect if z is greater than zero. On the other hand, higher equilibrium output will reduce long-term inflation, because:

$$\frac{d\pi_t}{dY^*} = -\frac{(2+\theta)g_t}{\Omega Y^{*2}} < 0 \quad (33)$$

The effect of parameter Ω of the interest rate rule will be negative when equilibrium inflation is positive, since

$$\frac{d\pi_t}{d\Omega} = -\frac{((2+\theta)(g_t-\tau Y^*)+z(1-\tau)Y^*)}{\Omega^2 Y^*} < 0 \quad (34)$$

By (32) it is possible to know that equilibrium inflation is positive when $(2+\theta)(g_t-\tau Y^*)+z(1-\tau)Y^*>0$.

With an elasticity of substitution in consumption equal to one, the policy of increasing the interest rate through an increase in z is inflationary both in the short term and in the long term. Also, a Taylor-type rule destabilizes the system. All this constitutes unpleasant arithmetic.

If the elasticity of substitution in consumption were less than the unit, the problem analyzed in this section would most likely be exacerbated, since in that case an increase in the real interest rate increases consumption in the short term as well as in the long run. That is, the income effect dominates the substitution effect at all times (Note 10).

5. Monetary Policy When the Elasticity of Substitution in Consumption Is Greater than One

Perhaps the most interesting case occurs when the elasticity of substitution in consumption is greater than one, because there an increase in the real interest rate reduces present consumption, generating a drop in aggregate demand and inflation at least in short term.

Even considering the previous explanation, the effect on future consumption of an increase in the present interest rate may be positive. In fact, equation (22) of the simple Keynesian model in this text shows that in most cases long-term income responds positively to the increase in the real interest rate, which suggests that even in the case where ρ is greater than one Taylor's rule can destabilize the system.

We carry out a simulation of the equations IS (18), the Phillips curve (25) and the Taylor monetary policy rule (27), as well as the modified one (29). In this case, starting in a period which we establish as the year 2019 the independent parameter of the interest rate rule increases by 10 percentage points (0.1) in 2021.

The values of the initial z parameter were calibrated so that the two exercises start from the same steady state values for output, inflation and the real interest rate. The periods were considered years. As we said before the starting year is 2019.

The assumptions and values of the steady state are shown in Table 1:

Table 1. Assumptions, Steady State and Equilibrium Values of the Simulation Exercise When the Elasticity of Substitution of Intertempora Consumption Is Greater than One

Parameters of the model	With the Taylor' rule (27)	With the modified rule (29)
Elasticity of substitution in consumption ρ	4	4
Subjective discount rate θ	0.1	0.1
Income's tax rate τ	0.2	0.2
Public expenditure	2	2
Response of inflation to the output gap in the Phillips curve γ	0.1	0.1
Natural output Y^*	12.35	12.35
Independent parameter of the monetary rule z (initially)	0	0.2

Parameter that multiplies inflation in the monetary rule Ω	0.1	-0.1
Steady state values		
Inflation	1.0 (100%)	1.0 (100%)
Output	12.35	12.35
Real interest rate	0.1	0.1
Equilibrium values of the simulation		
Inflation	There is not equilibrium it grows without limit.	2.0 (200%)
Output	There is not equilibrium it grows without limit.	12.35
Real interest rate	There is not equilibrium it grows without limit.	0.1

Perhaps the most surprising result of these exercises is that even if an increase in the short-term interest rate will lead to a fall in consumption, income and inflation, in equilibrium the traditional Taylor rule will lead to a situation that does not reach a steady state and where the reversal of greater future consumption leads to an increasing and unstable inflation.

Another result of interest is that in the case in which the modified monetary rule operates (29) the system returns to equilibrium, but there is, in any case, a permanent increase in inflation, indicating that the reversal of future consumption over compensates for the initial effect of a drop in present consumption, as was also the case in the simple Keynesian model.

The trajectories of output, inflation and the real interest rate are shown in Figures 1, 2 and 3.

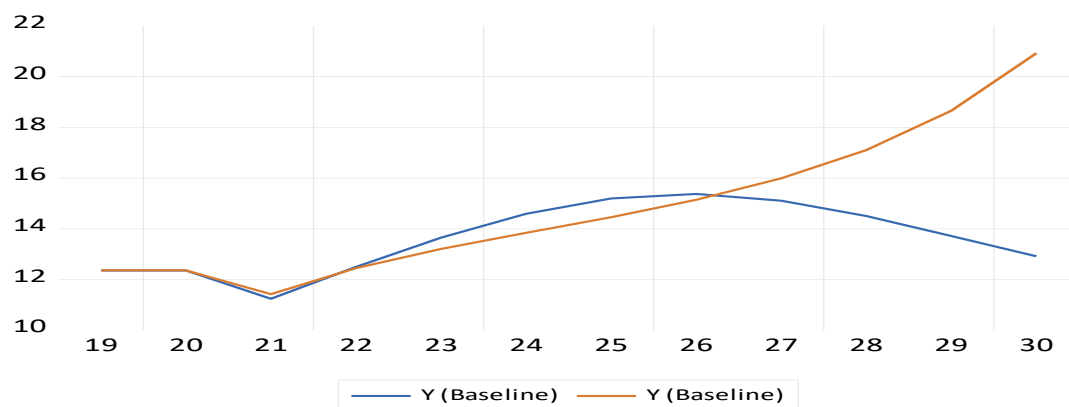


Figure 1. Output's Trajectory When the Independent Value of the Interest Rate Rule Increases and the Elasticity of Substitution in Consumption Is Four (4)

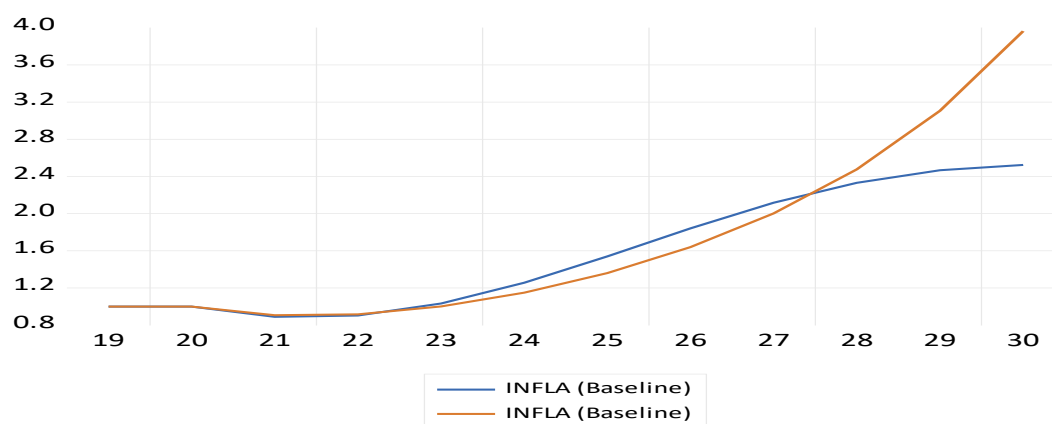


Figure 2. Inflation Trajectory When the Independent Value of the Interest Rate Rule Increases and the Elasticity of Substitution in Consumption is Four (4)

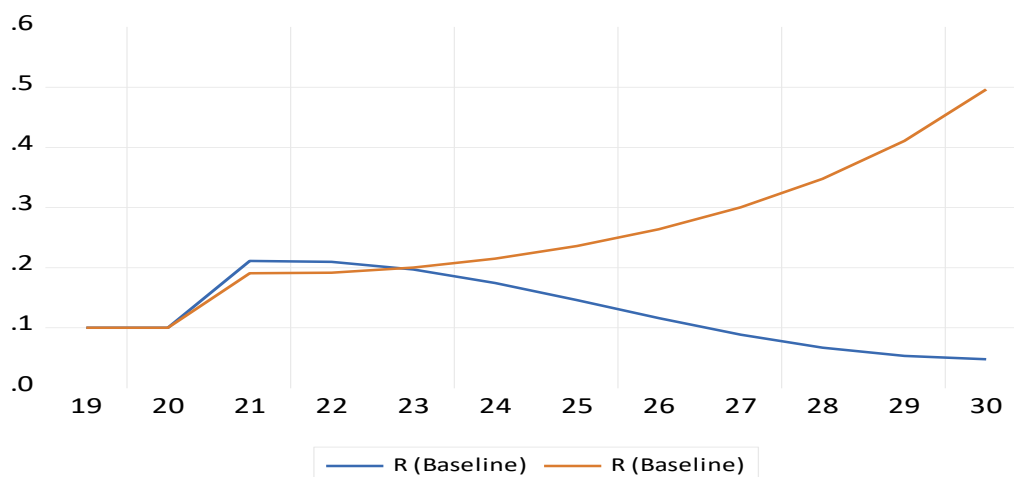


Figure 3. Real Interest Rate Trajectory When the Independent Value of the Interest Rate Rule Increases and the Elasticity of Substitution in Consumption Is Four (4)

In the particular case of inflation, it is advisable to analyze the short run effects in Figure 4.

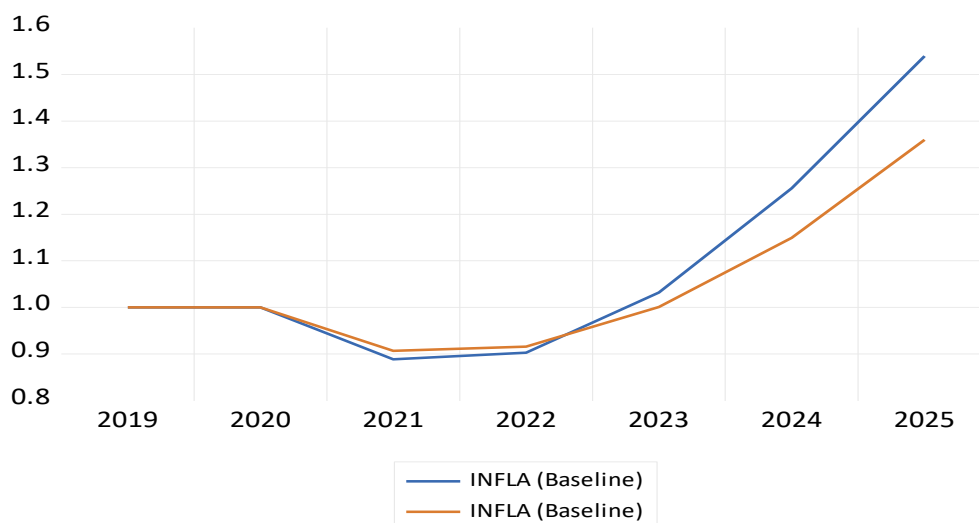


Figure 4. Short Run Trajectory of Inflation When the Independent Value of the Interest Rate Rule Increases and the Elasticity of Substitution in Consumption Is Four (4)

Orange: Trajectories with Taylor's type rule (equation (27))

Blue: Trajectories with modified rule (equation (29))

The increase in the real interest rate originally has the effects that are normally assumed to occur: inflation falls from a level of 100% to a level close to 90% both under a normal Taylor rule and under a modified rule. After the first year, it begins to rise and returns to the level it was originally two years later. From there it continues to rise in both cases, although in the case of having a modified monetary rule it stabilizes and in the other case it continues growing without limits.

Again, in this case the problem of unpleasant arithmetic appears: a seemingly restrictive monetary policy leads to an increase in long-term inflation.

6. The Necessary Fiscal Adjustment When There Is Instability

An unstable solution, in which output, inflation, the real interest rate and public debt are increasing steadily cannot last forever. Sargent and Wallace (1981) establish a maximum level where public debt may reach. It would be also possible to set a limit level for output, because resources are limited. Suppose that, in the situation described in the previous section, when the central bank raises the independent component of the Taylor type interest rate rule z_t , a time after the system begins to enter an unstable route the government adjusts the expenditure so that the debt remains constant. At that time this adjustment does not affect old people, because according to equation (4) of this text:

$$(1 + r_{t-1})b_{t-1} = C_t^b \quad (35)$$

Old people receive a capitalized debt that is already given at an already agreed rate. The adjustment will not affect the generation that was born the previous period, but those that were born the period in which the described adjustment is made and those that continue. Therefore, the aforementioned adjustment does not affect all previous generations at all and, then, everything that happened before remains valid (Note 11).

At some point, with the Taylor rule in effect, the government sets the expenditure g so that the public deficit is zero, which implies:

$$g_t = \tau Y_t - r_{t-1} b_{t-1} \quad (36)$$

By equations (17) and (35) this means

$$g_t = \tau Y_t - \frac{r_{t-1}(1-\tau)Y_{t-1}}{(1+(1+\theta)^\rho(1+r_{t-1})^{1-\rho})} \quad (37)$$

This equation is added to the IS (18), the Phillips curve (25) and the Taylor type rule (27) from the moment the government makes the adjustment.

In the simulation shown in the previous section, when the central bank raises the z parameter of the Taylor-type rule (27), a period begins where inflation and output go down and then, when they go up, they do so unstably. The parameter z is supposed to increase in 2021 and by 2023 inflation begins to increase. In 2024 it is already above where it began in 2019. In this new simulation we assume that the government makes the adjustment that involves equation (37) starting in 2025.

The results for output, inflation, the real interest rate and public expenditure from 2020 onwards are shown in Figures 5, 6, 7 and 8. These results are shown together with those that appear in the previous section for the case using the modified monetary policy rule (equation (29)), but in this one with a greater time interval (Note 12).

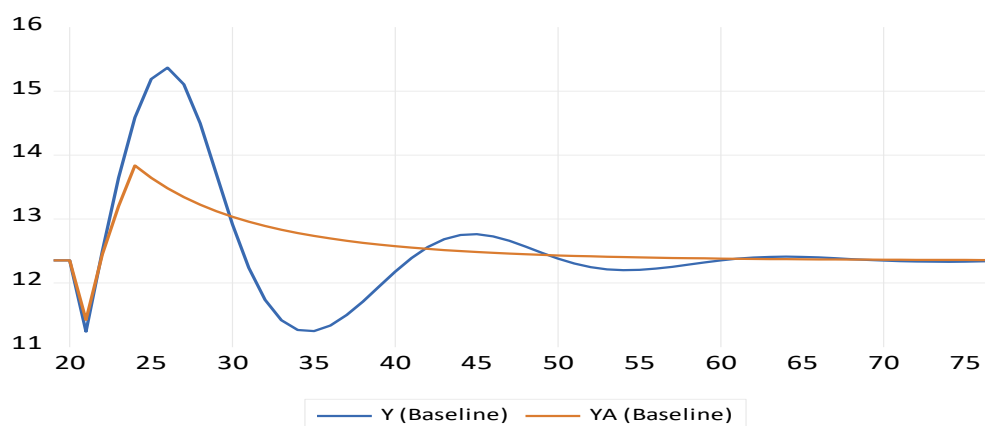


Figure 5. Output Trajectories

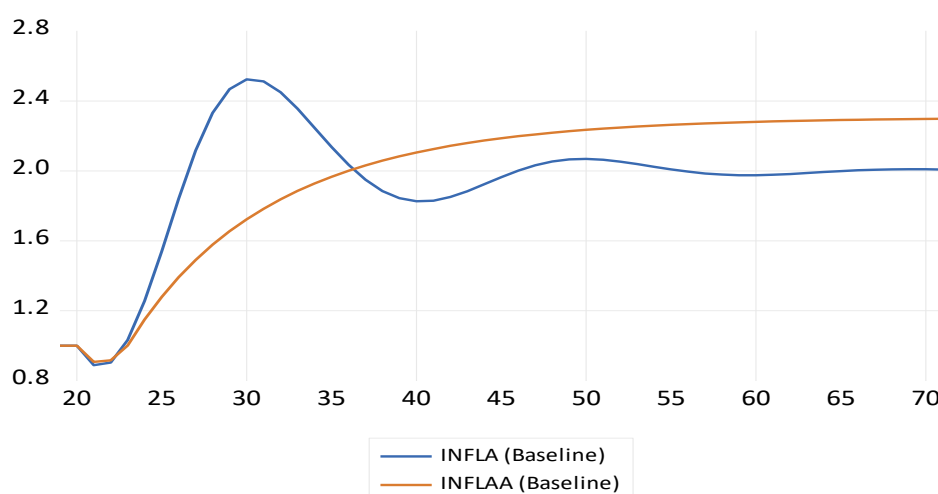


Figure 6. Inflation Trajectories

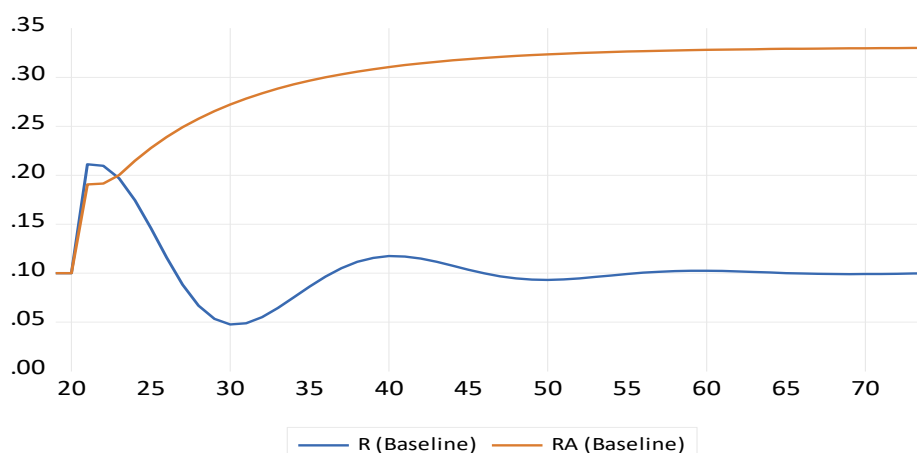


Figure 7. Trajectories for the Real Interest Rate

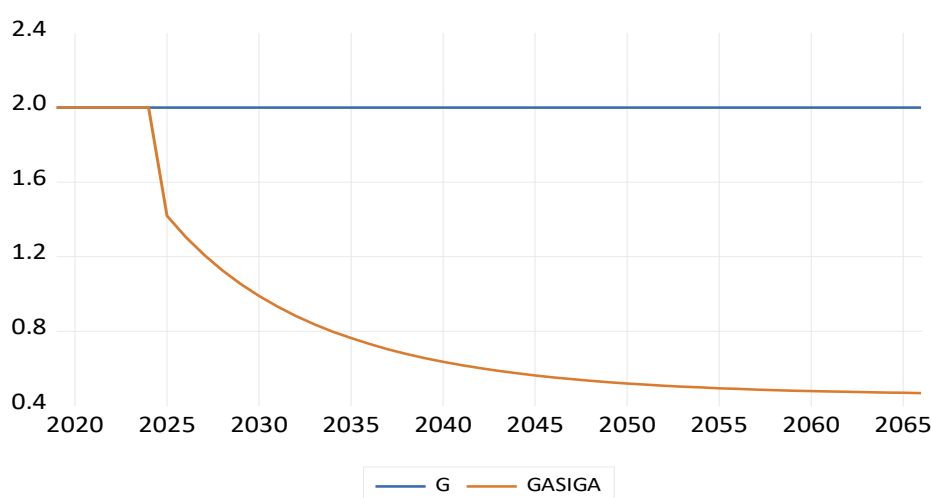


Figure 8. Trajectories for Government Expenditure

Blue: trajectories with the modified monetary policy rule (29)

Orange: trajectories with the Taylor's rule (27) and the adjustment of public expenditure starting in 2025 (equation (37)).

The fiscal adjustment of expenditure through equation (37) generates stability in the macroeconomic system. Unlike the case where the modified monetary policy rule is used, the adjustment does not generate oscillations (see figures in orange and blue). However, this type of adjustment results in higher long-term inflation than what occurs when the modified rate rule is used. On the other hand, public spending has to be reduced to more than half to stabilize the system (see orange line in Figure 8).

It should be noted that if the adjustment were made after 2025, the public expenditure that balances the mathematical model would be negative, which from the economic point of view cannot occur. However, this would imply the need to receive transfers from abroad to generate imports that would be the ones that, together with the fall in public spending, would generate stability.

It should also be noted that an exercise was attempted where the variable that balances the deficit is the tax rate, so that in equation (37) what clears is τ . The results in that case show possibly chaotic trajectories of the endogenous variables of the model.

7. Conclusions

This work finds that, when the government maintains its fiscal policy parameters and inflation is inertial, an increase in the real interest rate, which may be temporary or permanent, will, in general, lead to higher inflation in the long term. Moreover, the establishment of a monetary policy rule that contains the so-called Taylor principle will be destabilizing. On the other hand, a rule that does not comply with this principle, but allows the real interest rate to fall when inflation rises, will stabilize the system. Pegging the nominal interest rate constitutes a type of stabilizing rule.

The main reason for the phenomenon described is the long-term effect of the real interest rate on total consumption, which, in case the private sector is creditor of the public sector and the real interest rate is greater than zero, it will be positive in the long term. So even if the substitution effect exceeds the income effect in the short term and consumption falls because of an increase in the interest rate, the future rebound of this aggregate will eventually lead to higher inflation.

The results of the work are undoubtedly related to the assumptions of the model. In particular, the fact that the private sector is a creditor of the public sector and also that only the first period of people's lives is productive. Modifying these assumptions could change the conclusions in the same context of a model of overlapping generations.

Another problem of consideration is that there is no investment in this model. Including it may lead to the traditional solution, in which aggregate demand falls in the present and future in the face of an increase in the real interest rate.

In models of a representative agent that has an infinite horizon of time, and where there is a context of inertial inflation, it has been concluded that the establishment of a monetary rule that complies with the Taylor principle will be stabilizing. An increase in the independent parameter of the interest rate rule will generate a temporary increase in the real interest rate, a temporary recession and a permanent fall in inflation (Koenig, 2008). This happens whenever the elasticity of substitution in intertemporal consumption is positive, including cases in which it is less than one. Why in this context does the rebound in consumption not cause a reversal of the fall in inflation as in the model of overlapping generations analyzed in this work?

Perhaps the answer to the question in the previous paragraph is that in those contexts it is assumed, at least implicitly, that the government carries out or will carry out a fiscal adjustment that will prevent the public debt from destabilizing, something that does not happen in the context analyzed in this work. Therefore, it seems relevant to analyze how the solution would be in the case of the representative agent when the fiscal policy parameters remain constant, as is the case in this paper.

The premise that higher interest rates reduce inflation is almost a religion, especially for the general public. In the last years, some academic papers have shown that raising interest rates may be inflationary because of a cost channel (see Chowdhury et al., 2006; Tillman, 2008). If the Phillips curve is related positively to the real interest rate, then a seemingly restrictive monetary policy may lead to higher inflation. Other works show that pegging nominal interest rates in an environment where inflation is almost zero may yield to stability, a result that is found also in this work. Moreover, in this situation when interest rates increase, inflation also rises (see Cochrane, 2016). This paper shows another channel by which higher interest rates may increase inflation: the response of consumption to the real interest rate, which may be even positive in the short term but that almost always is greater than zero in the long run.

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Notes

Note 1. Other authors analyzed the problem of unpleasant arithmetic, among them Leviatan (1984) and Drazen (1985).

Note 2. It should be noted that Drazen (1985) shows that there are times when a reduction in inflation does not require a fiscal adjustment. In particular, this happens when the target inflation is greater than zero and the elasticity of the demand for money with respect to inflation is, in absolute value, greater than unity. Drazen (1985) also suggests that, in some cases, even if the demand for money has an elasticity with respect to inflation lower than one, inflation reduction may not be accompanied by fiscal adjustment if open market operations are carried out.

Note 3. The problem arises as follows: If the nominal interest rate rises and initially the inflation rate remains constant, the real rate also increases and demand falls, which then reduces inflation. However, if the nominal rate does not change more, the real rate has an additional increase, which again reduces demand, causing inflation to fall further and increasing the real rate again in a cycle where the real rate rises in an explosive way and inflation falls the same way.

Note 4. See: Clarida, Gal í and Gertler (1999), McCallum and Nelson (1999), King (2000), Romer (2000), Taylor (2000), Woodford (2001, 2003), Blanchard (2008), Koenig (2008).

Note 5. A relatively recent work by John Cochrane (2016) shows cases where higher nominal interest rates generate higher inflation. This work also shows that pegging the nominal interest rate may stabilize the macroeconomic system, which goes against Wicksell's cumulative causation problem.

Note 6. Numerous problems have been addressed in models of overlapping generations: pensions, public debt, economic growth and dynamic macroeconomics (see for example Diamond (1965), Lucas (1972), Azariadis (1981), Sargent and Wallace (1981), Cass and Shell (1983), Blanchard (1985), Hahn (1985), Hahn and Solow (1995), Geanakoplos (2008)). On the subject of our interest is of course the work of Sargent and Wallace (1981). Likewise, Otaki (2007) obtains several of the propositions of the simple Keynesian model. Gal í(2017) analyzes the emergence of speculative bubbles in this type of models.

Note 7. When the Ω parameter is equal to the one, the monetary policy rule is simply to set the nominal interest rate.

Note 8. We carry out a large number of simulations not reported with both the Taylor type rule (27) and the modified rule (29). In all the cases analyzed, the exercises using the Taylor-type rule were unstable and those that assumed the modified rule achieved stable results.

Note 9. Making $\rho=1$ in (35) and solving for Y_t .

Note 10. Also, in the case where $\rho<1$ all the simulation exercises we carried out were unstable with the Taylor type rule and stable with the modified rule.

Note 11. This would not happen in a Ricardian model in which a planned adjustment in the future would modify the behavior of economic agents who have an infinite life before it is realized.

Note 12. In the graphs of the previous section, the time interval is more limited because the solutions of the unstable model increase at high speed, which makes putting more periods completely lose the results of the stable solution and all the details that happen in the short term.