# Original Paper 

# Stock Return Autocorrelation and Individual Equity Option 

## Prices

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#### Abstract

This study demonstrates empirically the impact of stock return autocorrelation on the prices of individual equity option. The option prices are characterized by the level and slope of implied volatility curves, and the stock return autocorrelation is measured by variance ratio and first-order serial return autocorrelation. Using a large sample of U.S. stocks, we show that there is a clear link between stock return autocorrelation and individual equity option prices: a higher stock return autocorrelation leads to a lower level of implied volatility (compared to realized volatility) and a steeper implied volatility curve. The stock return autocorrelation is more important in explaining the level of implied volatility curve for relatively small stocks. The relation between stock return autocorrelation and option price structure is more pronounced when market is volatile, especially during financial crisis. The stock return autocorrelation is more important in explaining the level of implied volatility curve for relatively small stocks. Thus, stock return autocorrelation can help differentiate the price structure across individual equity options.


## Keywords

Autocorrelation, Implied Volatility, Realized Volatility, Slope of Implied Volatility Curve

## 1. Introduction

Empirical studies have uncovered some intriguing features, among others, of index and individual equity option prices: (i) the option implied volatility is higher than the realized volatility (e.g., Rubinstein, 1994; Jackwerth \& Rubinsten, 1996; Carr \& Wu, 2009; Duan \& Wei, 2009), and (ii) the implied volatility curve consistently exhibits pronounced smile effects (see Jackwerth \& Rubinsten, 1996; Bates, 2000; Bakshi et al., 2003; Yan, 2011; Xing et al., 2011).
The asset returns are assumed to be distributed independently of each other, in various option pricing models, including (i) jump-diffusion model of Merton (1976), Bates (1991), etc., (ii) the stochastic volatility models of Heston (1993), Hull and White (1987) etc., (iii) stochastic volatility and jump-diffusion models of Bates (1996), etc. However, the evidence of persistence autocorrelation in asset returns of both the short-term (see, for example, Lo and Mackinlay (1988, 1990), Conrad and Kaul (1988)) and long-term period (see, for example, Fama and French (1988) and Poterba and Summers (1988)) contradicts the assumption made in the option pricing models. Following this line of research, literature documents that stock return autocorrelation enters into the option pricing formula through
adjustments in volatility and/or expected asset price (Lo and Wang (1995), Jokivuolle (1998), Mezrin (2004), Liao and Chen (2006), etc.).

It is an empirical question, as to the extent that stock return autocorrelation affects option prices. To the best of our knowledge, there exists no empirical studies exploring the relative contribution of stock return autocorrelation in option pricing. In this paper, we fill the gap by investigating the impact of stock return autocorrelation on the individual equity option price structure and demonstrate a clear link between them. Our hope is that a better understanding of the sources of option price structure will help guide us in the future as we work to improve option pricing models and option trading strategies.
Following, Lo (2004), Griffin, Kelly, and Nardari (2010), and Cao et al. (2018), we employ two measures of stock return autocorrelation: variance ratios and first-order stock return autocorrelation. We start by testing the random walk hypothesis for optional stocks. At the $10 \%$ ( $5 \%$ ) significance level, the random walk assumption is rejected for about one third (one quarter) of stocks in our sample, suggesting the discrepancy between the assumption of the option pricing model and the data. Using 6,137 stocks from $01 / 1996$ to $04 / 2016$, we demonstrate a clear link between the stock return autocorrelation and individual equity option prices. Specifically, stocks with higher return autocorrelation exhibit a lower level of implied volatility (compared to realized volatility) and a steeper implied volatility curve. We find that the stock return autocorrelation is more important in explaining the level of implied volatility curve for relatively small stocks. We also find that the relation between stock return autocorrelation and option price structure is more pronounced when market is volatile, especially during financial crisis. The level of stock price autocorrelation can help differentiate the price structure across individual equity options.
We contribute to the extant literature which documents the impact of stock return autocorrelation on option prices empirically. Lo and Wang (1995) demonstrate that option value is a function of the absolute value of the first-order autocorrelation coefficient, with the increase in autocorrelation decreasing Black-Scholes option prices. They also suggest that predictability also affects option prices nontrivially for option pricing models with stochastic volatility or jump component. By modeling the index autocorrelation by the ARMA model and incorporating the autocorrelation into Rubinsten (1996) results, Jokivuolle (1998) shows that the autocorrelation enters into the option pricing formula by adjusting volatility and underlying index value, both of which enters into option pricing models. Mezrin (2004) incorporates stock return autocorrelation into Black Scholes Model and demonstrate that return autocorrelation affects the volatility and expected asset price, therefore, the option prices. By allowing stock returns to follow a first order moving average process, Liao and Chen (2006) show that the impact of autocorrelation is significant to option prices even when the autocorrelation between asset returns is weak.

Our paper also contributes to the strand of empirical studies that examine the factors or firm characteristics related to individual equity option prices. Duan and Wei (2009) illustrate the impact of the systematic risk of stocks on the option prices. With daily option quotes on the S\&P 100 index and its 30 largest component stocks, they show that a higher amount of systematic risk leads to a steeper implied volatility curve and a higher level of implied volatility. Using data on 1,421 individual firms, Dennis and Mayhew (2002) find that firm size, $\beta$, and stock trading volume help explain the risk-neutral skewness. Chou et al. (2011) illustrate the impact of both spot and option liquidity levels on option prices using options on component stocks of the DJIA Index. With a decrease (increase) in stock (stock option) liquidity, there is an increase in the level of the implied volatility curve. Our empirical results demonstrate that even after controlling for those variables, a clear link remains
between the individual equity prices and stock return autocorrelation.
The rest of the paper is organized as follows. Section 2 discusses variable definitions. Section 3 presents our empirical results including univariate soring, Fama-Macbeth regressions and robustness checks. Section 4 concludes.

## 2. Variable Construction

### 2.1 Option Price Structure

Following An et al. (2014) and Yan (2011), we use the interpolated volatility surface computed by OptionMetrics to construct individual equity option price structure. OptionMetrics computes option implied volatility using binomial trees and the interpolated volatility surface is then constructed using a kernel smoothing algorithm. One advantage of using the volatility surface is that it avoids having to make potentially arbitrary decisions on which strikes or maturities to include in computing an implied call or put volatility for each stock (see An et al., 2014). Since we are looking at the monthly frequency, we use the 30-day interpolated volatility surface at the last trading day of each month following An et al. (2014).

Bollen and Whaley (2004) use the Black-Scholes deltas to measure moneyness. Based on their deltas, options are placed into five moneyness categories. Yan (2011) uses options whose deltas equal to the average of the upper and lower bound of each moneyness category, as in Bollen and Whaley (2004), to define the moneyness of options. Following Bollen and Whaley (2004) and Yan (2011), we use standard options on the implied volatility surface with deltas equal to $0.5,-0.5$, and -0.25 as ATM calls, ATM puts, and OTM puts on the last trading day of each month.

### 2.1.1 IVol - RVol: Implied-Realized Volatility Spread

The options implied volatility, $I V o l_{i, t}$, for stock $i$ in month $t$, is defined as the average implied volatility of ATM calls and ATM puts, with deltas equal to 0.5 and -0.5 on the implied volatility surface on stock $i$ at the end of month $t$, respectively. Follow Bali and Hovakimian (2009) and Ang et al. (2006), the realized volatility, $R V o l_{i, t}$, of each month is calculated as the annualized standard deviation of daily returns over the past 12 months. We then calculate the spread between implied and realized volatility as follows.

$$
I V o l_{i, t}-R V o l_{i, t}=\frac{I V o l_{i, t}^{A T M C}+I V o l_{i, t}^{A T M P}}{2}-R V o l_{i, t}
$$

### 2.1.2 Slope: Slope of Implied Volatility Curve

Following Xing et al. (2010), we define the slope of implied volatility curve, Slope, as the difference between the implied volatility of options of ATM calls and OTM puts. To be specific, we define the Slope $_{i, t}$, for stock $i$ in month $t$, as the difference of the implied volatility of options with delta of 0.50 (ATM calls) and -0.25 (OTM puts) on the implied volatility surface on stock $i$ at the end of month $t$.

$$
\text { Slope }_{i, t}=I V o l_{i, t}^{A T M C}-I V_{i, t}^{A T M P}
$$

### 2.2 Measures of Stock Return Autocorrelation

Following Lo and MacKinlay (1988), Lo (2004), Griffin, Kelly, and Nardari (2010), and Cao et al. (2018), we derive two measures of stock return autocorrelation from stocks' daily quotes: variance ratios and first-order stock return autocorrelation. Those two measures capture patterns in stock returns discrepancies between the variance of long-term and short-term returns and serial dependency in stock returns, respectively.

### 2.2.1 Variance Ratio

Our main measure of stock return autocorrelation is variance ratio, proposed by Lo and MacKinlay (1988). An important property of stock return independence is that the variance of its increments must be proportional to the time interval over which the returns are sampled. In line with Lo and MacKinlay (1988) and Cao et al. (2018), we compute the following main measure of stock return autocorrelation:

$$
\begin{equation*}
\operatorname{Dev}_{-} V R_{i, t}=\left|1-\frac{\sigma_{2}^{2}}{2 \sigma_{1}^{2}}\right| \tag{1}
\end{equation*}
$$

where $\sigma_{2}^{2}$ and $\sigma_{1}^{2}$ are the return variances measured over 2-weeks and 1-week intervals.
The weekly stock returns are derived from the CRSP daily returns file. Following Lo and MacKinlay (1988), the weekly return of each security is computed as Wednesday-Wednesday return. If the following Wednesday's closing price is missing, then Thursday's closing price (or Tuesday's if Thursday's is also missing) is used. If both Tuesday's and Thursday's closing prices are missing, the return for that week is reported as missing.
This measure captures the absolute deviation of the ratio of return variances measured over 2 weeks to those measured over 1 week from 1 , which is the expected value of the ratio under the random walk hypothesis. Greater deviation of the variance ratio from 1 signals higher serial return autocorrelation. Balancing between estimation efficiency from a larger sample and the relatively shorter option maturities, we opt for a one-year ( 52 weeks) rolling window. Specifically, at the end of each month, we calculate variance ratio and the absolute deviation of variance ratio from 1 using return data in the past one year ( 52 weeks).

### 2.2.2 First-order Stock Return Autocorrelation

As an alternative measure of stock return autocorrelation, we also examine the absolute value of first-order monthly stock return autocorrelation with 3-year rolling windows, following Lo and Wang (1995), Lo (2004) and Cao et al. (2018). Following Lo and Wang (1995), we focus on the absolute value of the autocorrelation to avoid confusion in making comparisons between results for negatively autocorrelated and positively autocorrelated asset returns. Higher absolute value of autocorrelation proxies for greater deviation from random walk.

### 2.3 Control Variables

In order to rule out the possible effects on option prices from other firm-specific characteristics and factors, we include a number of control variables in our empirical tests.
Duan and Wei (2009), Dennis and Mayhew (2002) and Chou et al. (2011) document that systematic variance risk ratio, firm size, $\beta$, stocks' trading volume, spot and option liquidity can explain option prices. Following Duan and Wei (2009), we assume a standard one-factor market model for stock $i$. At the end of each month, we run daily, one-year rolling window OLS regression and estimate the systematic risk proportion as the $R^{2}$. We calculate a stock's $\beta$ by regressing the stock's excess return on the excess return of S\&P 500 for the past 24 months. Following Fama and French (1992), the firm size from July of year $t$ to June of year $t+1$ are measured based on the market equity in June of year $t$. Stock trading volume is defined as the total number of shares traded in the month. We adopt 12-month moving average Amihud (2002) illiquidity ratio to proxy for stock illiquidity. For each firm, we sum the trading volumes of all options that meet our requirements from each day in each month, as our measure of option liquidity.

## 3. Empirical Results

### 3.1 Summary Statistics

Our sample runs from January 1996 to April 2016. Options data are from OptionMetrics. Stock data are from CRSP, and accounting data are from Compustat. We first calculate the summary statistics over the cross section for each month, and then average the statistics over the monthly time series. There are 449,849 observations in our sample in total. The total number of stocks is 6,137 and the average number of stocks per month is 1,851 . Table 1 presents the summary statistics of the variables used in our empirical tests over the sample period from January 1996 to April 2016. We first calculate the summary statistics over the cross section for each month, and then average the statistics over the monthly time series.
Panel A reports the summary statistics on option price structures, namely $\mathrm{IVol}-\mathrm{RVol}$ and Slope. Several observations are in order. First, implied volatility is about $1.60 \%$ greater than realized volatility on average, consistent with Duan and Wei (2009) and Carr and Wu (2009). Second, the volatility smile is clearly present for majority of stocks in our sample. The curve is mostly downward sloping. The sample average of slope is $-4.81 \%$, consistent with Xing et al. (2010), whose sample average Slope is $6.4 \%$ in the period of January 1996 to December 2005. Panel B reports the summary statistics of measures of stock return autocorrelation. On average, the biweekly return variance is about 2.26 times of the weekly return variance. The mean and median of |Autocorrelation| are 0.10 and 0.12 respectively. Panel C describes the summary statistics on the control variables, including systematic risk proportion, firm size, $\beta$, stock trading volume, Amihud illiquidity ratio and option trading volume. The mean and median systematic risk proportion is 0.22 and 0.20 respectively, consistent with Duan and Wei (2009). The mean/median firm size in our sample is $\$ 4.30 / 1.04$ billion, whereas the mean/median firm size is $\$ 2.53 / 0.20$ billion for all the stocks in CRSP. The mean and median stock trading volume for firms in our sample are 28.09 and 8.91 million shares, respectively, comparing with 14.98 and 1.99 million shares for all stocks in CRSP. It is not surprising that firms in our sample are bigger and more liquid than the average firm trading in the stock market, since we utilize stocks with options. Firms in our sample have an average $\beta$ of 1.3.

Table 1. Summary Statistics
Panel A: Option Price Structure

|  | Mean | 5\% | 25\% | 50\% | 75\% | 95\% | S.D. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IVol - RVol (\%). |  | 1.60 |  | -22.39 | 7.09 | 27.30 | 19.99 |
| -4.99 0.93 |  |  |  |  |  |  |  |
| Slope (\%) | -4.81 | -22.08 | -7.43 | -3.50 | -0.75 | 7.51 | 13.76 |
| Panel B: Measures of Stock Return Autocorrelation |  |  |  |  |  |  |  |
|  | Mean | 5\% | 25\% | 50\% | 75\% | 95\% | S.D. |
| Variance Ratio | 1.13 | 0.62 | 0.82 | 0.97 | 1.11 | 1.34 | 8.07 |
| Dev_VR | 0.34 | 0.01 | 0.07 | 0.15 | 0.25 | 0.44 | 8.00 |
| \|Autocorrelation| | 0.10 | 0.01 | 0.06 | 0.12 | 0.20 | 0.33 | 0.10 |
| Panel C: Control Variables |  |  |  |  |  |  |  |
|  | Mean | 5\% | 25\% | 50\% | 75\% | 95\% | S.D. |
| Systematic Risk | 0.22 | 0.03 | 0.12 | 0.20 | 0.30 | 0.52 | 0.15 |
| Size (\$ Billion) | 4.30 | 0.11 | 0.39 | 1.04 | 2.98 | 16.70 | 13.30 |

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| $\beta$ | 1.31 | -0.04 | 0.68 | 1.19 | 1.83 | 3.11 | 1.02 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Stock Volume (in millions) | 28.09 | 1.16 | 3.80 | 8.91 | 23.04 | 103.93 | 92.46 |
| Amihud Illiquidity (\%) | 0.03 | 0.00 | 0.00 | 0.00 | 0.01 | 0.08 | 0.29 |
| Option Volume | 20286 | 18 | 212 | 1299 | 7977 | 77634 | 115599 |

### 3.1 Variance Ratio Test

We start our empirical studies with variance ratio tests. Following Lo and MacKinlay (1988) and Cao et al. (2018), we define the following estimator:

$$
J_{r}(q)=\frac{\sigma_{q}^{2}}{q \sigma_{1}^{2}}-1
$$

Where $\sigma_{q}^{2}$ and $\sigma_{1}^{2}$ are the return variance measured over $q$ weeks and 1 week intervals, respectively. In light of the Theorem 1 of Lo and MacKinlay (1988), for $q \geq 2$, the $z$ score is computed as

$$
Z_{q}=\sqrt{n q} \frac{J_{r}(q)}{\sqrt{2(q-1)}} \sim N(0,1)
$$

Where $n$ is the maximum number of non-overlapping $q$-week returns.
For each stock in our sample, we calculate the $z$ score using all the daily data. Table 2 report the percentage of stocks for which the random walk hypothesis is rejected, for $q$ from 2 to 16 , with weekly stock returns from January 1996 to April 2016. At $\$ 10 \%$ significance level, the random walk (stock return is unpredictable) hypothesis is rejected for about one third of stocks in our sample. At $5 \%$ significance level, the random walk hypothesis is rejected for about one quarter of stocks in our sample. The discrepancy between the data and the assumption of the option pricing model and the data indicates that the measure of stock return autocorrelation may help explain the individual equity option price structure.

Table 2. Variance Ratio Test

|  | $q$ | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Percentage of Stocks with | $10 \%$ | 34.16 | 36.28 | 34.37 | 33.58 | 32.89 |
| Rejection of Random Walk (\%) | $5 \%$ | 26.96 | 28.11 | 26.52 | 25.89 | 24.91 |
| Percentage of Stocks with | $10 \%$ | 32.98 | 33.34 | 32.98 | 31.60 | 32.41 |
| Rejection of Random Walk (\%) | $5 \%$ | 25.21 | 24.92 | 24.85 | 23.51 | 24.41 |
| Percentage of Stocks with | $10 \%$ | 31.16 | 31.68 | 33.40 | 30.86 | 33.11 |
| Rejection of Random Walk (\%) | $5 \%$ | 23.81 | 23.74 | 23.81 | 23.31 | 24.20 |

### 3.2 Fama-MacBeth Regressions

In this part, we use the Fama-MacBeth (see Fama and Macbeth, 1973) regressions to examine the cross-sectional relation between stock return autocorrelation and the option price structure with or without control variables. Table 3 and 4 present the Fama-MacBeth regression results of $\mathrm{IVol}-\mathrm{RVol}$ ānd Slope on Dev_VR, respectively, from January 1996 to April 2016. Column (1) of Table 3 and Table 4 report the univariate regression results of $\operatorname{VVol}-R V o l$ and Slope on Dev_VR. Column (2)-(5) report the regression results of $\mathrm{IVol}-\mathrm{RVol}$ and Slope on Dev_VR with control variables, including systematic risk proportion, firm size, $\beta$, stock trading volume, stock liquidity
(Amihud Illiquidity) and option liquidity (Option Volume).

Table 3. Fama-MacBeth Regressions of Implied and Realized Volatility Spread (IVol - RVol) on Measure of Stock Return Autocorrelation (Dev_VR)

|  | (1) | (2) | (3) | (4) | (5) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Dev_VR | $\begin{gathered} -0.031 \\ (-8.98) \end{gathered}$ | $\begin{aligned} & -0.032 \\ & (-9.78) \end{aligned}$ | $\begin{aligned} & -0.025 \\ & (-8.79) \end{aligned}$ | $\begin{aligned} & -0.033 \\ & (-10.12) \end{aligned}$ | $-0.027$ |
|  |  |  |  |  | (-10.84) |
| Systematic Risk |  | $\begin{aligned} & 0.059 \\ & (8.73) \end{aligned}$ |  |  | $0.012$ |
|  |  |  |  |  | (2.02) |
| Size |  |  | $\begin{aligned} & 0.000 \\ & (9.75) \end{aligned}$ |  | $\begin{aligned} & 0.000 \\ & (7.93) \end{aligned}$ |
| $\beta$ |  |  | $\begin{gathered} 0.016 \\ (6.86) \end{gathered}$ |  | $\begin{aligned} & 0.016 \\ & (6.78) \end{aligned}$ |
| Stock Volume |  |  | 0.000 |  | $\begin{aligned} & 0.000 \\ & (6.71) \end{aligned}$ |
| Amihud Illiquidity |  |  | (8.49) | $\begin{aligned} & -0.086 \\ & (-2.94) \end{aligned}$ | -0.084 |
|  |  |  |  |  | (-2.77) |
| Option Volume |  |  |  | 0.000 |  |
|  |  |  |  | (8.31) | -0.000 |
|  |  |  |  |  | (-3.07) |
| Adjusted $\mathrm{R}^{2}(\%)$ | 0.37 | 1.25 | 4.36 | 1.95 | 6.76 |

Before we interpret the regression results of $I V o l-R V o l$ on $D e v_{-} V R$ in Table 3, please note that the implied volatility slope is negative for most stocks in our sample. When we regress $\mathrm{IVol}-\mathrm{RVol}$ only on $\operatorname{Dev} v_{-} V R$, the coefficient estimate is -0.031 with a $t$-statistic of -8.98 , indicating that a stronger stock return autocorrelation leads to a lower $I V o l-R V o l$, a lower option implied volatility compared to realized volatility. To separate the explanatory power of stock return autocorrelation from other firm characteristics, we consider 6 control variables in regression (2)-(5). Inclusion of the control variables does not reduce the explanatory power of stock return autocorrelation on $\mathrm{IVol}-\mathrm{RVol}$. When we regress Slope only on $\operatorname{Dev} v_{-} V R$, as in column (1) of Table 4, the coefficient estimate is -0.005 with a statistically significant $t$-statistic of -3.26 . The coefficient of $\operatorname{Dev} V R$ remains negatively significant after we include control variables into the regressions, as in column (2)-(5). This indicates that a higher stock return autocorrelation leads to a steeper downward-sloping implied volatility curve. The coefficient of $D e v_{-} V R$ remains negatively significant after we include control variables into the regressions.

Table 4. Fama-MacBeth Regressions of Implied Volatility Slope (Slope) on Measure of Stock Return Autocorrelation (Dev_VR)

|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Dev_VR | $\mathbf{- 0 . 0 0 5}$ | $\mathbf{- 0 . 0 0 5}$ | $\mathbf{- 0 . 0 0 4}$ | $\mathbf{- 0 . 0 0 4}$ | $\mathbf{- 0 . 0 0 4}$ |
|  | $\mathbf{( - 3 . 2 6})$ | $\mathbf{( - 3 . 3 4 )}$ | $\mathbf{( - 2 . 9 6 )}$ | $\mathbf{( - 3 . 2 4 )}$ | $\mathbf{( - 2 . 9 7 )}$ |
| Systematic Risk |  | 0.000 |  |  | $\mathbf{0 . 0 0 6}$ |
| Size |  | $(0.09)$ |  |  | $\mathbf{( 3 . 1 5 )}$ |
|  |  |  | $\mathbf{0 . 0 0 0}$ |  | $\mathbf{0 . 0 0 0}$ |
| $\beta$ |  |  | $\mathbf{( 1 2 . 5 6}$ |  | $\mathbf{( 1 4 . 4 1 )}$ |
|  |  |  | 0.000 |  | $\mathbf{- 0 . 0 0 1}$ |
| Stock Volume |  |  | $(0.03)$ |  | $\mathbf{( - 2 . 1 7 )}$ |
|  |  |  | 0.000 |  | $\mathbf{0 . 0 0 0}$ |
| Amihud Illiquidity |  |  |  |  | $-0.14)$ |
|  |  |  |  | $(-1.01)$ | $(0.19)$ |
| Option Volume |  |  |  | -0.000 | $\mathbf{- 0 . 0 0 0}$ |
|  |  |  |  | $(-1.07)$ | $\mathbf{( - 6 . 8 7 )}$ |
| Adjusted $R^{2}$ (\%) | 0.08 | 0.22 | 0.57 | 0.57 | 1.24 |

### 3.3 Univariate Sorting

To gauge the magnitude of impact of stock return autocorrelation on option price structure, we perform univariate sorting. On the last trading day of each month, stocks are ranked, in ascending order according to $D e v_{-} V R$, into quintiles, and five portfolios are formed by equally weighting the stocks within that quintile. We then record the implied and realized volatility spread and the slope of implied volatility curves. Repeating these steps for every month in the sample period of January 1996 to April 2016 generates the time series of monthly options price structure for the five quintiles. We then calculate the time-series average of the monthly portfolio option prices and report them in Table 5. Each quintile portfolio has 370 stocks per month, on average. Portfolio Q1 contains stocks with lowest Dev_VR, which is only 0.03 for Q 1 , indicating that the variance ratio for stocks in Q 1 is very close to 1 . Portfolio Q5 contains stocks with highest $D e v_{-} V R$ of 1.19.

Table 5. Quintile Portfolios of Option Price Structures Sorting on Measure of Stock Return Autocorrelation (Dev_VR)

| (Dev_VR) |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | IVol - Rvol <br> $\%$ | Slope | Dev_VK | $\beta$ | Size <br> (\$ Billion) |
|  |  | $\%$ |  |  |  |
| Q1 | 1.15 | -4.43 | 0.03 | 1.29 | 4.60 |
| Q2 | 1.13 | -4.48 | 0.08 | 1.31 | 4.34 |
| Q3 | 1.09 | -4.50 | 0.15 | 1.31 | 4.33 |
| Q4 | 1.01 | -4.47 | 0.23 | 1.31 | 4.20 |
| Q5 | 0.26 | -4.64 | 1.19 | 1.33 | 4.03 |
| Q5-Q1 | $\mathbf{- 0 . 9 0}$ | $\mathbf{- 0 . 2 1}$ |  |  |  |
| $t($ Q5-Q1) | $\mathbf{- 6 . 7 0}$ | $\mathbf{- 3 . 1 0}$ |  |  |  |

With the increases of $\operatorname{Dev}, V R$, i.e., higher serial correlation of stock returns, the option implied volatility becomes smaller compared to realized volatility and the implied volatility curve becomes more negatively skewed. For example, portfolio Q1 has a monthly average $\mathrm{IVol}-\mathrm{RVol}$ of $1.15 \%$ and a monthly average Slope of $-4.43 \%$, while portfolio Q5 has a monthly average IVol -RVol of $0.26 \%$ and a monthly average Slope of $-4.64 \%$. The average difference of $\mathrm{Vol}-\mathrm{RVol}$ between Q5 and Q1 is $-0.9 \%$, statistically and economically significant. The Slope spread between Q5 and Q1 is $-0.21 \%$ with a $t$-statistic of 3.1. The univariate regression results are consistent with the Fama-MacBeth regressions. The impact that the stock returns autocorrelation has on the level of implied volatility curve is stronger than that on the slope of the implied volatility curve. It is also noticeable that the patterns of monthly $\mathrm{IVol}-\mathrm{RVol}$ and Slope are somewhat flat from Q1 from Q4. To make sure that our results are not driven by outliers, we eliminate the stocks whose $\operatorname{Dev} v_{-} V R$ is in the top or bottom $1 \%$ for each month and the results remain quantitatively similar.

### 3.4 Robustness Check

We also perform several robustness checks. We start by replicating the Fama-MacBeth regressions using alternative measure of stock return autocorrelation, absolute value of first-order stock return autocorrelation and find that our main empirical results remain: stocks with strong return autocorrelation exhibit a lower level of implied volatility (compared to realized volatility) and a steeper implied volatility curve.
We then repeat univariate sorting for different subsample. To visualize the results, we plot $\mathrm{IVol}-$ $R V o l$ and Slope and the corresponding t -values of the long-short stock portfolio formed on Dev_VR v.s. different subsamples in Figure 1. The solid blue lines show the $I V o l-R V o l$ or the Slope spread between the top and bottom portfolio sorting by $\operatorname{Dev} v_{-} V R$ within subsample and the solid read lines show the corresponding t values. The blue dotted lines represent the $\mathrm{IVol}-\mathrm{RVol}$ or the Slope spread between the top and bottom portfolio sorting by $\operatorname{Dev} V R$ for all stocks.


Panel A Size: Small, Medium-Size and Big Stocks


Panel B Before, During and After Financial Crisis


Panel C Volatile and Involatile Market
Figure 1. The IVol-RVol, Slope and the Corresponding T-Valuesof the Long-Short Stock Portfolio Formed on Dev_VR Within Different Subsamples

To check if our main results are driven by relatively small stocks (Note 1) in our sample, we repeat the univariate sorting within small, median and large stocks in our sample. We find that the relationship between $\mathrm{IVol}-\mathrm{RVol}$ and stock return autocorrelation is most significant for smaller stocks in our sample. As in Panel A of Figure 1, the $I V o l-R V o l$ spread between Q5 and Q1 increases from relatively small firms to big firms. For example, the $I V o l-R V o l$ spread sorting by $D e v_{-} V R$ is -1.90 , and -0.60 for small and large stocks respectively. The measure of stock return autocorrelation is more important in explaining the level of implied volatility curve for relatively small stocks. The Slope spread between top and bottom portfolio sorting by $D e v_{-} V R$ is statistically significantly for medium-size and large stocks in our sample.
We then perform the univariate sorting before, during and after financial crisis. Our main conclusions hold in all subsamples, with the results being most significant during financial crisis, followed by before crisis for $\mathrm{IVol}-\mathrm{RVol}$ and after crisis for the Slope, as shown in Panel B of Figure 1. For example, the difference of $\mathrm{IVol}-\mathrm{RVol}$ between Q 5 and Q 1 sorting by $D e v_{-} V R$ is $-1.01,-1.53$, and -0.56 before, during and after crisis, with $t$-statistics of $-7.84,-3.45$ and -1.80 , respectively. The difference of Slope between Q 5 and Q 1 are $-0.09,-1.02$, and -0.23 before, during and after crisis, with $t$-statistics of $-0.79,-2.67$, and -1.93 respectively.

It is also interesting to see if the results differ under different market conditions, especially when market is volatile. We then separate our data into two subsamples by median VIX of our entire sample and repeat the univariate soring for both samples. We plot the $I V o l-R V o l$ and Slope of the long-short stock portfolio formed on $\operatorname{Dev} v_{-} V R$ for the two subsamples in Panel C of Figure 1. We find that the relation between stock return autocorrelation and implied-realized volatility spread is much stronger when market is volatile. The measure of stock return autocorrelation is more important in explaining the level of implied volatility curve when the_market is volatile. The relations between the measure of stock return autocorrelation and implied volatility slope does not differ when market is volatile or not.

## 4. Conclusions

The asset returns are assumed to be distributed independently of each other, in various option pricing models. However, the evidence of persistence autocorrelation in asset returns of both the short-term and long-term period contradicts the assumption made in the option pricing models. Following theses researches, literature documents that stock return autocorrelation enters into the option pricing formula through adjustments in volatility and/or expected asset price.

It is an empirical question, as to the extent that stock return autocorrelation affects option prices. To the best of our knowledge, there exists no empirical studies exploring the relative contribution of stock return autocorrelation in option pricing. In this paper, we fill the gap by investigating the impact of stock return autocorrelation on the individual equity option price structure and demonstrate a clear link between them. With the increases of serial correlation of stock returns, the option implied volatility becomes smaller compared to realized volatility and the implied volatility curve becomes more negatively skewed. The relation between stock return autocorrelation and option price structure is more pronounced when market is volatile, especially during financial crisis. The stock return autocorrelation is more important in explaining the level of implied volatility curve for relatively small stocks. Our hope is that a better understanding of the sources of option price structure will help guide us in the future as we work to improve option pricing models and option trading strategies.

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## Note

Note 1 . Firms in our sample are relatively larger and more liquid stocks compared with average stocks trading in the U.S. stock market, since we utilize stocks with options.

