Original Paper

The Private Housing Market Cyclical Price Dynamics

SUN, Jingbo¹ & HO, Kim Hin / David²

¹ Consultant, Deloitte & Touche LLP, Singapore
² Honorary Professor (Development Economics & Land Economy) (University of Hertfordshire, UK); Immediate Past Tenured Associate Professor (International Real Estate), Department of Real Estate, School of Design and Environment, National University of Singapore, Singapore

Received: May 1, 2020          Accepted: May 13, 2020          Online Published: May 30, 2020
doi:10.22158/jepf.v6n2p173 URL: http://dx.doi.org/10.22158/jepf.v6n2p173

Abstract

Two types of heterogeneous investors (momentum and disposition) form a unique difference model to interpret housing price dynamics. Three parameters are crucial: auto-correlation, the rate of mean reversion and the contemporaneous adjustment towards long-term equilibrium price. For Singapore, we examine the dynamic structures that oscillate and/or diverge from equilibrium. Disposition investors predominate although the interaction between momentum and disposition investors acts as a key determinant of private housing price dynamics for a given time in a specific market. Key implication is that Singapore's private housing market is low risk, offering stable returns owing to virtually no divergence even in the speculative 1990s. The best way to invest is to consider the momentum strategy and avoid the herd behavior for profit sustainability. For policy-makers, the Singapore private housing market is over-damped in the long run. Predominating disposition investors contribute to the market mechanism, which automatically adjusts private housing market prices. It is imperative to relax government intervention in Singapore's private housing market to enhance its efficiency.

Keywords

Housing market price dynamics, momentum investors, disposition investors, Singapore

1. Introduction

Housing prices largely affect the consumption and asset portfolios of households (Flavin and Yamashita, 2002), the financial sector (Note 1) and the macro economy (Note 2). The resulting volatility and its driving forces constitute a core issue in housing economics (Mankiw & Weil, 1991) that compels
researchers to seek answers in economic fundamentals. The temporary deviation of accrual prices from economic fundamentals (e.g., Clayton, 1996) and the positive autocorrelation of housing prices in the short run (e.g., Capozza & Seguin, 1996) have merely supported the belief that investors’ irrational expectations and investor psychology can be held accountable. Although both explanations have been repeatedly documented in previous empirical studies (see Case & Shiller, 1989, 1990; Cutler et al., 1991; Abraham & Hendershott, 1996; Malpezzi, 1999; Meen, 2002), there has been no systematic theory explaining private housing market price dynamics features within a behavioral context (Note 3). This paper models the different kinds of investors’ behavior in the private housing market and derives a second-order difference equation. Disposition behavior refers to the tendency to sell winners quickly and hold onto losers. Momentum behavior refers to the tendency to buy winners and sell losers. We further translate the disposition-momentum behavioral difference equation into the standard empirical formulation of autocorrelation and mean reversion. Specifically, the autocorrelation at a one-year frequency depends on the proportion of the effect from momentum behavior in the sum effect from both disposition and momentum behaviors. The mean reversion over a longer period is determined by the proportion of the effect from disposition behavior in the sum effect from both disposition and momentum behaviors. Therefore, the interaction (trades) between the two types of investors becomes a key determinant of private housing market price dynamics. Referring to the definitions by Capozza et al. (2004), we further identify four types of dynamic structures, namely, convergent without oscillation, divergent without oscillation, convergent oscillation and divergent oscillation, but define them in terms of the disposition and momentum coefficients. Consequently, we provide more rigorous definitions of “cyclical dynamics” and “bubble” within the disposition-momentum behavioral theory. When the complex roots occur for the general solution to the difference equation, the dynamics is oscillatory. When the proportion of the effect from momentum behavior in the sum effect from both disposition and momentum behaviors exceeds one, it leads to divergent dynamics. These constitute one of the main differences between our study and Capozza et al. (2004).

Nevertheless, as more real estate funds and other institutional investors allocate capital into Asian real estate, Singapore emerged as the world’s “hottest” real estate market in 2007, and is securely among the top favorites of real estate investors (the Economic Times, 2007) (Note 4). In addition, Singapore is granted a highest real estate transparency rating and the transaction cost is generally low (Cruz, 2008). Therefore, we continue the empirical study in the context of the Singapore private housing market. Over the sample period from 1982 Q1 to 2007 Q3, the market experienced more than two boom and bust movements on different scales (Figure 1(a)). This raises some questions. Why does the upturn around 2006 differ from the 1990s’ boom, i.e., a lower magnitude of price gain and a gradual recovery (Morgan Stanley report, 2007)? What are the underlying causes of such differences? Singapore’s average nominal earnings per employee show a clear increasing and oscillating trend while the variable housing loan rate has remained relatively smooth, even during the 1990s (Figures 1(b) and (c)). What were the causal behavioral price dynamics during the 1990s and from 1982 to 2007? Our empirical
analysis shows that the autocorrelation at a one-year frequency is around 0.7 in both periods. The market prices only converge from 0.02 to 0.03 of the total adjustment each year from 1982 to 2007, and from 0.03 to 0.04 during the 1990s. In the long run (1982 to 2007), the price dynamics are convergent to equilibrium prices without oscillations (being over-dampened). In the 1990s, the prices still fluctuated in a convergent and oscillating way, without showing divergence. Thus, the behavioral characteristics of price dynamics vary over time in the Singapore private housing market. The effect of disposition investors dominates that of their momentum counterparts in both periods. The comparative magnitude of the disposition effect to the momentum effect is larger during the 1990s than it is over the total sample period. The value of the composite autocorrelation parameter during the 1990s is larger than that over the total sample period. Both results explain the difference between the upturn around 2006 and the one in the 1990s in terms of amplitude and frequency.

Figure 1. Housing Prices, Earnings and Housing Loan Rates. 1(a). Singapore Private Housing Property Price Index

Notes. 1998 Q4 = 100; the nominal price index is computed based on fixed weights before 1998 Q4; the weights used to compute the index are updated every quarter from 1998 Q4. The URA publishes the data named the residential price index, which refers to the nominal private housing price index in the text.

Source: Singapore Urban Redevelopment Authority (URA), 2008.

Figure 1(b). Singapore Average Nominal Earnings per Employee

Published by SCHOLINK INC.
Notes. The series is computed using data from the Central Provident Fund (CPF) Board and complied using 5-digit fields instead of the 4-digit fields from 1998; it includes bonuses, if any, but excludes employers’ CPF contributions and pertains to all full- and part-time employees who contribute to the CPF, but excludes all identifiable self-employed persons from 1992.

Source: Singapore Ministry of Manpower, 2008.

![Figure 1(c). Singapore Variable Housing Loan Rate for 15-year](image)

Notes. The housing loan rate refers to the average rates compiled from those quoted by 10 leading finance companies.

Source: Monetary Authority of Singapore, 2008.

Our contribution is firstly a generic and rigorous modeling of private housing market price dynamics which relaxes the homogenous investors’ assumption. Second, we interpret the characteristics of private housing market price dynamics within the disposition-momentum behavioral theory and define four dynamic structures including the price bubble. Third, we relate the disposition-momentum behavioral theory to the stylized facts of private housing market price dynamics, i.e., autocorrelation and mean-reversion. Consequently, this study sheds light on the observed positive correlations between private housing market prices and trading volumes, as well as the effects of past housing price changes on housing turnover. Our fourth contribution in this paper is to test the bubble conjecture. All of the above contributions are meaningful for policy makers and investment managers in enhancing their in-depth understanding of the strength of policy intervention and the efficiency of real estate portfolio management.

2. Review of the Related Literature

2.1 Real Estate Cyclical Dynamics

Several studies emphasize the existence of the real estate cycles, for e.g., office construction and vacancy cycle (Wheaton, 1987), real estate returns cycle (Kaiser, 1997), the first global cycle (Renaud, 1997). As for the real estate price cycle, different types of real estate exhibit various cyclical patterns over time and across markets (see Wheaton, 1999; Case & Mayer, 1995). Empirical studies try to pin
the determinants of real estate price cycles on economic fundamentals (see Muellbauer & Murphy, 1997; Quigley, 1999). However, they present inconsistent results with respect to specific economic determinants, based on a variety of methodologies and across diverse areas. Another line of relevant studies concerns the estimation of housing supply elasticity and price adjustments (Note 5) (see Blackley, 1999; Goodman, 2005; Wigren & Wilhelmsson, 2007). These studies also include an opposite perspective from which to analyze the effects of supply constraints on housing prices (see Aura & Davidoff, 2008). Hence, the theoretical models of real estate cyclical dynamics have been patchy and relatively scarce.

Traditionally, there are two types of theories: the representative agent model and the popular stock-flow (four-quadrant) model. Both accord due attention to economic fundamental determinants. Briefly, the representative agent model considers the new dwelling market and homeowners act as both consumers and investors. The market equilibrium condition is that homeowners obtain the same return from investing in houses in relation to other assets. As for the stock-flow model, two sub-markets are considered, with the interaction between supply and demand determining housing prices. Therefore, a similarly reduced form is derived from the two different theoretical models, but the theoretical development of housing cycles remains limited. Some further research efforts relate to the causes or explanations of housing cyclical fluctuations.

The “honeycomb cycle” theory of Janssen et al. (1994) reiterates the relevance of the market conditions that trigger housing market cyclical dynamics, particularly concerning the interaction between housing price and transaction volume dynamics. More importantly, the empirical tests suggest that transaction volume dynamics are more closely related to changes in market conditions, compared with housing price dynamics. According to Chinloy (1996), the apartment rental rate is a function of the vacancy and the space absorption expectation. Rent expectations and construction lags are empirically found to be significant determinants of housing cycles. Dokko et al. (1999) link economic fundamentals to real estate value and income cycles via the general relationship between real estate value and the capitalization of expected future rents. Capozza et al. (2004) investigate housing price dynamics by focusing on the stylized facts, i.e., autocorrelation and mean reversion. Using data from 62 metro markets, they conclude that local economic variables, construction costs, the size and the growth of a metro area can explain housing price fluctuations.

2.2 The Disposition Effect of Investors’ Behavior in the Asset Market

The tendency to sell winners quickly and hold onto losers, as a prominent portfolio puzzle in the rational expectations paradigm, denotes the disposition effect established by Shefrin and Statman (1985). This tendency is found in a variety of markets with different considerations, such as the Finnish stock market (Grinblatt & Keloharju, 2001), the Taiwanese stock market (Barber et al., 2006), the Australian stock market (Brown et al., 2006) and in exercising company stock options (Heath et al., 1999). The disposition effect has also been found in specific periods, i.e., from January to November compared with December when tax-motivated selling prevails (Odean, 1998). Odean (1998) also finds
that individual investors demonstrate a significant disposition behavior that does not seem to consider the importance of trading costs or rebalancing portfolios. In contrast, Ferris et al. (1988) present overwhelming evidence that the disposition behavior exists throughout the year, inclusive of the year-end. The evidence from the actual trading records of professional traders also exhibits their myopic loss aversion (Locke & Mann, 2000).

Although the existence of the disposition effect seems undisputed, there is only partial consensus on the explanation for it. Favorable behavioral explanations include prospect theory (Kahneman & Tversky, 1979), regret aversion (Loomes & Sugden, 1982), mental accounting (Thaler, 1985) and the cognitive dissonance theory (Shefrin & Statman, 1985). Muermann and Volkman (2007) show that an investor seeking pride and avoiding regret exhibits the disposition trading behavior. As for formal testing, several papers formalize the explanations for the influence of prospect theory and loss aversion on the disposition effect (see Kyle et al., 2006). However, Rangelova (2001) finds that the larger the market capitalization of the firm, the more likely the disposition behavior. He proclaims that individual beliefs rather than preferences lead to the disposition behavior. Hens and Vlcek (2005) query the prediction of the disposition effect based on prospect theory. Barberis and Xiong (2006) also find that prospect theory does not always produce the disposition effect. A behavioral alternative focuses on rational explanation, such as the portfolio rebalancing consideration (Lakonishok & Smidt, 1986) and the transactions cost consideration (Glosten & Harris, 1988).

2.3 The Momentum Effect of Investors’ Behavior in the Asset Market

Jegadeesh and Titman (1993) popularize the asset strategies of buying winner and selling loser due to their good performance. However, the consistent profitability of such momentum strategies reveals puzzling anomalies in modern finance theory by violating the central theme of the efficient market hypothesis. Thus, the momentum effect is universally noted and appears robust to methodological tweaking. Momentum strategies have been found to be effective in twelve European countries (Rouwenhorst, 1998), and Asian markets with the exception of Japan and Korea (Chui et al., 2000), because momentum profits persisted throughout the 1990s (Jegadeesh & Titman, 2001). In addition, momentum behavior is clearly identified in Michigan housing market by Piazzesi and Schneider (2009) based on the survey data. They find that a small cluster of investors believe it is time to buy a house as its price will keep increasing. Moreover, the size of such momentum cluster varies along the housing market dynamics.

Various explanations have alluded to the momentum phenomenon, a strategy denoting psychological phenomena and based on irrational behavior, such as the representative heuristic and conservatism bias in Barberis et al. (1998) and the self-attribution bias in Daniel et al. (1998). “Irrational decisions may lead to the systematic under- or over-reaction of prices relative to their fundamental value, whatever that may be” (Swinkels, 2004, p. 122). Hong and Stein (2000) argue that communication frictions cause under-reaction in the short term and over-reaction in the long term, in keeping with momentum behavior.
A risk-based explanation proposed by Jegadeesh and Titman (1993) fails to provide evidence. The same is true for Fama and French’s (1996) three-factor unconditional asset pricing model. Ang et al. (2001) provide evidence of the momentum profits as compensation for exposure to downside risk. Other explanations include the cross-sectional variation in the unconditional expected returns, instead of the predictable time-series variation in returns (Conrad & Kaul, 1998) and the industry effects (Moskowitz & Grinblatt, 1999). However, the inconsistent results challenge both explanations (see Grundy & Martin, 2001). Antoniou et al. (2007) consider both risk factors and behavioral biases in a two-stage model specification to explain the momentum effect. They imply that risk factors could explain the momentum effect but behavioral factors do not matter much.

However, we do not aim to discuss the explanations for the disposition and momentum behavior. We draw on the extensive existence of disposition and momentum effects to construct a housing price dynamics model that investigates the characteristics of the housing price time path and identifies its cyclical or speculative bubble movement.

3. The Housing Price Dynamics Model

The housing market has long been suspected to be inefficient (Note 6) (Case & Shiller, 1989, 1990; Tirtiroglu, 1992; Meese & Wallace, 1994). “The apparent predictability in housing prices, at least in the short run, leaves open the possibility of speculative purchases in the housing market” (Riddel, 1999, p. 272). “Past researchers have shown that a mix of fundamental and feedback traders in a market may lead to price volatility over and above that driven by rational price forecasts” (Riddel, 1999, p. 273). Hence, housing price dynamics are deemed to be determined by investors’ behavior and economic conditions. Practitioners and academic scholars have indicated that individual investors suffer from behavioral biases, such as insufficient diversification, excessive trading and some relatively simple trading strategies as reviewed by Barberis and Thaler (2002). These strategies have been divided into two major categories: the disposition strategy (effect), which relies on price reversals, and the momentum strategy (effect), which is based on price continuation (Shen et al., 2005).

Among the behavioral biases, the disposition effect has gained the most attention. Although several studies use financial assets, the disposition effect has been documented in the Finnish apartment market (Einio & Puttonen 2006), individuals’ behavior in the sale of housing (Genesove & Mayer, 2001) and among professional investors at an Israeli brokerage house (Shapira & Venezia, 2001). In Genesove and Mayer (2001), sellers with nominal losses require higher asking prices and have a lower hazard rate for selling. However, conditional on selling, they would receive higher prices. Given micro evidence that varies, higher pricing levels do prompt housing equity accumulation before the sale but might, during a brief spell, lower the marginal probability to sell a unit, whereas the influence on the downside is stronger. Hence, in housing market terms, the investors’ disposition effect should affect their realized housing prices.

Intuitively, if only the disposition investors exist in the private housing market at any point in time,
then all investors would behave as sellers or potential sellers. In reality, housing transaction prices need only require that “price and volume are simultaneously determined in equilibrium”, so that “whatever process… generates price could give rise to the accompanying trading volume” (Lee & Swaminathan, 2000, p. 2065) while “past trading volume also predicts both the magnitude and persistence of price momentum” (Lee & Swaminathan, 2000, p. 2017). Thus, there must be some investors in the market engaging in contrary behavior. The literature identifies the momentum strategy as that unique, contrarian behavior (e.g., Shen et al., 2005). According to Strobl (2003), the disposition effect is consistent with the price momentum. Massa and Goetzmann (2000) offer evidence that trades between the disposition investors and their counter-parties (the momentum investors) influence relative prices. It is therefore reasonable to characterize housing price dynamics by considering both types of investor behavior within the disposition and momentum behavioral theory.

Although the housing market is suspected to be an inefficient one due to the asymmetrical information and high transaction costs, similar investor’s behavior in financial market, for e.g., the momentum behavior, which is observed in housing market (see Piazzesi & Schneider, 2009). Hence, we hypothesize that heterogeneous investors prevail in the housing market because it is more of an asset market, by nature, than a common commodity market. This implies that investors, even speculators, play the most important role in the housing market. In contrast, if few investors are in the housing market, then it becomes a common commodity market. No government intervention is expediently assumed. “Housing markets are inefficient and house prices, at times, deviate from fundamental or intrinsic values. A sharp run-up in housing price is partly due to irrational expectations (fads, noise traders, trend chasing) and signals a future correction, as housing prices are ultimately anchored by (i.e., cointegrated with) market fundamentals” (Clayton, 1997, p. 359-360). Because investor expectations are not rational, they are not homogeneous. Thus, there are two types of investors in the housing market: the disposition investors and the momentum investors.

Figure 2 depicts the framework for our theoretical model, in which the disposition investor seeks risk when faced with possible losses and avoids risk when a certain gain is possible (Kahneman & Tversky, 1979). Such behavior is equivalent to a utility function, which is steeper for losses than for gains (Tversky & Kahneman, 1992) unless it is defined on gains and losses as opposed to levels of wealth (Odean, 1998). The nature of the disposition behavior proposes an asymmetric S-shaped value function. This function is a departure from the standard, expected utility maximization framework in that an S-shaped value function for investors is centered around a profit of zero on a given trading position. According to prospect theory (Kahneman & Tversky, 1979), the disposition investors have already experienced gains or losses. Their initial state is not zero when they make decisions to hold or sell their housing units. A motivated seller’s marginal probability to sell the unit is assumed to be the carrier of loss aversion (Genesove & Mayer, 2001; Engelhardt, 2003).
Figure 2. Outline of the Theoretical Model

Source: Authors, 2018.

Figure 3 depicts the behavior of the disposition investors’ value functions. Figure 3(a) displays the state of gains: from a gain point M, to increase x revenue that brings less happiness to the investors than the sorrow caused by an increasing x deficit. Figure 3(b) displays the state of losses: from a loss point N, to increase the x deficit that brings less sorrow to the investors than the happiness caused by an increasing x revenue.

However, when the market is in a good condition, the momentum investors expect the housing market to behave well, whereas when the market is in a bad condition, they expect the housing market to behave badly. The value function of the momentum investors is concave in the domain of capital gains and convex in the domain of capital losses, as depicted in Figure 4. However, their value function is different from that of the disposition investors; that is, zero becomes the reference point for the strategies of the momentum investors when the initial states of gains and losses are zero.

Figure 3. Value Function of Disposition Investors in Gains and Losses Condition

Source: Tao Guan, 2007 (unpublished); Authors, 2018.
3.1 The Model Construction

Disposition investors sell the unit when the housing price increases. However, the price changes do not definitively lead the disposition effect because disposition investors refer to a price when making decisions. This price acts as a benchmark. In this study, we selected the fundamental price as the reference point. The disposition investor then gains the characteristics of the “fundamental investor” as defined by Riddle (1999), “…who bases price forecasts on expected economic conditions in the area. This type of investor would be more likely to purchase a home when prices are low relative to expected fundamentals and to sell when the converse is true”. In the short run, this should be the linear relationship between the changes in disposition investors’ demand and housing prices, as expressed in Eq (1) (Note 7):

\[
D_t^D = -\alpha (P_t - P^*) ; \, \alpha > 0 .
\]  

(1)

where \( D_t^D \) is the change in the disposition investors’ demand in period \( t \); \( P_t \) is the housing price in period \( t \); \( P^* \) is the reference price in the log of the equilibrium value per unit, \( p^* = p(X_t) \); \( X_t \) is a vector of exogenous explanatory variables that serves as a proxy for economic condition; and \( \alpha \) indicates the sensitivity of disposition investors to housing price changes. With regard to momentum investors, the changes in their demand depend on housing price changes in every term of Eq (2). The momentum effect appears when the housing price changes exhibit inertia by increasing or decreasing continuously. We assume, for at least two periods, i.e., this period and the last period, that the linear function is

\[
D_t^M = \beta_1 (P_t - P_{t-1}) + \beta_2 (P_{t-1} - P_{t-2}) ; \, \beta_1, \beta_2 > 0 .
\]  

(2)

where \( D_t^M \) is the change in momentum investors’ demand in period \( t \); \( P_t \), \( P_{t-1} \) and \( P_{t-2} \) are the housing price in periods \( t \), \( t-1 \) and \( t-2 \), respectively; and \( \beta_1 \) and \( \beta_2 \) indicate the sensitivities of momentum investors to different periods’ price changes. The relative magnitude between \( \beta_1 \) and \( \beta_2 \) need not be defined because although sensitivity is diminishing, i.e., the marginal value of both the gains and losses decreases with increasing changes in housing prices, as shown in Figure 4, \( P_t - P_{t-1} \).
and \( P_{t-1} - P_{t-2} \) are only required to keep up its inertia (with the same sign), rather than be larger or smaller (comparing their absolute values).

The resulting market state can be expressed as the sum of all of the disposition and momentum investors’ demand changes, plus the supply of new units in period \( t \):

\[
D^D_t + D^M_t + N = 0. \tag{3}
\]

where \( N \) is the supply of new units.

Following the logic of Mayer and Somerville (2000), we assume that supply of new units occur only when housing market experiences transition from one equilibrium to another, a period identified by the increase of the price. Hence, all else being equal, assuming a linear supply schedule with the changes of equilibrium price, it is mathematically described as,

\[
P_t^* = n\Delta P^*_t, \tag{4}
\]

where \( n \) is constant and \( n > 0 \). \( \Delta \) is the difference operator.

Substituting \( D^D_t \), \( D^M_t \) and \( N \) in Eq (3) with Eq (1), Eq (2) and Eq (4), respectively, produces

\[
\beta_1(P_t - P_{t-1}) + \beta_2(P_{t-1} - P_{t-2}) - \alpha(P_t - P_t^*) + n\Delta P^*_t = 0. \tag{5}
\]

Eq (5) can be rewritten as,

\[
P_t^* = \frac{\alpha P_t^* + n\Delta P^*_t}{\alpha - \beta_1} = \frac{n\Delta P^*_t + \alpha P_t^*}{\beta_2 - \beta_1 - \beta_2 - \beta_1 - \alpha} \tag{6}
\]

Eq (6) is a second-order difference equation and its solution includes a particular integral and complementary functions. \( P_t^* \) is generally stochastic. In order to investigate the dynamic characteristics of the difference equation, for an illustrative example, let \( P_t^* \approx P^*; \Delta P_t^* \rightarrow 0 \). Let \( P_t = P_{t-1} = P_{t-2} = C \), where \( C \) is a constant, then the particular integral of Eq (6) is obtained as (Note 8).

\[
C \approx \frac{\alpha P_t^* + n\Delta P^*_t}{\alpha - \beta_1} \approx P_t^* + \frac{n\Delta P^*_t + \alpha P_t^*}{\alpha} \approx P^* \tag{7}
\]

A difference equation with convergent variables tends to ultimately arrive at the particular integral. As previously mentioned, \( P^* \) is a benchmark price for the disposition investors’ behavior when they take economic conditions into account to make decisions. Eq (7) shows the benchmark price for disposition investors, i.e., economic conditions, to be the determinant of the final state of the housing market. Eq (7) also provides self-proof that the equilibrium price \( P^* \) will be the final state of the housing market if no changes of equilibrium price \( P^* \) in a short run.

The complementary functions of Eq (6) are obtained by applying the “Z-transform” \( \lambda' = P_t \). A quadratic equation for the unknown \( \lambda \) is

\[
\lambda^2 + \frac{\beta_2 - \beta_1}{\beta_2 - \beta_1} \lambda - \frac{\beta_2}{\beta_2 - \beta_1} = 0 \tag{8}
\]

Eq (8) is the characteristic equation from which the characteristic values (roots) can be obtained.
\[ \lambda_1, \lambda_2 = \frac{1}{2} \left( \frac{\beta_1 - \beta_2}{\beta_1 - \alpha} + \sqrt{\left( \frac{\beta_1 + \beta_2}{\beta_1 - \alpha} \right)^2 - 4\alpha\beta_2} \right) \] (9)

When \((\beta_1 + \beta_2)^2 - 4\alpha\beta_2 > 0\), there are two real unequal characteristic roots \(\lambda_1, \lambda_2\), hence, the solution to Eq (6) is,

\[ P_t = A_1\lambda_1^t + A_2\lambda_2^t + C \] (10)

\(A_1, A_2\) are constants. When \((\beta_1 + \beta_2)^2 - 4\alpha\beta_2 = 0\), there are repeated real roots \(\lambda = \frac{1}{2} \left( \frac{\beta_1 - \beta_2}{\beta_1 - \alpha} \right)\), then, the solution to Eq (6) is

\[ P_t = A_3\lambda^t + A_4t\lambda^t + C \] (11)

\(A_3, A_4\) are constants. When \((\beta_1 + \beta_2)^2 - 4\alpha\beta_2 < 0\), the roots of the characteristic equation are complex conjugates and the solution to Eq (6) is,

\[ P_t = A_5r^t \cos(\theta t) + A_6r^t \sin(\theta t) + C \] (12)

where, \(r = \sqrt{\frac{\beta_1 - \beta_2}{\alpha - \beta_1}}; \ \theta = \arccos\left( \frac{\beta_1 - \beta_2}{2\sqrt{\beta_1(\alpha - \beta_1)}} \right)\)

Or \(P_t = Ar^t \cos(\theta t - \phi) + C \) (13)

Eq (13) is the phase-amplitude form of the general solution. \(A = \sqrt{A_5^2 + A_6^2}\); phase \(\phi = \arctan \frac{A_6}{A_5}\). All \(A_i, i=1,...,6\), \(A\), \(C\) and \(\Phi\) are constants that are determined by the initial conditions. The constructed model implications are subsequently discussed in terms of four key propositions and the specific derivations are provided in Appendix 1.

3.2 The Model Propositions

Proposition 1. Interaction between the disposition and momentum investors results in the time path of housing prices featured in the autocorrelation and mean reversion. The autocorrelation and mean reversion parameters are expressed by composite coefficients: the proportion of the momentum coefficient to the last period’s price changes in the sum effect from disposition behavior and momentum behavior to this period’s price changes; and the proportion of the disposition coefficient to this period’s price changes in the sum effect from disposition behavior and momentum behavior to this period’s price changes, respectively.

Specifically, with the autocorrelation parameter \(\tilde{\alpha} = \frac{\beta_1}{\alpha - \beta_1}\) and the mean reversion parameter \(\tilde{\beta} = \frac{\alpha}{\alpha - \beta_1}\), Eq (6) is rewritten in Eq (14).

\[ P_t - (1 + \tilde{\alpha} - \tilde{\beta})P_{t-1} + \tilde{\alpha}P_{t-2} - \tilde{\beta}P^*_t - \tilde{\gamma}\Delta P^*_t = 0, \] (14)
where $\tilde{y} = \frac{\alpha}{\alpha - \beta_1}$. Substituting $P_t - P_{t-1}$ and $P_{t-1} - P_{t-2}$ in Eq (14) with $\Delta P_t$ and $\Delta P_{t-1}$, respectively, results in the characteristics of the time path of the housing price dynamics being represented in Eq (15),

$$\Delta P_t = \tilde{\alpha}\Delta P_{t-1} + \tilde{\beta}(P_{t-1} - P_{t-2}) + (\tilde{\beta} + \tilde{\gamma})\Delta P_t^*$$  \hspace{1cm} (15)

Eq (15) reinterprets the key stylized facts of the housing market: the positive autocorrelation of price changes at one-year frequencies (Glaeser & Gyourko, 2006), and a long-term tendency toward “fundamental reversion” with prices responding to contemporaneous economic shocks (Lamont & Stein, 1999). Distinctively, grounded in the disposition and momentum theory, Eq (15) can be duly explained. The autocorrelation parameter is determined by the proportion of the momentum coefficient $\beta_2$ to the last period price changes $\Delta P_{t-1}$ in the sum effect $(\alpha - \beta_1)$ from the disposition behavior and the momentum behavior to this period’s price changes $\Delta P_t$, i.e., $\tilde{\alpha} = \frac{\beta_2}{\alpha - \beta_1}$. It is the momentum effect that mainly contributes to continuous price changes, and hence it serves as a numerator in defining the autocorrelation. Following similar logic regarding the autocorrelation, the mean reversion parameter is determined by the proportion of the disposition coefficient $\alpha$ to this period’s price changes $\Delta P_t$ in the sum effect $(\alpha - \beta_1)$ from the disposition behavior and the momentum behavior to this period’s price changes $\Delta P_t$, i.e., $\tilde{\beta} = \frac{\alpha}{\alpha - \beta_1}$. It is the disposition effect that mainly leads housing price changes in a reversion manner, and hence it serves as a numerator in defining the mean reversion. Both composite parameters imply that the interaction between the two types of investors affects the autocorrelation and mean-reversion.

Proposition 2. The characteristics of the housing price dynamics can be anatomized into four types based on two critical conditions: oscillating (cycling), to be determined by $(\beta_1 + \beta_2)^2 - 4\alpha \beta_2 = 0$ or $(1 + \tilde{\alpha} - \tilde{\beta})^2 - 4\tilde{\alpha} = 0$ and convergent to the long-run equilibrium, to be determined by $\beta_2 = \alpha - \beta_1$ or $\tilde{\alpha} = 1$.

Mathematically, a necessary condition for the housing price dynamics in oscillations (cycles) is that the complex roots occur: $(\beta_1 + \beta_2)^2 - 4\alpha \beta_2 < 0$ or $(1 + \tilde{\alpha} - \tilde{\beta})^2 - 4\tilde{\alpha} < 0$. With restrictions from the economics of the propositional problem, the absolute value of the autocorrelation $\tilde{\alpha}$ being less than one serves as a necessary condition for convergence to equilibrium (Capozza et al., 2004; Capozza & Israelsen, 2007). Subsequently, housing price dynamics can be categorized into four types under the two foregoing conditions. Here, the autocorrelation parameter $\tilde{\alpha}$ and the mean reversion parameter $\tilde{\beta}$ are composite parameters derived from $\alpha, \beta_1, \beta_2$, which are the sensitivity coefficients of the disposition and momentum investors. Hence, the space of composite parameters with coefficients $\alpha, \beta_1, \beta_2$ can be graphically divided into four regions, as shown in Figure 5. The various combinations of both parameter values generates dynamic patterns when the equilibrium is “shocked”. Therefore, our difference model proclaims that the interaction between both types of investors is a
crucial force in housing market price dynamics and determines its features.

Specifically, \((\beta_1 + \beta_2)^2 - 4\alpha\beta_2 = 0\), i.e., \(\alpha = \frac{(\beta_1 + \beta_2)^2}{4\beta_2}\) acts as a critical condition for oscillation: if \(\alpha > \frac{(\beta_1 + \beta_2)^2}{4\beta_2}\), then there is oscillatory behavior (overshooting); if \(\alpha < \frac{(\beta_1 + \beta_2)^2}{4\beta_2}\), then there is damped behavior (no overshooting).

With regard to each region, the following cases provide useful insights:

Case (1). When \(\alpha < \beta_1\), and \(\alpha < \frac{(\beta_1 + \beta_2)^2}{4\beta_2}\), i.e., \((1 + \alpha - \beta_1)^2 - 4\alpha > 0\), then the dynamics show no oscillations. When \(\alpha < \beta_1\), it also ensures that \(\alpha < \beta_1 + \beta_2\), which suggests that the dynamics are divergent, as in Region III (no oscillations-divergence dynamics). The theoretical explanation is that when the disposition investors’ sensitivity to the price change is smaller than the composite sensitivity of the momentum investors, i.e., \(\alpha < \frac{(\beta_1 + \beta_2)^2}{4\beta_2}\), then the strength of the oscillation fluctuation contributed by disposition investors is not enough to overwhelm that of continuous rise or decline contributed by momentum investors. The total sensitivity to price change of the disposition investors is smaller than that of the momentum investors, i.e., \(\alpha < \beta_1 + \beta_2\), such that if the price rises, then the demand of the disposition investors (selling) is not enough to satisfy the demand of the momentum investors (buying), leading to price divergence.

Case (2) a. When \(\alpha > \beta_1\), and \(\alpha < \frac{(\beta_1 + \beta_2)^2}{4\beta_2}\), i.e., \((1 + \alpha - \beta_1)^2 - 4\alpha > 0\), it can be rewritten as \(\beta_1 < \alpha < \frac{(\beta_1 + \beta_2)^2}{4\beta_2}\).

If \(\beta_1 < \beta_1 + \beta_2 < \alpha < \frac{(\beta_1 + \beta_2)^2}{4\beta_2}\), then it is in Region IV (no oscillations-convergence dynamics). The theoretical explanation is that when the total sensitivity to price change of the disposition investors is larger than that of the momentum investors, then the resistance to housing price emerges from the disposition investors sufficiently enough to counteract the impetus to the housing price from the momentum investors, leading to price convergence.

If \(\beta_1 < \alpha < \frac{(\beta_1 + \beta_2)^2}{4\beta_2}\) and \(\alpha < \beta_1 + \beta_2\), it suggests Region III (no oscillations-divergence dynamics). The theoretical reasons are the same as those in Case (1).

Case (2) b. When \(\alpha > \beta_1\), and \(\alpha > \frac{(\beta_1 + \beta_2)^2}{4\beta_2}\), i.e., \((1 + \alpha - \beta_1)^2 - 4\alpha < 0\), if \(\frac{(\beta_1 + \beta_2)^2}{4\beta_2} < \alpha < \beta_1 + \beta_2\), it suggests Region II (oscillations-divergence dynamics);
if \( \alpha > \frac{(\beta + \tilde{\beta})^2}{4\beta_1} \) and \( \alpha > \beta_1 + \beta_2 \), it suggests Region I (oscillations-convergence dynamics). The abovementioned logic provides the corresponding explanation.

**Proposition 3.** On housing price dynamics in oscillations, the amplitude increases sharply and concussively with the increase of \( \frac{\beta_2}{\alpha - \beta_1} \) or the autocorrelation parameter \( \tilde{\alpha} \), but becomes ambiguous in \( \frac{\alpha}{\alpha - \beta_1} \) or the mean reversion parameter \( \tilde{\beta} \); the frequency decreases steeply with the increase of \( \frac{\beta_2}{\alpha - \beta_1} \) or the autocorrelation parameter \( \tilde{\alpha} \), but is also ambiguous in \( \frac{\alpha}{\alpha - \beta_1} \) or the mean reversion parameter \( \tilde{\beta} \).

As for the oscillations, the amplitude and frequency are expressed as,

\[
\text{Amplitude} = 2|P_0 - P^*| \cdot \frac{\alpha(\alpha - \beta_1)}{\sqrt{4\alpha\beta_1 - (\beta_1 + \beta_2)^2}} \cdot \left( \frac{\beta_2}{\alpha - \beta_1} \right)^2.
\]

\[
= 2|P_0 - P^*| \cdot \frac{\tilde{\beta}}{\sqrt{4\tilde{\alpha}\tilde{\beta} - (\tilde{\alpha} + \tilde{\beta} - 1)^2}} \cdot \left( \sqrt{\tilde{\alpha}} \right) \quad (16a)
\]

where \( P_0 \) is a constant from initial conditions \( P(0) = P(1) = P_0, P_0 \neq 0 \) and \( P_0 \neq P^* \).

\[
\text{Frequency} = \frac{1}{2\pi} \cdot \arccos \left( \frac{\beta_2 - \beta_1}{2\sqrt{\beta_1(\alpha - \beta_1)}} \right) = \frac{1}{2\pi} \cdot \arccos \left( \frac{\tilde{\alpha} - \tilde{\beta} + 1}{2\sqrt{\tilde{\alpha}}} \right) \quad (17)
\]

Eq (16a) states that the amplitude of oscillations depends on the distance of the system, at the initial point, from equilibrium as well as the sensitivity coefficients of both the disposition and momentum investors. In Eq (17), the frequency is determined by the sensitivity coefficients of both investors. According to Eq (16b) and Eq (17), the relationships connecting frequency and amplitude with the two composite parameters, \( \tilde{\alpha}, \tilde{\beta} \), are graphed in Figure 6.

![Figure 6. Amplitude and Frequency of the Oscillations with the Composite Parameters](image)

**Notes.** The Amplitude is portrayed at t=2.

**Source:** Authors, 2018.
The graphs in Figure 6 clearly depict proposition 3. Intuitively, if the momentum coefficient $\beta_2$ to the last period’s price changes is a large proportion of the sum effect ($\alpha - \beta_1$) from the disposition behavior and the momentum behavior to this period’s price changes, it implies that the momentum behavior greatly affects the housing market. Because the momentum effect continuously leads housing prices to keep increasing or decreasing, the larger the proportion of $\frac{\beta_2}{\alpha - \beta_1}$, the larger scale (amplitude) of housing prices increase or decrease at the same time; and the longer period (1/frequency) for which housing prices continue to increase or decrease. As for the proportion of $\frac{\alpha}{\alpha - \beta_1}$, it does not capture the sum of the competing effects from the disposition and momentum behavior in the housing market, so the trend of amplitude and frequency changes is ambiguous with this parameter.

Proposition 4. Other relevant parameters remaining invariant, the speed at which the amplitude decreases increases while the duration of convergence to the equilibrium is shorter, in line with the increase in the disposition coefficient $\alpha$ or the mean reversion parameter $\beta$.

Based on Eq (8), the damped ratio $\zeta$ in our model is given in Eq (18).

$$\zeta = \frac{\beta_2 - \beta_1}{2\sqrt{\beta_2(\alpha - \beta_1)}} = \frac{\bar{\alpha} - \bar{\beta}}{2\sqrt{\bar{\alpha}}} + 1$$

It is reversed from the disposition coefficient $\alpha$ and mean reversion parameter $\beta$, whereas if other parameters are kept invariant, then the damped ratio $\zeta$ decreases with the increase in $\alpha$ or $\bar{\beta}$. Specifically, the damped ratio characterizes the time length of price that is converging to its long-term equilibrium level. If housing prices increase, then the disposition investors tend to sell their housing units, which increases the market supply and increasing prices are restrained. The larger the disposition coefficient $\alpha$, the greater the strength of its counteraction to the housing price fluctuations, i.e. the less time that housing prices need to be back at the long-term equilibrium level. More intuitively, in the autocorrelation and mean reversion domain, the larger the mean reversion parameter $\bar{\beta}$, the faster prices must return to equilibrium, i.e., the smaller the damped ratio $\zeta$.

3.3 Model of Reference Price

In model estimation, our aim is to investigate the characteristics of Singapore private housing market price dynamics within the disposition-momentum behavioral theory, and to test the differences in different periods of shock to the local economy. These differences are captured by examining those points, determined by various values of $\bar{\alpha}, \bar{\beta}$ from the studied periods, which appear in different Regions as illustrated in Figure 5.
$p^*$ is important in estimating the parameters $\alpha, \beta$ in Eq (15). $p^* = f(X)$, where $X$ is a vector of independent variables that considers economic conditions. Hence, the key consideration lies in the selection of $p^* = f(X)$. Relevant studies have adopted a reduced form price equation, which they estimate based on some underlying notion of the determinants in the context of supply and demand. Generally, the fitted regression of housing price on a set of the potential determinants is interpreted as the price level justified by fundamental factors (forces) within the economy. The priori and important factors are sometimes insignificant, have opposite signs or are significant. The finance-based approach features an underlying notion of arbitrage, typifying the ratio of rental income to house prices as a standard metric.

However, the underlying supply and demand factors, such as income, are not modeled. In addition, the adjustment path of housing prices compared to their fundamental level is beyond this approach. Recent theoretical models even highlight that borrowing can make asset prices more sensitive to fundamental shocks (see Lamont & Stein, 1999). Housing loans and housing prices are interdependent in the long run and they have a positive contemporaneous effect on each other in the short run, according to Gimeno and Martínez-Carrascal (2006). Moreover, the variable mortgage rate influences the growth rate of housing prices (Otto, 2007). Income and interest rates can explain housing price movements through time (Case & Shiller, 2003). Therefore, we select the hybrid method from McQuinn and O’Reilly (2008) because their model captures the significant roles of credit, income and interest rate as drivers of housing demand in Eq (19).

$$HL_t = kY_t \left( \frac{1 - (1 + R_t)^{-\tau}}{R_t} \right),$$

(19)

where $HL_t$ is the housing loan amount that can be borrowed in period $t$; $k$ is the proportion of household income that goes into mortgage repayments; $Y_t$ is the disposable income per household; $R_t$ is the mortgage interest rate; and $\tau$ is the duration of the mortgage. After nesting Eq (19) for the purpose of a general housing market model, the resulting expression is simplified in Eq (20) (Note 9).
\[ P_t^e = \zeta + \psi X_t, \quad (20) \]

where \( X_t \) is defined as the time-varying component of \( HL_t \).

Two advanced regression models are adopted to estimate long-term equilibrium prices in Eq (20): the dynamic ordinary least squares (DOLS) model of Stock and Watson (1993) and the fully-modified OLS (FM-OLS) of Phillips and Hansen (1990). Recently, the single equation DOLS approach has been popular in different models in the housing market studies, such as those by Muellbauer and Murphy (1997), Fitzpatrick and McQuinn (2007) and McQuinn and O’Reilly (2008). The potential correlation between the explanatory variables (factors) and the error process are explicitly permitted in the DOLS model. The expression is

\[ y_t = a_0 + a_1 x_u + \sum_{j=1}^{k} \phi_j \Delta x_{t+j} + \varepsilon_t, \quad (21) \]

where \( x_u \) is endogenous. As Eq (21) reveals, and to correct for correlations, the DOLS involves the leads and lags of the differenced regressors in the specification. The FM-OLS is more complex and its advances lie in correcting the OLS for possible autocorrelation and endogeneity in the regressors caused by the existence of a cointegrating relationship.

4. Results and Analysis

4.1 The Data

Our data set includes three time series: the nominal private housing price index, the average monthly nominal earnings per employee and the variable housing loan rate for 15 years, a long enough period to enable meaningful analysis. The data are quarterly span 1982 Q1 to 2007 Q3. The quarterly disposable incomes of households are not available for Singapore and the average monthly nominal earnings per employee from all of the industries are selected as a proxy (Figure 1(b)). The percentage of housing in the household expenditure is 22% (SingStat, 2005, p. 6), which is well below the widely accepted and cautious notion that the average monthly nominal earnings per employee for Singaporeans should exclude the CPF (the central provident fund form of social security), and that the average proportion of earnings going into housing loan repayments should not exceed 30% (Note 10). The variable housing loan rate is selected because it captures the economy changes better than its fixed counterpart. The amount of the housing loan is obtained from Eq (19).

4.2 Preliminary Tests and Long-term Equilibrium Estimation

To prepare for the DOLS and FM-OLS regressions, the unit roots in both the logarithmic levels and the logarithmic levels of the first differences for each variable are tested. The ADF, DF-GLS (Generalized Least Squares) and PP (Phillips-Perron) tests are conducted, and the results for the log level of the private housing price index and the housing loan are reported in Table 1. All of the cases fail to reject the unit root hypothesis at the 1% level of significance, and at the log level of the first differences, almost all of tests reject the unit root hypothesis at the 1% level of significance.
Table 1. Unit Root Tests

<table>
<thead>
<tr>
<th></th>
<th>Private Residential Price Index</th>
<th>Housing Loan</th>
<th>1%</th>
<th>5%</th>
<th>Stationary</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Level &amp; Intercept</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ADF t-test</td>
<td>-0.962</td>
<td>-3.452</td>
<td>-3.496</td>
<td>-2.890</td>
<td>no</td>
</tr>
<tr>
<td>DF-GLS</td>
<td>-0.105</td>
<td>0.577</td>
<td>-2.588</td>
<td>-1.944</td>
<td>no</td>
</tr>
<tr>
<td>PP-GLS</td>
<td>-1.311</td>
<td>-2.237</td>
<td>-3.496</td>
<td>-2.890</td>
<td>no</td>
</tr>
<tr>
<td><strong>1st Difference &amp; Intercept</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ADF t-test</td>
<td>-4.055</td>
<td>-3.655</td>
<td>-3.497</td>
<td>-2.891</td>
<td>yes</td>
</tr>
<tr>
<td>DF GLS</td>
<td>-3.969</td>
<td>-1.782*</td>
<td>-2.588</td>
<td>-1.944</td>
<td>yes</td>
</tr>
<tr>
<td>PP GLS</td>
<td>-4.067</td>
<td>-41.905</td>
<td>-3.496</td>
<td>-2.890</td>
<td>yes</td>
</tr>
</tbody>
</table>

Notes. For the ADF, DF tests, the lag length for the test regressions is chosen using Ng and Perron’s Modified AIC procedure; the maximum lags are eight; keeping all of these settings consistent, we also conduct the tests based on Trend and Intercept and all of the results report the I(1) process. * shows the I(1) process at the 10% level of significance with critical value (-1.614487).

The correlation between the private housing market price index and the housing loan is 0.704181 (0.812251 in the logarithm form), which implies a long-term relationship between both series. The cointegration tests are presented in Table 2, and to avoid spurious results from a single test, the Johansen and the Engle-Granger (1987) cointegration tests are conducted for a robust conclusion. As Table 2 shows, the results from the Johansen tests provide evidence of one cointegrating vector at the 5% significance level, while the Engle and Granger test rejects the null hypothesis of no cointegration at the 5% level.

Table 2. Cointegration Tests for Private Residential Price Index and Housing Loan

<table>
<thead>
<tr>
<th>Johansen tests</th>
<th>Hypothesized no. of cointegration equation</th>
<th>5% critical values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No intercept or trend</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max-Eig. Stat.</td>
<td>16.083</td>
<td>3.548</td>
</tr>
<tr>
<td><strong>Intercept and no trend</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trace</td>
<td>27.899</td>
<td>4.561</td>
</tr>
<tr>
<td>Max-Eig. Stat.</td>
<td>23.338</td>
<td>4.561</td>
</tr>
<tr>
<td><strong>Intercept and trend</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trace</td>
<td>23.858</td>
<td>3.627</td>
</tr>
<tr>
<td>Max-Eig. Stat.</td>
<td>20.231</td>
<td>3.627</td>
</tr>
</tbody>
</table>

Summary of Johansen tests

<table>
<thead>
<tr>
<th>Data trend</th>
<th>None</th>
<th>Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test type</td>
<td>No Intercept, No Trend</td>
<td>Intercept, Trend</td>
</tr>
<tr>
<td>Trace</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Maximum Eigenvalue</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Engle-Granger Cointegration Test

<table>
<thead>
<tr>
<th>Stat.</th>
<th>5% Critical Values</th>
<th>10% Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.503</td>
<td>-3.40</td>
<td>-3.09</td>
</tr>
</tbody>
</table>

Published by SCHOLINK INC.
Because “the DOLS estimator falls under the single-equation Engle Granger (1987) approach to cointegration while allowing for endogeneity within the specified long-run relationships” (McQuinn & O’Reilly, 2008, p. 384), the above cointegration results enable us to proceed to the DOLS regression. Table 3 reports the results from the DOLS, FM-OLS and OLS for the housing price index and housing loan in the long run. As expected, the estimators from each method correspond closely. The coefficient of the housing loan shows the expected sign. In particular, the housing loan calculated from the housing loan rate and average earnings as a proxy for the determinants of housing demand under certain economy conditions is positively and significantly related to private housing price.

Table 3. Long-Run Model DOLS, FM-OLS and OLS Estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>DOLS estimate</th>
<th>FM-OLS estimate</th>
<th>OLS estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.76***</td>
<td>0.78***</td>
<td>0.75*</td>
</tr>
<tr>
<td></td>
<td>(1.4)</td>
<td>(1.6)</td>
<td>(8.3)</td>
</tr>
<tr>
<td>Log of Housing loan</td>
<td>0.62**</td>
<td>0.63**</td>
<td>0.62*</td>
</tr>
<tr>
<td></td>
<td>(2.3)</td>
<td>(2.4)</td>
<td>(15.5)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.67</td>
<td>n.a.</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Notes. Values in parenthesis are t-statistics of each estimate; $R^2$ is centered $R^2$ for both DOLS and OLS; the $R^2$ of FM-OLS is not calculated because to do so would not make sense in a cointegrating regression; the results of DOLS, OLS are obtained from the RATs 7.0 program; the FM-OLS results are obtained from the Matlab program. * denotes significance at the 0.001 level; ** denotes significance at the 0.05 level; *** denotes significance at the 0.2 level.

Furthermore, the parameter stability for equations containing the I(1) processes is investigated under Hansen’s (1992) method (Note 11). The results of the FM-OLS estimators are presented in Table 4. All of the test statistics fail to reject the null hypothesis of parameters’ stability at the 5% significance level, as Figure 7 reveals.

Table 4. Applying Hansen (1992) Test of Parameter Stability in Regression with I(1) Series

<table>
<thead>
<tr>
<th>Stability Test Stat.</th>
<th>P value of rejecting stability null*</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC</td>
<td>0.132</td>
</tr>
<tr>
<td>MeanF</td>
<td>1.588</td>
</tr>
<tr>
<td>SupF</td>
<td>4.917</td>
</tr>
</tbody>
</table>

Notes. * $p \geq 0.200000$ is restricted to $p = 0.200000$; the estimations in Table 4 and Figure 7 are obtained using the Matlab code programmed by Professor Bruce Hansen; the pre-whitened and Bartlett kernel are adopted for each test.
4.3 Dynamic Responses Estimation

We investigate the characteristics of Singapore private housing market price dynamics during two periods: the whole sample period from 1982 Q2 to 2007 Q3, and the sub-period from 1990 Q1 to 2001 Q1. Using the results from the equilibrium estimation from DOLS for both period samples, we estimate Eq (15) together with the form proposed by Capozza et al. (2004) in Eq (22) adopting the OLS.

\[
\Delta P_t = \sum_i \tilde{\alpha}_i (X_i - \bar{X}_i) \Delta P_{t-1} + \sum_i \tilde{\beta}_i (X_i - \bar{X}_i)(P^*_t - P_{t-1}) + \tilde{\eta} \Delta P^*_t
\]

(22)

In this paper, \(X_t = HL_t\) and \(\tilde{\eta} = \tilde{\beta} + \tilde{\gamma}\).

The results are reported in Table 5. First, for both periods, \(\tilde{\eta}\) denotes the contemporaneous adjustment of prices to current shocks and \(\tilde{\alpha}\) represents the autocorrelation. According to efficient market theory, \(\tilde{\eta}\) should be 1 and \(\tilde{\alpha}\) would be zero. However, several studies obtain the autocorrelation of housing price dynamics to be more than zero, such as the range from 0.25 to 0.5 suggested by Case and Shiller (1989) for four cities; 0.4 for a panel of 29 cities with 0.2 for the inland cities and 0.5 for the coastal cities by Abraham and Hendershott (1993); around 0.45 for 15 OECD countries in Englund and Ioannides (1997); and a range from -0.2 to 1.7 for 992 metro areas in Capozza et al. (2004). Here, \(\tilde{\alpha}\) is significant at around 0.7, which is consistent with existing studies. \(\tilde{\eta}\) is almost zero with a large p-value, suggesting that during both periods, almost 100% of housing price adjustments occur gradually over time. Both values of \(\tilde{\alpha}\) and \(\tilde{\eta}\) imply that the Singapore private housing market was inefficient from 1982 to 2007. With regard to the mean reversion parameter \(\tilde{\beta}\) (Note 12), no theory predicts its estimated value (Capozza et al., 2004). However, if housing prices converge to their equilibrium values in the long run, \(\tilde{\alpha} > 0\) implies \(\tilde{\beta} > 0\) (Note 13) (Capozza et al., 2004).

In this paper, the pairs of \(\tilde{\alpha}\) and \(\tilde{\beta}\) in Table 5 are significant and consistent with previous observations. Owing to the zero value of \(\tilde{\eta}\), market prices converge 2% (0.02) to 3% (0.03) of the total adjustment each year from 1982 to 2007, according to the value of \(\tilde{\beta}\); and 3% (0.03) to 4% (0.04) from 1990 to 2001. Our findings are consistent with those of Abraham and Hendershott (1996),
who report a value of zero for Midwestern cities in the U.S.

The results of Eq (22) in Table 5 shed light on the endogenous adjustments of housing price dynamics. The changes in housing loans (Note 14) consider the loans’ influence on the autocorrelation and mean reversion parameters, denoted as $\tilde{\alpha}_1, \tilde{\beta}_1$. However, for the whole and sub-periods, $\tilde{\alpha}_1$ and $\tilde{\beta}_1$ are statistically insignificant with high probability values (see Table 5). Thus, the changes in housing loans do not lead to statistically significant differences in autocorrelation and mean reversion in Singapore private housing market price dynamics. This implies that housing loans, which represent economic conditions, affect housing market price dynamics by comprehensively entering the equilibrium price $P_t^*$ for the Singapore private housing market. Our model offers a stylized investment market in which the fundamental housing value changes exogenously. This means that housing market price dynamics should be explained beyond general economic conditions, providing support for our Eq (15) on the basis of the disposition and momentum behavioral theory.

**Table 5. Price Dynamic Responses Regressions**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Equation (15)</th>
<th>Equation (22)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.72*</td>
<td>0.66*</td>
</tr>
<tr>
<td>$\tilde{\alpha}_1$</td>
<td>0.31</td>
<td>0.04</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.03***</td>
<td>0.02***</td>
</tr>
<tr>
<td>$\tilde{\beta}_1$</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.51</td>
<td>0.52</td>
</tr>
</tbody>
</table>

**Sub-period: 1990 Q1 to 2001 Q1**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Equation (15)</th>
<th>Equation (22)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.77*</td>
<td>0.77*</td>
</tr>
<tr>
<td>$\tilde{\alpha}_1$</td>
<td>-0.63</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.04***</td>
<td>0.03****</td>
</tr>
<tr>
<td>$\tilde{\beta}_1$</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-0.05</td>
<td>-0.05</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.58</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Notes. $\tilde{\alpha}_1, \tilde{\beta}_1$ are the changes in housing loans plus autocorrelation and mean reversion, respectively. * denotes significance at the 0.001 level; ** denotes significance at the 0.05 level; *** denotes significance at the 0.1 level; **** denotes significance at the 0.2 level.

Given the locations of the points determined by $\tilde{\alpha}$ and $\tilde{\beta}$, as plotted in the “Region” map (see Figure 8), it is clear that for the whole sample period, housing market price dynamics lie in Region IV (convergent but no oscillations). However, for the shorter sub-period, both of the models point toward
Region I (convergent with oscillations). It can be concluded that from 1982 to 2007, Singapore private housing market price dynamics are convergent with equilibrium prices without oscillations (being over-damped). From 1990 to 2001 when the Singapore private housing market is deemed speculative, the housing market prices fluctuating is in a convergent and oscillating manner without showing divergence. This implies that Singapore private housing market price dynamics are far from a speculative price bubble.

Figure 8. Parameters Allocation in the Region Map

Notes. The dot below the line is obtained from the regression based on Eq (22) for the whole sample period, because the model of Eq (22) has a higher $R^2$ than that of Eq (15); two dots above the line are obtained from the regression based on Eq (15) and Eq (22) for the sub-period.

Source: Authors, 2018.

According to Proposition 1 and within our disposition and momentum framework, every pair of significant $\tilde{\alpha}$ and $\tilde{\beta}$ shows the positive sign, suggesting $\alpha > \beta_1$. Our results show that the housing market price dynamics lie in Region IV for the whole sample period, and it should be in Proposition 2, Case (2)a, i.e., $\beta_1 < \beta_1 + \beta_2 < \alpha < \frac{(\beta_1 + \beta_2)^2}{4\beta_2}$. Thus, the Singapore private housing market is strikingly dominated by the disposition investors, compared with the momentum investors, from 1982 to 2007. In the long run, the housing price dynamics converge to equilibrium. In terms of the shorter sub-period, the housing price dynamics lie in Region I, which corresponds to the situation in Case (2)b, i.e., $\alpha > \frac{4(\beta_1 + \beta_2)^2}{4\beta_2}$ and $\alpha > \beta_1 + \beta_2$. Once again, the disposition investors dominate and the housing market price dynamics do not show divergence even in the so-called speculative period (1993-1996, 2000-2001). Our results here are consistent with those of extensive domestic sources, such as financial advisory firms, online comments and academicians. For example, at the IPAC (2007) panel, three presentations reiterate that the housing price bubble is not in the offing (Note 15). Abeysinghe (2007)
mentions that the rise in housing prices is below the long-term equilibrium level, i.e., the pace of housing price rises is still below the expected level based on market fundamentals. Because $\alpha$ is larger than $\frac{\left(\beta_1 + \beta_2\right)^2}{4\beta_1}$ in the sub-period and smaller in the whole period, the comparative magnitude of $\alpha$ to $\beta_1, \beta_2$ for the sub-period is larger than it is for the whole period. In Table 5, the value of $\tilde{\beta}$ in the sub-period is slightly larger than that in the whole period.

According to Proposition 4, a larger $\alpha$ or $\tilde{\beta}$ contributes to faster recovery, which explains the fast recovery from the 1990s’ boom and bust. The value of $\tilde{\alpha}$ in the sub-period is larger than that in the whole period, according to Proposition 3, for which the amplitude of the 1990s’ upturn is higher. Thus, combining the estimates of $\alpha$ and $\tilde{\alpha}$ from Propositions 4 and 3, respectively, explains the 1990s’ boom and bust in terms of the autocorrelation-mean reversion and investors’ behavior. In short, Singapore private housing market boom in the 1990s’ differs from the other upturns observed from 1982 to 2007: the recovery from the 1990s’ bust was faster and the magnitude of the price gain was significantly higher. These characteristics are consistent with Morgan Stanley’s (2007) analysis of Singapore private housing market. The patterns of housing market price dynamics can vary over periods. According to Cappozza et al. (2004), 26% of the observed housing price dynamics exhibit convergence with no oscillation (i.e., in Region IV) while 67% exhibit convergence with oscillations (i.e., in Region I), whereas zero% lie in Region III.

Hence, Singapore private housing market price dynamics, while consistent, exhibits a uniqueness that can be attributed to a variety of price dynamics corresponding with various periods. From 1982 to 2007 in Singapore, the damped convergence is the reaction to price shocks (in Region IV) while in the sub-period and inclusive of the “speculative” period (1990 to 2001), the convergent oscillation is the reaction to price shocks (in Region I). Finally, although we have not included the influence of the public policy effect, it is deemed to be part of investors’ behavior, i.e., policy acts as an exogenous variable and does not affect the model structure. This is primarily because the Singapore government has regularly and strongly intervened in the housing market with the overall aim of housing price stabilization (Note 16). Thus, the over-damped convergence of the Singapore private housing price dynamics is caused by the aggregate effects of the disposition and momentum investors’ behavior and long-term government policy.

4.4 The Robustness Checks

To enable the robustness of our results, a number of alternative specifications were attempted for the six specific cases. The first case addresses the price dynamic response regressions for the shorter (relative to the equilibrium) results from the DOLS for the same period. Although $R^2$ increases, the values of $\tilde{\alpha}, \tilde{\beta}$ are consistent with the earlier results. The second case arises because new private housing supply in the whole sample period is limited (Note 17), N in Eq (3) is set to zero. The third case concerns the proportion of income to mortgage payment, which is adjusted to a lower proportion of 26%. For the fourth case, the data is seasonally adjusted, adopting the Census X12 statistical mode.
The fifth case notes certain variables in real terms to explain economic phenomena. The sixth case adjusts the data set in real terms. All the results of the long-term equilibrium that are not reported in the tables are very similar to Table 3.

All the results of the dynamic response regressions for the six different cases are provided in Table 6. It is clear that the variables in seasonally adjusted real terms exhibit less autocorrelation and similar mean reversion compared with their “un-seasonally” adjusted data. However, for the whole period, the points determined by $\tilde{\alpha}$, $\tilde{\beta}$ from Eq (22) are associated with a higher $R^2$, compared to Eq (15), and all are located in Region IV (convergence with no oscillations). Interestingly, all the points determined by $\tilde{\alpha}$, $\tilde{\beta}$ from Eq (15) lie close to the oscillations’ critical line. Regarding the sub-period, both equations exhibit similar results and all the points are located in Region I (convergence with oscillations). Overall, our results are robust with respect to the main features of the Singapore private housing market price dynamics.

### Table 6. Robust Checks of Price Dynamic Response Regressions

<table>
<thead>
<tr>
<th>Total sample period: 1982 Q1 to 2007 Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Based on sub-period sample</td>
</tr>
<tr>
<td>Equation on</td>
</tr>
<tr>
<td>(15)</td>
</tr>
<tr>
<td>$\tilde{\alpha}$</td>
</tr>
<tr>
<td>$\tilde{\beta}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The selected sub-period: 1990 Q1 to 2001 Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Based on sub-period sample</td>
</tr>
<tr>
<td>Equation on</td>
</tr>
<tr>
<td>(15)</td>
</tr>
<tr>
<td>$\tilde{\alpha}$</td>
</tr>
<tr>
<td>$\tilde{\beta}$</td>
</tr>
</tbody>
</table>

**Notes.** Equation (15’) is from Equation (15) when N=0: $\Delta P_t = \tilde{\alpha} \Delta P_{t-1} + \tilde{\beta} (P^*_t - P_{t-1})$; Equation (22’) is from Equation (22) when N=0: $\Delta P_t = \sum \tilde{\alpha}_i (x_{t,i} - \bar{x}_i) \Delta P_{t-1} + \sum \tilde{\beta}_i (x_{t,i} - \bar{x}_i) (P^*_t - P_{t-1})$. 

Published by SCHOLINK INC.
5. Conclusion
This paper develops a rigorous model of private housing market price dynamics within behavioral theory. A key assumption is that the investors are a heterogeneous mix of the disposition and momentum types. The decision-making of the two types' investors shows different sensitivities to housing market price changes. This paper sheds light on the behavioral explanation of empirical estimates for housing market price’s autocorrelation and mean reversion time path, as in the study by Gao, Lin and Na (2009) and Titman, Wang and Yang (2014). The interaction between the two types of investors and the aggregate effect of their behavior are important determinants of the private housing market price dynamics.

This paper highlights the definition and interpretation of the patterns (features) of the private housing market price dynamics, in accordance with the disposition-momentum behavioral theory. The paper categorizes the private housing market price dynamics into four patterns, including the price bubble via the composite autocorrelation and mean-reversion parameters, within the disposition-momentum domain, i.e. convergent or divergent and oscillatory or not oscillatory, as established by Capozza et al. (2004).

The paper empirically investigates the features of the Singapore private housing market price dynamics and interprets them within both the autocorrelation-mean reversion and the disposition-momentum domains. The private housing market price dynamics exhibit a variety of features over different periods with variant autocorrelation and mean reversion parameters. Damped convergence is the reaction to price shocks during the longer period (1982 to 2007). The price dynamics display convergent oscillation rather than divergence in the shorter, so-called “speculative” period (1990 to 2001). It is found that the characteristics of the private housing market upturn around 2006 differ from those of the 1990s’ boom-and-recovery, which had been slower and that the magnitude of the price gain tend to be lower. In both periods (around 2006 and the 1990s’ boom-and-recovery), the disposition investors prevail, as compared to the momentum investors in the Singapore private housing market.

Furthermore, this paper’s model offers a stylized investment market, in which the fundamental value changes exogenously. Such a stylized investment market provides potential evidence that investor behavior is endogenously crucial in explaining the private housing market price dynamics. The average autocorrelation parameter in this paper is approximately 0.7. The instantaneous adjustment parameter is almost zero with a large p-value, suggesting that during both periods, i.e., around 2006 and the 1990s' boom-and-recovery, almost 100% of housing price adjustments occur gradually over time. The housing market prices merely converge to the 2-3% range of the total adjustment each year, from 1982 to 2007; to the 3-4% range from 1990 to 2001. A key implication for investors is that the boom around 2006 of the Singapore private housing market does not offer as large a magnitude as that from the price gain in the 1990’s boom-and-recovery from a long-term perspective. However, the Singapore private housing market seems to be low risk, offering stable returns, thanks to virtually no divergence, even in the speculative 1990s.
Given that the disposition investors prevail in the private housing market, the best way to invest is to consider the momentum strategy and to avoid the herd behavior for profit sustainability. For policy-makers, the Singapore private housing market is over-damped in the long run. Moreover, the disposition investors predominate this private housing market and their behavior contributes to the market mechanism, which automatically adjusts the private housing market prices. The implication is to consider the appropriateness of relaxing government intervention in the Singapore private housing market, in order to make it more efficient.

References


Appendix 1: Features Derivation of the Second-Order Difference Equation

**Solutions of the Second-Order Difference Equation**

Let \( P_t^* \approx P^*; \Delta P_t^* \rightarrow 0 \), then Eq (6) can be rewritten as,

\[
P_t + \frac{\beta_2 - \beta_1}{\beta_1 - \alpha} P_{t-1} - \frac{\beta_2}{\beta_1 - \alpha} P_{t-2} = -\alpha P_t^* \]

Initial Conditions,

Let \( P(0) = P(1) = P_0 \), where \( P_0 \) is a constant, but \( P_0 \neq 0 \) and \( P_2 \neq P^* \).

The roots of the characteristic equation based on the Eq (6) are,

\[
\lambda_1, \lambda_2 = 1 + \frac{1}{2} \left[ \frac{\beta_1 - \beta_2}{\beta_1 - \alpha} \pm \sqrt{\left( \frac{\beta_1 + \beta_2}{\beta_1 - \alpha} \right)^2 - 4\alpha \beta_2} \right]
\]

When there are complex roots, i.e., \((\beta_1 + \beta_2)^2 - 4\alpha \beta_2 < 0\), the solution of the Eq (6) is,

\[
P_t = A_0 r^t \cos(\theta t) + A_0 r^t \sin(\theta t) + C \text{ , where , } C = P^* .
\]

\[
A_0 = P_0 - P^* ;
\]
Let $\tilde{\alpha} = \frac{\beta_2}{\alpha - \beta_1}$, $\tilde{\beta} = \frac{\alpha}{\alpha - \beta_1}$, all the solutions can be expressed in terms of $\tilde{\alpha}, \tilde{\beta}$. For example,

The roots of the characteristic equation based on Eq (6) are,

$$\lambda_1, \lambda_2 = \frac{1}{2} \left[ \beta_1 - \beta_2 \pm \sqrt{(\beta_1 + \beta_2)^2 - 4\alpha \beta_2} \right]$$

$$= \frac{1}{2} \left[ (\tilde{\alpha} - \tilde{\beta} + 1) \pm \sqrt{(\tilde{\alpha} + \tilde{\beta} - 1)^2 - 4\tilde{\alpha}\tilde{\beta}} \right].$$

When there are complex roots, i.e., $(\beta_1 + \beta_2)^2 - 4\alpha \beta_2 < 0$, i.e., $(1 + \tilde{\alpha} - \tilde{\beta})^2 - 4\tilde{\alpha} < 0$, the solution of Eq (6) is,

$$P_t = A_0 r^t \cos(\theta t) + A_0 r^t \sin(\theta t) + C,$$ where, $C = P^*,$

$$A_0 = (P_0 - P^*) \left[ \frac{2\alpha - \beta_1 - \beta_2}{\sqrt{4\alpha \beta_2 - (\beta_1 + \beta_2)^2}} \right];$$

$$= (P_0 - P^*) \left[ \frac{1 - \tilde{\alpha} + \tilde{\beta}}{\sqrt{4\tilde{\alpha}\tilde{\beta} - (\tilde{\alpha} + \tilde{\beta} - 1)^2}} \right];$$

$$r = \sqrt{\frac{\beta_2}{\alpha - \beta_1}} = \sqrt{\tilde{\alpha}},$$

$$\theta = \arccos \left[ \frac{\beta_1 - \beta_2}{2\sqrt{\beta_1(\alpha - \beta_1)}} \right] = \arccos \left[ \frac{\tilde{\alpha} - \tilde{\beta} + 1}{2\sqrt{\tilde{\alpha}}} \right],$$

Amplitude $= |P_0 - P^*| \cdot \frac{\sqrt{4\alpha(\alpha - \beta_1)}}{\sqrt{4\alpha \beta_2 - (\beta_1 + \beta_2)^2}} \cdot \left( \sqrt{\frac{\beta_2}{\alpha - \beta_1}} \right);$
Frequency = \frac{1}{2\pi} \cdot \arccos \left( \frac{\beta - \beta}{2\sqrt{\beta(\alpha - \beta)}} \right)

= \frac{1}{2\pi} \cdot \arccos \left( \frac{\tilde{\alpha} - \tilde{\beta} + 1}{2\sqrt{\tilde{\alpha}}} \right)

Appendix 2: The Singapore Housing Property Demand and Supply Dynamics


Notes. Area shaded in grey are for periods when there is oversupply; incremental demand is calculated based on the increase in the number of households; incremental property supply includes both private housing property and public housing property. Supply data up till 2006 refers to additional supply net of demolition. Supply data after 2006 refers only to the gross private housing property supply.

Notes
Note 3. To our knowledge, only Glaeser and Gyourko (2006) have mathematically deduced the form of the autocorrelation and mean reversion for housing price dynamics. Their model fails to explain high-frequency positive autocorrelation.
Note 5. Wigren and Wilhelmsson (2007) provide a detailed review on this issue based on recently published articles, while DiPasquale (1999) offers a more complete review.
Note 7. An incunabular work modeling the demand functions of both disposition and momentum investors can be found in Tao Guan’s unpublished paper.
Note 8. The detailed explanations on solution of second-order difference equations can be found in
most mathematical textbooks.

Note 9. See page 380 in McQuinn and O’Reilly (2008) for the linear format and the detailed derivation.

Note 10. This is calculated according to the relevant reports from Singstat. In this study, we set it 30%. In McQuinn and O’Reilly (2008), the value is also 30% for Irish.

Note 11. More details can be found in Gregory and Hansen (1996).

Note 12. Here, $\frac{\alpha}{\alpha - \beta} = \tilde{\beta} = 0.02$ is controversial, relating to our hypothesis stating that $\alpha$, $\beta$, are positive. We explain this issue from two perspectives. Regarding the alternative hypothesis, the policy effect, which is excluded by our hypothesis, plays an important role in the Singapore housing market. Regarding the market clearing condition, it is theoretical. In reality, the real estate market is at a disequilibrium most of the time (see Riddel, 2004; Ho, 2006).

Note 13. This implication can also be deduced from our disposition and momentum model: because all of the parameters, $\alpha$, $\beta$, are assumed to be positive, the $\tilde{\alpha}$, $\tilde{\beta}$ must share the same sign. Hence, if one of $\tilde{\alpha}$, $\tilde{\beta}$ is positive, then the other is positive as well.

Note 14. The changes in housing loan rates and earnings are also considered, but not reported. Their effect on the autocorrelation and mean reversion parameters is consistent with that from housing loans.

Note 15. See the lushhomeonline comment entitled “Analysts see no property bubble” at http://www.lushhomemedia.com/category/property-bubble/


Note 17. See Appendix 2 for the detailed trend.