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The Price Dispersion of Consumer Products

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Abstract

Presented is an analytic dynamic model of the price dispersion of consumer products. The theory is based on the idea that sellers offer product units for a profit maximizing price, denoted p_m. Product units not sold at p_m are called excess units. Based on the conservation equation of offered units, it can be shown that the stationary price distribution of consumer products consists of a Dirac-delta peak at p_m surrounded by a fat-tailed Laplace distribution from the excess units. A good quantitative agreement with empirical data can be obtained with a fit of the two free parameters of the theory.

Keywords

consumer products, market dynamics, price, price dispersion, Laplace distribution

1. Introduction

A consumer product can be defined as an item that serves as a solution to a specific consumer problem having the same features. A consumer good is a collection of consumer products. Based on the idea that both, the demand and the supply side of a market can take advantage of arbitrage opportunities, the neo-classic microeconomic theory suggests that a product must sell for the same price, known as "the law of one price" (Hens & Rieger, 2010). However, even for consumer products, empirical investigations show the existence of a price dispersion (Eden, 2014). Economists give four popular explanations for the existence of a price dispersion: amenities, heterogeneous costs, intertemporal price discrimination and search frictions. The first explanation suggests that identical products sell at different prices because they are bundled with different amenities in different transactions (Sorensen, 2000). The second states that firms at different locations have different costs causing prices to vary for similar goods (Golosov & Lucas, 2007). Time dependent fluctuations of the price in order to satisfy different consumer groups (Conlisk et al., 1984; Sobel, 1984; Klenow and Malin, 2010; Albrecht et al., 2013) and the limited ability of buyers to search the entire market (Butters, 1977; Varian, 1980, Burdett & Judd, 1983).

Independent of the microeconomic origin of the price dispersion, an analytic model is presented that yields a price distribution equivalent to distributions found in empirical studies (Kaplan & Menzio, 2014). The key idea of the model is that for a time interval Δt, a consumer product has a unique profit maximizing price, denoted p_m. The "law of one price" applies if all supplied product units can be sold at this price. However, there are units supplied, but not sold at p_m, called excess units. They generated a current of available units on the price scale. This flow is the origin of the price dispersion in this model. Based on the conservation equation of available units it is shown analytically that, under market equilibrium conditions and a random drift of the excess units, the price dispersion has the form of a Laplace distribution with a central peak at p_m.

The remainder of the paper is organized as follows. The first chapter presents the general framework for the description of the price distribution. Based on the conservation equation of supplied units and three simplifying assumptions, the stationary price distribution is established in the subsequent chapter. After a comparison with empirical data the paper completes with a conclusion.

2. Theory of the Price Distribution of Consumer Products

2.1 General Framework

The scope of this paper is to establish the basic equations for the price distribution of a consumer product. The probability density function (pdf) of sold units is defined as:

$$P_y(t,p) = \frac{y(t,p)}{\tilde{y}(t)} \tag{1}$$

where $y(t,p)$ is the number of sold units per unit time at time step t in a price interval p and $p+dp$. The total unit sales reads (Note 1):

$$\tilde{y}(t) = \int_0^\infty y(t,p)dp \tag{2}$$

Further introduced is the number of available units $z(t,p)$ at time step t in the price interval p and $p+dp$. It is governed by the following conservation equation:

$$\frac{\partial z(t,p)}{\partial t} = s(t,p) - y(t,p) - \chi z(t,p) - \frac{\partial j(t,p)}{\partial p} \tag{3}$$

The first term on the right-hand side of the equation determines the supply rate $s(t,p)$ of units at time step t in the price interval p and $p+dp$. It quantifies the number of units entering the market per unit time at price p. The next term considers the sales process. It decreases the number of available units at p by the unit sales rate $y(t,p)$. Units can also withdraw from the market. This happens for example for non-durable consumer products if units increase the expiry date. This contribution is proportional to the current number of offered units $z(t,p)$ and a rate χ. However, since unsold units increase the costs, the rate χ must be very small. It is considered to be sufficiently small to be neglected in further considerations. The last term in this relation takes the flow of units on the price scale into account. It is governed by the current $j(t,p)$ (Note 2).

The total number of offered units is defined by:

$$\tilde{z}(t) = \int_0^\infty z(t, p)dp \tag{4}$$

and the total supply rate is:

$$\tilde{s}(t) = \int_0^\infty s(t, p)dp \tag{5}$$

A time interval Δt is chosen such that the profit $r(t,p)$ of the consumer product has a maximum at a price p_m with:

$$\frac{\partial r(t, p_m)}{\partial p} = 0 \tag{6}$$

while $\frac{\partial^2 r(t, p_m)}{\partial p^2} < 0$.

The derivation of the price distribution $P_y(p)$ is based on four assumptions. The first two assumptions specify the price dependent supply rate $s(t,p)$:

i) Sellers try to maximize their profit by supplying all units at p_m. The supply rate $s(t,p)$ of the product is therefore located at profit maximum.

This assumption implies that:

$$\frac{\partial s(t, p_m)}{\partial p} = 0 \tag{7}$$

The distribution of supplied units can be approximated by a Dirac δ-function (Note 3):

$$s(t, p) = \tilde{s}(t)\delta(p - p_m) \tag{8}$$

If all units are sold during Δt at p_m, the price distribution reduces to (Note 4):

$$P_y^{\,0}(p) = \delta(p - p_m) \tag{9}$$

However, if this is not possible, sellers offer units for a price $p \neq p_m$. The number of these excess units is denoted $z_{ex}(t,p)$. Therefore, the total number of offered units consists in general of two contributions:

$$\tilde{z}(t) = \tilde{z}_0(t) + \tilde{z}_{ex}(t) \tag{10}$$

where $\tilde{z}_0(t)$ is the total number of units available at p_m. The total number of excess units available at $p \neq p_m$ is denoted $\tilde{z}_{ex}(t)$. It can be obtained from:

$$\tilde{z}_{ex}(t) = \int_{p \neq p_m} z(t, p)dp \tag{11}$$

Equivalently to eq.(10) the total unit sales can be written as:

$$\tilde{y}(t) = \tilde{y}_0(t) + \tilde{y}_{ex}(t) \tag{12}$$

where $\tilde{y}_0(t)$ is the total number of sold units per unit time at p_m and $\tilde{y}_{ex}(t)$ are the unit sales at $p \neq p_m$. The stable price distribution from the excess sold units is denoted $P_y^{ex}(p)$. The stationary price distribution of a product consists in this model of two components:

$$P_y\ (p) = \begin{cases} 1-a & for \quad p = p_m \\ \\ aP_y^{ex}(p) & for \quad p \neq p_m \end{cases} \tag{13}$$

where $0 \leq a \leq 1$ is a free parameter that quantifies the magnitude of excess units. The first term of eq.(13) stems from units sold at p_m and the second term is due to the excess units. The remainder of the paper aimed at deriving $P_y^{ex}(p)$ for $a \neq 0$.

2.2 The Derivation of the Price Distribution of Excess Units

The derivation starts by rewriting eq.(3) in the form (Note 5):

$$\frac{\partial z(t,p)}{\partial t} = \alpha(t,p)z(t,p) - \frac{\partial j(t,p)}{\partial p} \tag{14}$$

where the growth rate $\alpha(t,p)$ of available units is introduced by (Note 6):

$$\alpha(t,p) = \frac{1}{z(t,p)}\left[s(t,p) - y(t,p)\right] \tag{15}$$

The evolution of the total number of offered units becomes:

$$\frac{d\tilde{z}(t)}{dt} = \overline{\alpha}(t)\tilde{z}(t) \tag{16}$$

with the mean growth rate of supplied units:

$$\overline{\alpha}(t) = \frac{1}{\tilde{z}(t)}\left[\tilde{s}(t) - \tilde{y}(t)\right] \tag{17}$$

The mean growth rate can be obtained from:

$$\overline{\alpha}(t) = \frac{1}{\tilde{z}(t)}\int_0^\infty \alpha(t,p)z(t,p)dp \tag{18}$$

For further use the price scale is separated into three price regions: $p<p_m$, $p=p_m$ and $p>p_m$. Based on eq.(18), mean growth rates can be established for the corresponding price regions as follows.

For $p=p_m$:

$$\overline{\alpha}_{p_m}(t) = \frac{1}{\tilde{z}(t)}\int_0^\infty \alpha(t,p)z(t,p)\delta(p-p_m)dp \tag{19}$$

For $p<p_m$:

$$\bar{\alpha}_{p<p_m}(t) = \frac{1}{\tilde{z}(t)} \int_{0 \le p < p_m} \alpha(t,p) z(t,p) dp \tag{20}$$

and for $p>p_m$:

$$\bar{\alpha}_{p>p_m}(t) = \frac{1}{\tilde{z}(t)} \int_{p_m < p < \infty} \alpha(t,p) z(t,p) dp \tag{21}$$

The second statement of the model assumes that the market is in a stationary state (Note 7):

$$\frac{d\tilde{z}(t)}{dt} = 0 \tag{22}$$

ii) For the considered time interval Δt total unit sales equal total supply rate, $\tilde{s}(t) = \tilde{y}(t)$ (Note 8).

With eq.(17) this assumption is equivalent to:

$$\bar{\alpha}(t) = 0 \tag{23}$$

From the definitions eq.(19)- eq.(21) follows:

$$\bar{\alpha} = \bar{\alpha}_{p_m} + \bar{\alpha}_{p<p_m} + \bar{\alpha}_{p>p_m} = 0 \tag{24}$$

The case:

$$\bar{\alpha} = \bar{\alpha}_{p_m} = 0 \tag{25}$$

corresponds to the situation $a=0$, where all units are sold at p_m. For $a \neq 0$, the growth rate of offered units at p_m must be positive (Note 9):

$$\bar{\alpha}_{p_m} > 0 \tag{26}$$

and hence $\tilde{z}_{ex}(t) > 0$. The evolution of the excess units $z_{ex}(t,p)$ on the price scale can be approximated with eq.(14) for $p \neq p_m$ by:

$$\frac{\partial z_{ex}(t,p)}{\partial t} \cong \begin{cases} \bar{\alpha}_{p<p_m} z_{ex}(t,p) - \dfrac{\partial j(t,p)}{\partial p} & \text{for} \quad 0 < p < p_m \\ \\ \bar{\alpha}_{p>p_m} z_{ex}(t,p) - \dfrac{\partial j(t,p)}{\partial p} & \text{for} \quad p > p_m \end{cases} \tag{27}$$

where the time dependent growth rates $\alpha(t,p)$ are approximated by their mean values $\bar{\alpha}_{p<p_m}$ and $\bar{\alpha}_{p>p_m}$.

In order to solve eq.(27) a last assumption is made concerning the flow process of excess units:

iii) Price variations of excess units can be approximated as random.

This assumption has two consequences:

A) It implies that the drift of excess units takes place in both directions of the price scale away from p_m with equal chance. The effective current of excess units can be written as:

$$j(p) = v z_{ex}(p) \tag{28}$$

where v indicates the mean flow velocity of the excess offered units on the price scale. The sign of the flow velocity is positive for units flowing on average to increasing and negative for units flowing on average to decreasing prices, relative to p_m (Note 10).

B) There is no preference for the price dependent mean growth rates. For $p \neq p_m$, they can be approximated as equal. Applying eq.(24) we obtain:

$$\bar{\alpha}_{p<p_m} = \bar{\alpha}_{p>p_m} = -\frac{1}{2}\bar{\alpha}_{p_m} \tag{29}$$

The stable distribution of excess units ($\partial z_{ex}(t,p)/\partial t=0$) is governed with eq.(27) for $p>p_m$ by:

$$v\frac{dz_{ex}(p)}{dp} = \bar{\alpha}_{p>p_m} z_{ex}(p) \tag{30}$$

while for the price region $p>p_m$, the price drift takes place with positive mean velocity, $v>0$. With eq.(29) we obtain for eq.(30):

$$\frac{dz_{ex}(p)}{dp} = -\frac{\bar{\alpha}_{p_m}}{2|v|} z_{ex}(p) \tag{31}$$

which leads to the stationary distribution of the excess units for $p>p_m$:

$$z_{ex}(p) = C\exp\left(-\frac{\bar{\alpha}_{p_m}}{2|v|}p\right) \tag{32}$$

with the integration constant C.

For the price range $p<p_m$, the mean flow velocity is negative, $v<0$. The stationary solution of eq.(27) turns with eq.(29) into:

$$\frac{dz_{ex}(p)}{dp} = \frac{\bar{\alpha}_{p_m}}{2|v|} z_{ex}(p) \tag{33}$$

with the solution

$$z_{ex}(p) = C'\exp\left(\frac{\bar{\alpha}_{p_m}}{2|v|}p\right) \tag{34}$$

and an integration constant C'. From the symmetry of the distribution (assumption iii)) follows $C=C'$. Taking advantage from the fact that the total number of excess units is $\tilde{z}_{ex}$, we obtain for the stationary distribution of excess units:

$$z_{ex}(p) = \frac{\tilde{z}_{ex}\bar{\alpha}_{p_m}}{4|v|}\exp\left(-\frac{\bar{\alpha}_{p_m}}{2|v|}|p-p_m|\right) \tag{35}$$

The stationary probability density function (pdf) of offered excess units has the form:

$$P_z^{ex}(p) = \frac{z_{ex}(p)}{\tilde{z}_{ex}} \tag{36}$$

and becomes with eq.(35) (Note 11):

$$P_z^{ex}(p) = \frac{1}{2\sigma_L}\exp\left(-\frac{|p-p_m|}{\sigma_L}\right) \tag{37}$$

with the standard deviation:

$$\sigma_L = 2|v|/\bar{\alpha}_{p_m} \tag{38}$$

This result suggests that the price distribution of excess units is governed by a Laplace distribution around p_m (Note 12).

2.3 The Price Distribution of Sold Units

Based on assumption i) all units are supplied at p_m and with eq.(8) is $s(p)=0$ for $p \neq p_m$. Therefore, eq.(15) suggests that the unit sales rate of excess units $y_{ex}(p)$ can be written in the stationary state as:

$$-y_{ex}(p) = \begin{cases} \overline{\alpha}_{p<p_m} z_{ex}(p) & for \quad 0 < p < p_m \\ \\ \overline{\alpha}_{p>p_m} z_{ex}(p) & for \quad\quad p > p_m \end{cases} \tag{39}$$

It means, that all excess units supplied at p_m are sold in the price region $p \neq p_m$. With eq.(29) the relation turns into:

$$y_{ex}(p) = \frac{1}{2}\overline{\alpha}_{p_m} z_{ex}(p) \tag{40}$$

Taking advantage from the definitions eq.(1), eq.(2) and eq.(10), the stationary distribution of excess sold units reads:

$$P_y^{ex}(p) = \frac{y_{ex}(p)}{\tilde{y}_{ex}} = \frac{\frac{1}{2}\overline{\alpha}_{p_m} z_{ex}(p)}{\frac{1}{2}\overline{\alpha}_{p_m} \tilde{z}_{ex}} = P_z^{ex}(p) \tag{41}$$

where we used eq.(36) and eq.(40). In other words, the price distribution of sold excess units is equivalent to the price distribution of available excess units.

The probability distribution function of sold units eq.(13), turns therefore with eq.(12), eq.(37) into:

$$P_y(p) = \begin{cases} 1-a & for \quad p = p_m \\ \\ \frac{a}{2\sigma_L}\exp\left(-\frac{|p-p_m|}{\sigma_L}\right) & for \quad p \neq p_m \end{cases} \tag{42}$$

with the standard deviation σ_L given by eq. (38). The price distribution consists of a central peak at p_m surrounded by a Laplace distribution. The mean price of the price distribution corresponds to the price of maximum profit p_m:

$$\overline{p} = p_m \tag{43}$$

The parameter a expresses the magnitude of the total number of excess units and has the form:

$$a = \frac{\tilde{y}_{ex}}{\tilde{y}} = \frac{\tilde{z}_{ex}}{\tilde{z}} \tag{44}$$

where we used eq.(11) and eq.(41).

3. Comparison with Empirical Data

The model is compared with empirical data of a comprehensive investigation of prices of consumer products performed by Kaplan and Menzio (2014). They studied data from the Kilts-Nielsen Consumer Panel Dataset (KNCP) and investigated price and quantity information for over 300 million transactions by 50,000 households. The panel covers over 1.4 million goods in 54 geographical markets for the time period 2004-2009. The investigators aggregated the data of products into four different categories of a good:

1.) UPC: A good is the set of products with the same barcode (Universal Product Code: UPC).

2.) Generic Brand Aggregation: According to this definition, a good is the set of products that share the same features, the same size and the same brand, but may have different UPC’s. Since the KNCP assigns the same brand code to all generic brands (regardless of the retailer), this definition collects all generic brand products that are otherwise identical.

3.) Brand Aggregation: According to this definition, a good is the set of products that share the same features and the same size but may have different brands and different UPCs.

4.) Brand and Size Aggregation: In this case a good is the set of products that share the same features but may have different sizes, different brands and different UPCs.

After scaling the data by the mean price of a product they aggregated the data to price distributions of the corresponding definitions of a good. Since the first three definitions are closest to that of a consumer product used in this model, we confine here to a comparison with empirical data of the first three definitions.

For this purpose, the empirical data reproduced from Kaplan and Menzio in terms of a pdf are transformed into the corresponding cumulative distribution function (cdf). It is defined by:

$$F_y(p) = \int_0^p P_y(p')dp' \tag{45}$$

The cdf of eq.(42) reads:

$$F_y(p) = \begin{cases} \dfrac{a}{2}\exp\left(\dfrac{p-p_m}{\sigma_L}\right) & for \quad 0 < p < p_m \\ 1-a/2 & for \quad p = p_m \\ 1-\dfrac{a}{2}\exp\left(-\dfrac{p-p_m}{\sigma_L}\right) & for \quad p > p_m \end{cases} \tag{46}$$

Since eq.(43) suggests that the mean price of a product is equivalent to p_m, the price distribution eq.(46) can be also written as:

$$F'_y(p') = \begin{cases} \dfrac{a}{2}\exp\left(\dfrac{p'-1}{\sigma_S}\right) & for \quad 0 < p' < 1 \\ 1-a/2 & for \quad p' = 1 \\ 1-\dfrac{a}{2}\exp\left(-\dfrac{p'-1}{\sigma_S}\right) & for \quad p' > 1 \end{cases} \tag{47}$$

with the scaled price $p' = p/\bar{p}$ and variance:

$$\sigma_S = \sigma_L / p_m \tag{48}$$

The fit procedure to the empirical investigations was performed by applying a least square fit of the cumulative distribution eq.(47) using the two free parameters a and σ_S. The cdf was then transformed back into the pdf and plotted in Figures 1-3 as fat lines together with the empirical price data (squares). A good quantitative coincidence of the empirical data can be obtained with this two-parameter fit (Note 13). In view of the presented model, the first definition of a good (UPC) is closest to the model assumptions. It has a pronounced central peak surrounded by a Laplace distribution as suggested by the model. The central peak disappears (decreasing factor a) and the standard deviation slightly increases with a broader definition of a good.

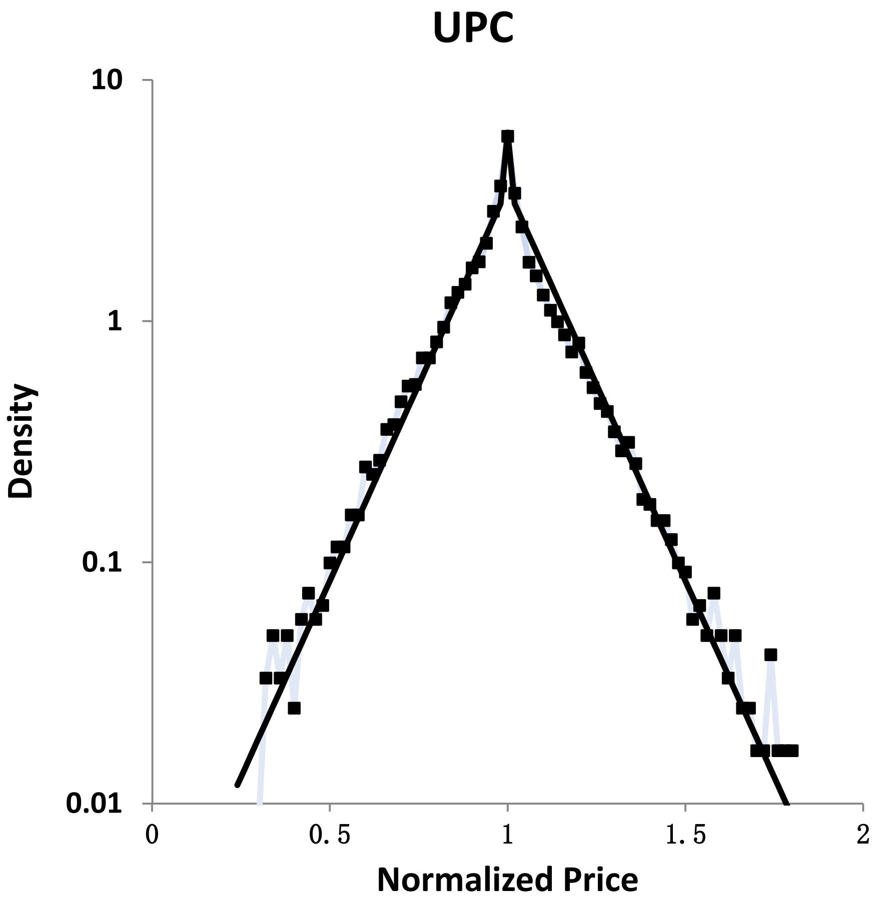

Figure 1. Displayed is the Empirical Price Distribution (Squares) for the First Definition of a Good, by Plotting the Distribution of Normalized Prices across All Investigated Markets, Goods and Quarters (Kaplan & Menzio, 2014). The Solid Line Indicates a Least Square Fit of eq.(47) with $a=0.05$, $\sigma_S=0.13$.

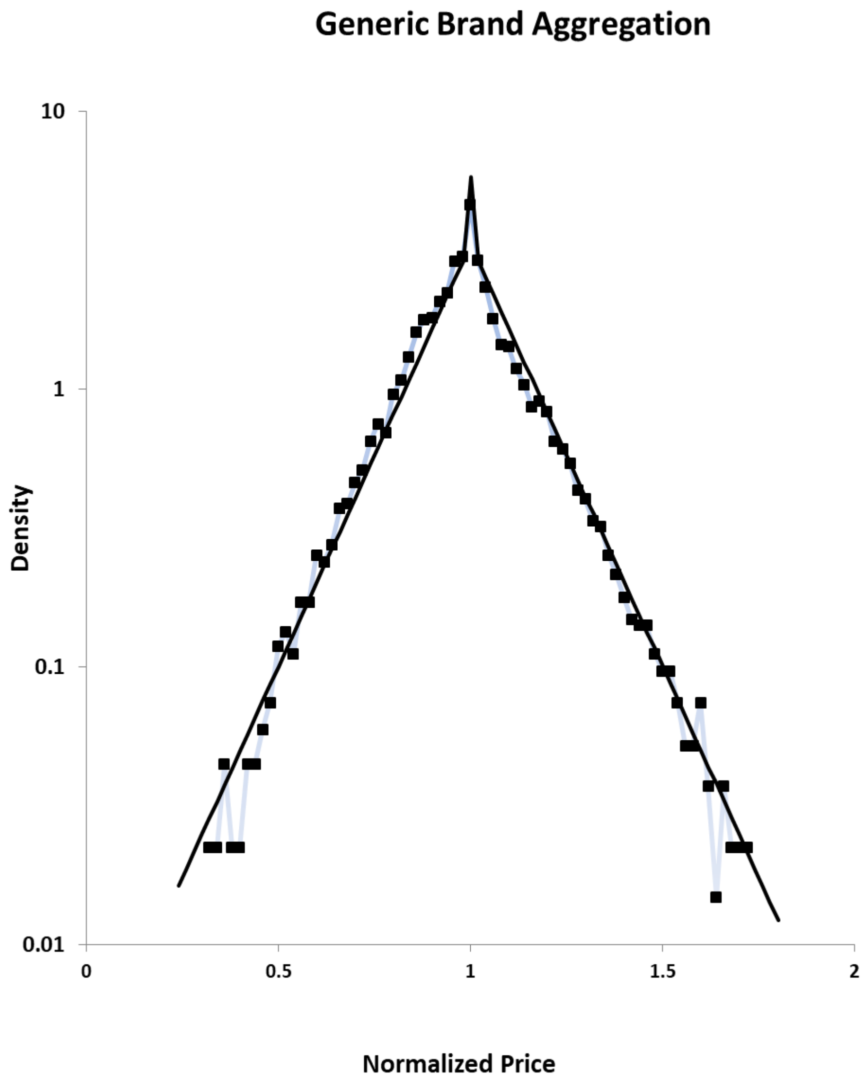

Figure 2. Displayed is the Empirical Price Distribution (Squares) for the Second Definition of a Good (Kaplan & Menzio, 2014). The Solid Line Indicates a Least Square Fit of eq.(47) with ***a*=0.05, σ_S=0.142.**

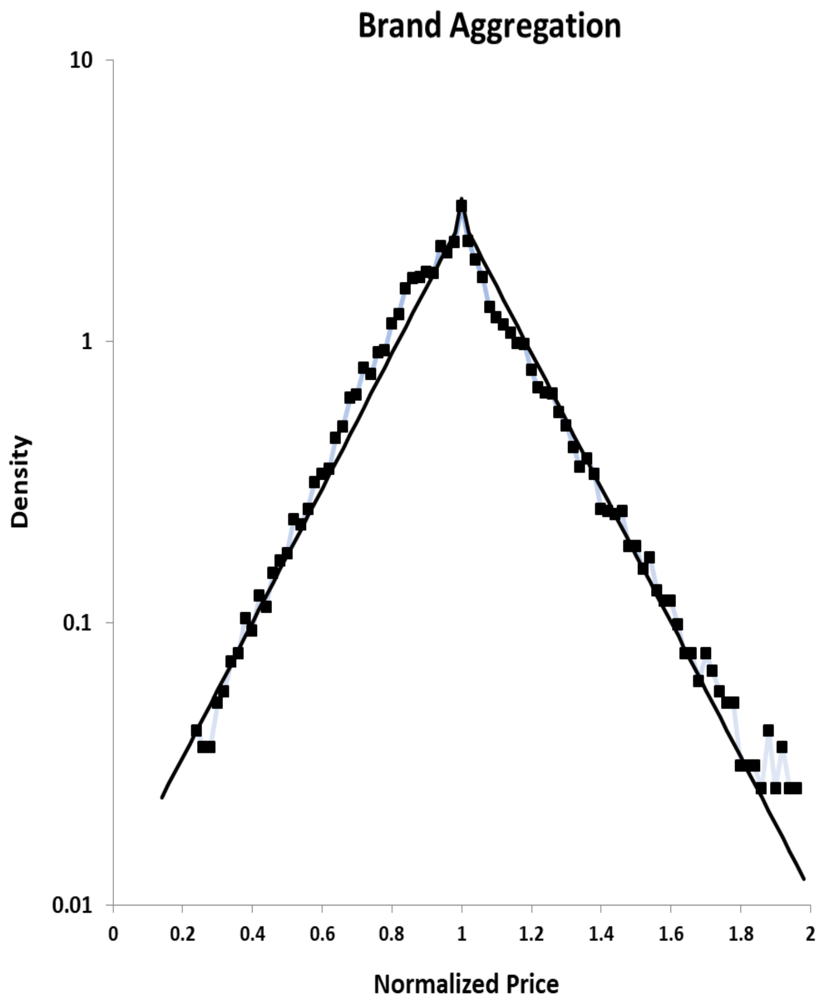

Figure 3. Displayed is the Empirical Price Distribution (Squares) for the Third Definition of a Good (Kaplan & Menzio, 2014). The Solid Line Indicates a Least Square Fit eq.(47) with $a=0.01$, $\sigma_S=0.182$.

4. Conclusion

The paper establishes a model for the price dispersion of consumer products. The presented model suggests that the price dynamics of a product is governed by a profit maximizing p_m. Excess units, supplied but not sold at p_m, suffer from a drift process on the price scale. The model suggests that a stable price distribution of sold units consists of a central peak at p_m, surrounded by a Laplace distribution as given by eq.(42). A comparison with empirical investigations of the price dispersion of different definitions of a good by Kaplan and Menzio yields a good quantitative agreement with a two-parameter fit. The data exhibit a decreasing central peak and a higher standard deviation of the price distribution with a broader definition of a good. This result can be explained with this model by the aggregation process of product variations with different p_m. Aggregating more product versions into a broader definition of a good, the profit maximizing price p_m can no longer be viewed as unique, but has to be described by a distribution. A broader definition of a good broadens therefore the price distribution and the central peak disappears, accompanied with an increase of the standard deviation. Note that the model is applicable only, if time-dependent variations of the profit maximizing price $p_m(t)$ are small during the time of investigation Δt, compared to the mean drift velocity v of excess units, $\Delta p_m(t)/\Delta t<<|v|$. It can be expected that the pressure to sell product units increases with a shorter mean offering time τ, hence $|v|\sim 1/\tau$. Therefore, the presented model rather applies to non-durables than to durables (Note 14).

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Notes

Note 1. A tilde over variables indicates total numbers.

Note 2. Note that the variable *z(t,p)* represents a density of available units. For a price interval p and $p+\Delta p$ the density of units changes in time by $(j(t,p)-j(t,p+\Delta p)/\Delta p$. For $\Delta p \rightarrow 0$ we obtain the differential contribution $-\partial j(t,p)/\partial p$. It can be interpreted as a term from a continuity equation for available units (Philipson & Schuster, 2009).

Note 3. Dirac-delta function: $\delta(p-p_m)=1$ for $p=p_m$ and 0 for $p\neq p_m$.

Note 4. This solution corresponds to the law of one price.

Note 5. Such an equation is known as a reaction-convection equation (Philipson & Schuster, 2009).

Note 6. For $\chi=0$.

Note 7. Assumption ii) can be interpreted as a finite total storage capacity of the sellers.

Note 8. Interpreting the total sales as total demand, assumption ii) is equivalent to the market equilibrium condition in standard microeconomics (Mankiw & Taylor, 2020).

Note 9. A negative growth rate of offered units at p_m implies that more units are sold than supplied at p_m, which is a contradiction with *i)* and therefore not possible in this model.

Note 10. In this approximation the magnitude of the velocity is treated as independent of the flow direction.

Note 11. The distribution is valid for $p\neq p_m$. However, including $p=p_m$ leads in this continuous description to an infinitesimal small error, that can be neglected.

Note 12. Note that assumption iii) suggests, that the drift process of excess units can be considered as a one-dimensional Brownian motion of the excess units on the price scale with a mean flow velocity v. However, the motion of the excess units is not just a slow diffusional flow. If this would be the case the Brownian motion would level out concentration differences of available units on the price scale and the price distribution would rather be a normal distribution.

Note 13. The relative error is always less than 10%.

Note 14. Durables usually exhibit a lognormal price distribution (Coad 2009). Electric current has a very short offering time. The price distribution for this homogeneous good can be described by a Laplace distribution (Sapio, 2004; Kaldasch, 2015).