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Abstract
This study estimate technical efficiency indices and examines evidence of economies of scope in Botswana agriculture for each 18 districts and commercial sector using a multiple-output multiple-input stochastic input distance function approach covering data from 1979 to 1996. The estimated model provides input-output relations, economies (diseconomies) of scope and technical inefficiency. All the production outputs (cattle, crops and goats/sheep) were significant with expected signs. The estimated mean technical efficiency of 0.885 for 18 districts and the commercial sector was obtained. This suggest the existence of inefficiency in Botswana agricultural production which indicates that there is opportunity to increase production with the same quantities of input factors, and through adaptation of improved technology such as irrigation, use of fertilisers, and improved high quality crops and livestock. There is significant in economies of scope between the production of cattle and goat/sheep, at the 1 percent level, and cattle and crops at 5 percent level. This existence of economies of scope indicates that higher economic returns are possible through efficient use of labour and livestock feeds, and reducing risk by not producing output (e.g., crops) that is easily affected by droughts and poor soils.

Keywords
technical efficiency, inefficiency, economies of scope, stochastic input distance function, trans-log, multi-input and multi-output function

1. Introduction
Botswana is a land-locked Southern African country surrounded by Zimbabwe, Namibia, Angola, and South Africa, with an area of 566,000 square kilometres and population of almost 2 million people, its population density being unusually low at 2.6 persons per square kilometre (World Bank, 2002). However, the carrying capacity is low because the soils are mostly poor and the climate is semi-arid with frequent droughts so most of the land is better suited for cattle ranching than arable agriculture (Thirtle & Irz, 2004). In the past beef export, especially to the European Union, was the major source
of foreign exchange. Since the advent of diamond mining in Botswana, diamond exports and tourism are the major sources of foreign exchange, which has rescued Botswana from being one of the poorest nations in the world. Mining does not provide enough employment for the whole nation, so the wealth is not shared and agriculture is still the main source of income for the rural population. Agriculture accounts for 5 percent of Gross Domestic Product (GDP) (World Bank, 2002). This means that the rural population who rely on agriculture for a living is relatively poor and there is unequal distribution within the sector.

The diamond revenue allows the government to spend as much as 40 percent of agricultural GDP on support schemes such as CEDA Young Farmer’s Fund, which aims to improve the welfare of the agricultural population and keep them in the rural areas, despite very harsh conditions (Thirtle et al., 2000). Despite efforts made by the government, growth of the agricultural sector has been decelerating which remains one of the most urgent objectives facing policy makers in developing countries, such as Botswana, where agricultural productivity is low and most of the food is imported, despite the amount being inadequate. Although international trade and food aid may alleviate short-term imbalances between the growth in demand and supply of food, it is likely that long-term food security will only be achieved by a sharp increase in domestic food production (Dadi et al., 2004).

This study aims to estimate the technical efficiency of 18 districts of Botswana and its commercial sector using an input distance function model. The fact that prices are not needed is an advantage for the analysis of productivity growth in developing countries where markets for major inputs, such as land and labour, are often not sufficiently developed for there to be meaningful prices (Irz & Thirtle, 2004). The approach used for this study accounts for noise which is important for the analysis of productivity growth in agriculture in general, the sector being subject to important production shocks, but it seems particularly relevant in the Botswana context where the variability of rainfall is well established (Seleka, 1999). Furthermore, it can determine the existence of economies (diseconomies) of scope between crops, cattle, and goats/sheep by considering the second partial derivative of the input distance function using Excel. That could indicate whether Botswana farmers should focus on the production of a single product or diversify into the production of two or more outputs. According to Coelli et al. (2006), the most important factors to consider are twofold: the degree to which diversification could reduce risks associated with output price volatility; and the degree to which diversification (or specialisation) results in higher or lower unit costs.

Technological advances in the farming sector inclined more towards on-farm specialization, the trend in reduced crop diversification has continued but at a much reduced rate, and the proportion of farms without livestock has significantly increased (Helmers & Shaik, 2003). Currently, beginning farmers tend to concentrate solely on crop production and encounter difficulties in assembling financial control over adequate sized units. Generally, there may well be a lack of understanding of the existing advantage of integrated operations relating to the agriculture sector, namely diversification and the analysis of what enterprises can be integrated for purposes of higher economic returns with reduced
risks (Helmers & Shaik, 2003).

In this study, the measures of economies of scope were derived from estimated coefficients of the input distance function, which are often derived from a multi-output cost function. The estimates of economies of scope were obtained using the estimated input distance function results, considering second partial derivatives of the input distance function with respect to the output of interest. This approach has the advantage that the input distance function does not require questionable behavioural assumptions, such as cost minimising behaviour, nor does it require input price data, which are often difficult to obtain (Coelli et al., 2006).

This study extends the work done by Irz and Thirtle (2004), by analysing the agricultural efficiency and productivity in the 18 districts of Botswana and the commercial sector, for the period from 1979 to 1996. It differs from their work in that it considers economies of scope comparing farmers who focus on the production of one output with those who diversify into two or more products. This study’s data set uses three outputs in estimation of the model instead of the two outputs employed by Irz and Thirtle (2004). Additionally, the number of inputs variables was increased from five to eight by splitting herd’s size into cattle and goats/sheep, as well as rainfall and animal feeds. Specifically, the aims of this report are to derive the estimates of the production relations of Botswana agriculture from 1979 to 1996 using an input distance function; estimate the district’s and commercial sector’s specific and mean technical efficiency; and obtain measures of evidence of economies of scope from the estimates of parameters of the second order derivatives of the input distance function with respect to outputs variables.

One question that was of particular interest in this study is: “to what degree can farmers of Botswana benefit from economies of scope by producing both crops and cattle; crops and goats/sheep; or cattle and goats/sheep”. The nature of the agricultural production process in Botswana is not highly managed and, therefore, technical inefficiency is expected to be high. A particular research interest in the analysis of this study is the determination of whether it is more beneficial for farmers to follow diversification, thereby producing two or more products; or for those who focus on a strategic production of only one output. In Botswana agriculture, some farmers traditionally choose to produce jointly, while others choose to specialize in the production of one output. In the long run, for competitive equilibrium, it would be expected that farmers will tend to joint production of output if the diversification hypothesis were correct and towards specialization if the strategic focus on the production of one output cannot be rejected, and is therefore correct. Farmers following an inefficiency strategy would be compelled by market forces to change their strategy or exit the industry (Cummins et al., 2003).

An economies of scope hypothesis argues that owning and operating a broader range of businesses can add value from exploiting cost scope economies by sharing inputs in joint production through factors such as managerial economies of scale (Teece, 1980). They also posit that earning diversification can lower the cost of capital and raise the debt capacity of the firm. Internal capital markets are said to allocate resources efficiently and to be less prone than external markets to information asymmetries and
other imperfections (Williamson, 1970; Gertner et al., 1994). These arguments suggest the following hypotheses for Botswana agricultural production:

H0a: Scope economies exist between crop and cattle production.
H1a: Scope economies do not exist between crop and cattle production.

H0b: Scope economies exist between crop and goats/sheep production.
H1b: Scope economies do not exist between crop and goats/sheep production.

H0c: Scope economies exist between cattle and goat/sheep production.
H1c: Scope economies do not exist between cattle and goats/sheep production.

The remainder of this paper is organised as follows. Section 2 discusses the agricultural production in Botswana, by evaluating the existing literature relating to the productivity performance of the whole of Botswana agriculture. Section 3 describes the research methodology and the data. It delineates how the data was used to estimate the input distance function, and the relationship between the second derivatives of the cost function and input distance function which is used to derive evidence of economies of scope in terms of the derivatives of the input distance function. Section 4 provides the results and discussion of them, beginning with tests to determine the appropriate model and the efficiency indices, while concluding remarks and policy implications are made in Section 5.

2. Agriculture Production in Botswana

2.1 Challenges and Role of Agriculture in Botswana

Botswana agricultural production is hampered by traditional farming methods, recurrent drought, soil erosion, and disease, which might cause the country’s food situation to reach crisis level if agriculture is not improved. Environmental factors have determined the kinds of crops and livestock that can be raised in the country. Most arable production activities take place in the eastern part (Northeast and Central Districts) of the country where the climate and soil conditions are more favourable.

The principal crops for domestic use are sorghum, maize, millet, and beans/pulses. Such crops are generally grown for immediate consumption. With an average farm size of 2.3 hectares, production does not meet subsistence needs and rural householders are compelled to supplement their food requirements. Irrigated crops have proved difficult to promote and the country has to import up to 80 percent of its food requirements. The sorghum and maize harvests comprise 10 percent of the annual requirement of 250,000 tonnes. In 2001, Botswana imported 174,198 tonnes of cereals, valued at $53 million from South Africa and Zimbabwe. The result of this stagnation of production is that Botswana farmers have not improved their farming technology in line with the level of output required by growing numbers and commercial economic settings of modern Botswana. In this country, large farmers have come to eschew crop farming in favour of livestock breeding. This means that farmers with the most adequate resources and greatest capacity to significantly alter the level of food crop production do not participate in crop production (Odell, 1980).

The livestock (cattle, goats and sheep) production is the most important in Botswana agricultural sector.
because its contribution averages 80 percent of the agriculture share of GDP. Livestock not only play a major role in agriculture GDP, but also act as a draught resource. Families with less than 40 herd of cattle or none at all, tend to plough small tracts of land, and plough and plant later as a result of having later access to draught power. Therefore, their harvests have low outputs, while those with large herds of cattle can plough large areas of land, thereby ensuring themselves higher output during normal rainy seasons (Alverson, 1979). The major part of the country is semi-desert and partly-savannah with erratic rainfall and poor soil conditions, making it more suitable for non-intensive cattle production. Estimates show that the animal population is close to being excessive to land carrying capacity at 3.2 billion. The Botswana Meat Commission (BMC) has a statutory monopoly on all beef exports. All of its abattoirs has a combined throughput of 1,320 cattle and 700 goats and sheep per day. Beef is one of the country’s major export items and it has an export quota of 19,000 tonnes per annum of boneless beef to the European Union. The Botswana dairy industry is still in its infancy so most fresh milk and dairy products are imported from South Africa.

Agriculture in Botswana is practised primarily to feed the country, rather than for export. Yet, agricultural production is not sufficient to meet domestic demand, Botswana agricultural exports being worth US$114.2 million in 1998, while agricultural imports for the same year totalled US$348.4. Though the majority of people in Botswana practise agriculture (80 percent), it contributes only 4 percent of the country’s GDP, and accounts for only 15.6 percent of formal employment (National Economies Encyclopedia, 2007).

2.2 Crops and Livestock Performance

![Figure 1. Trend of Major Agricultural Outputs](source: Botswana Agricultural Statistics, Central Statistics Office.)

Figure 1 shows the trend of total production of crops including sorghum, maize, millet and beans/pulses in tonnes, total number of cattle from sales and slaughter and total number of goats and sheep from sales and slaughter for traditional farming of 18 districts in Botswana and the commercial sector. The movement of crop production for the period covered in this study has been generally trending upward, though there was significant declines during 1989, 1990, 1992 and 1993. The 1981-1984 agricultural statistics show that yield rates generally decline with an increase in land area, but that a large farm’s output is a function of land cultivated rather than productivity per unit of land. Despite big areas
allocated to large farmers (i.e., larger cattle owners), there is a weak upward trend in yield per unit of land with cattle ownership up to 80 herds, followed by a sharp decline among the biggest cattle owners. On the whole, although the big farmers have considerable recourses to further improve their quality of crop production, their investment in arable farming has been relatively low.

The production of cattle was generally moving at a constant trend from 1979 to 1990, and then started trending downward until 1996. On the other hand, goats and sheep production from 1979 to 1990 was trending upward, then showed a downward trend until 1996. This was the result of long cycle of continued drought that dominated the country since 1990s. The livestock sector did not perform well after 1990 also due to various factors, including an outbreak of Foot and Mouth disease at Pandamutenga. Over the past few years the country has never produced enough to satisfy the dietary needs of its population, at best managing to produce only 50 percent of its cereal requirement.

3. Methods of Analysis and Data

3.1 Analytical Framework

3.1.1 Standard Stochastic Frontier Model

The stochastic frontier model is an alternative approach to the estimation of frontier functions using econometric techniques. Following Aigner and Chu (1968), the stochastic frontier production function model takes the form:

\[ \ln q_i = x_i' \beta + v_i - u_i \]  \hspace{1cm} (1)

Where \( q_i \) represents output of the \( i^{th} \) firm; \( x_i \) is a \( K \times 1 \) vector containing the logarithms of inputs; \( \beta \) is the vector of an unknown parameter; \( v_i \) is symmetric random error, which accounts for statistical noise; and \( u_i \) is a non-negative random variable associated with technical inefficiency.

Much of the stochastic frontier analysis is directed towards the prediction of technical inefficiency effects, which is the ratio of observed output to the corresponding stochastic frontier output for output oriented measures (Aigner et al., 1977; Meeusen & Broeck, 1977).

\[ TE_i = \frac{q_i}{\exp(x_i' \beta + v_i)} = \frac{\exp(x_i' \beta + v_i - u_i)}{\exp(x_i' \beta + v_i)} = \exp(-u_i) \]  \hspace{1cm} (2)

Technical efficiency measures the output of the \( i^{th} \) firm relative to the output that could be produced by a fully efficient firm using the same input vectors. The measure of technical efficiency takes a value between zero and one.

3.1.2 Stochastic Input Distance Function

A multiple-input multiple-output stochastic input distance function was used to calculate technical efficiencies indices for each sampled district and the mean technical efficiency across all districts, which allows us to describe production technology without specifying cost minimisation or profit maximisation behaviour. An input distance function is the minimal proportional contraction of input
vectors, given the output vector. Conversely, output distance function is the maximal proportional expansion of the output vector, given the input vector (Coelli et al., 1998).

Input distance function is used in preference to output distance function when farmers or firms have more control over inputs than outputs and vice versa (Coelli & Perelman, 1996; O’Donnell & Coelli, 2005). This study will use the stochastic input distance function because the number of animals needed for slaughter by the Botswana Meat Commission is fixed by export contracts and home consumption, and the amount of crops sold to the Botswana Agricultural Marketing Board (BAMB) is limited. Thus, it makes sense for farmers to adjust their inputs including fertiliser application and daily feed intake by animals, in order to produce the required number of animals and amount of crops. Therefore, a farm manager’s objective should be to produce the desired output level in the minimum number of days using the minimum amount of animal feed intake and fertiliser application within that period.

One of the aims of this study is to conduct hypothesis tests on whether significant economies of scope or diseconomies exist between outputs in the farm production process across Botswana districts. So in order to estimate the appropriate coefficient needed to calculate economies of scope for all output variables, it considers the use of an input variable on the left hand side of the equation to be estimated.

This is done to allow significant tests on all combination of crops, cattle and goats/sheep outputs.

3.1.3 Measuring Technical Efficiency
Prior to estimation, 18 zero-one dummy variables were created for 17 different Botswana districts and the commercial sector, with the Borolong district being the base. These dummy variables were added to allow the technologies to differ in level between different districts and the commercial sector. Furthermore, due to lack of data of technical efficiency variables, these 18 districts and commercial sector dummy variables were used also as inefficiency effects in the model to help explain differences in $u_i$. The unknown mean $u_i$ is defined by:

$$ u_i = \delta_0 + \sum_{m=1}^{18} \delta_m z_{mi} $$

(3)

Where $Z_1$ is the dummy variable for Ngwaketse; $Z_2$ for Ngwaketse; $Z_3$ for Bamalete/Tlokweng; $Z_4$ for Kweneng North; $Z_5$ for Kgaetleeng; $Z_6$ for Kweneng; $Z_7$ for Mahalapye; $Z_8$ for Phalapye; $Z_9$ for Serowe; $Z_{10}$ for Bobonong; $Z_{11}$ for Tutume; $Z_{12}$ for Tati; $Z_{13}$ for Ngamiland West; $Z_{14}$ for Ngamiland; $Z_{15}$ for Chobe; $Z_{16}$ for Ghanzi; $Z_{17}$ for Kgalagadi; and $Z_{18}$ for the commercial sector (Note: the eighteenth Botswana district, Borolong, serves as the base for comparisons).

3.1.4 Measuring Economies (Diseconomies) of Scope
Economies of scope occur when the joint output of a single firm is more than the output that could be achieved by two different firms each producing a single output, with equivalent production units allocated between two firms. However, if the production of two single firms each producing output separately is more than the joint production of a single firm diseconomies of scope occur, and this happens when the production of one output conflicts with the production of the second output (Pindyck & Rubinfeld, 2000). According to Panzar and Willig (1981), there are economies (diseconomies) of
scope where it is less (more) costly to produce two or more outputs jointly in a single firm rather than producing them separately. Economies (diseconomies) can be defined relative to the cost function—for example, in the case where a firm produces two outputs \(y_1\) and \(y_2\), there is a weak economies of scope between outputs 1 and 2 if:

\[
C(y_1, y_2, p) \leq C(y_1, 0, p) + C(0, y_2, p)
\]  

(4)

Coelli et al. (2006) state that economies of scope cannot be verified directly when the cost function assumes forms in which outputs appear in a logarithmic or a reciprocal form. In such cases, the following sufficient conditions can be used for testing the presence of economies of scope. They include the requirement that the cost function exhibit weak cost complementarities, namely:

\[
\frac{\partial^2 C(\tilde{y}, p)}{\partial y_i \partial y_j} \leq 0, \ i \neq j
\]  

(5)

For all \(\tilde{y}\), such that \(0 \leq \tilde{y} \leq y\) (Baumol et al., 1988).

According to (Deller et al., 1988) the second cross partial derivative of the estimated cost function is used to test evidence of economies of scope, meaning that with a cost function \(C=c(y, z, w)\) (where \(y\) is output; \(z\) is input quantities; and \(w\) is input prices) it can be observed that economies of scope exist between output \(i\) and \(j\) if:

\[
\frac{\partial^2 C}{\partial y_i \partial y_j} < 0, \ i \neq j, \ i, j=1\ldots m
\]  

(6)

Where \(C\) is the cost of \(m\) outputs; and \(y_i\) is the \(i\)-th output variable. Equation (6) states that the addition of an extra unit of output \((i)\) decreases the marginal costs of producing an extra unit of output \((j)\).

In this study, instead of using the standard approach, we use the estimate of the input distance function instead of the cost function. This is because there is no access to cost data due to the impossibility of obtaining prices of many of the inputs in the production system which is the focus of this study. Also, the input distance function is estimated to allow for the possibility of inefficiency in the production model.

Based on Coelli et al. (1998, p. 64), the input distance function is defined as:

\[
d(x, y) = \{ D : (x / D) \in L(y) \}
\]  

(7)

Where \(L(y)\) is the set of all fixed variables of input vectors \(x\) that can produce the output vector \(y\). The expression \(d(x, y)\) is non-decreasing in the input vectors \(x\), increasing in the output vector \(y\), and linearly homogenous and concave in \(x\). The value of the distance function is equal to 1, or more than 1 if \(x\) is an element of the feasible input set \(L(y)\). That is, \(d(x, y) \geq 1\) if \(x \in L(y)\). It is equal to 1 if \(x\) is located in the inner bound of the input set, where the firm is technically efficient and more than 1 if the firm is technically inefficient. As stated by Coelli and Fleming (2003), the measure of economies of scope is defined relative to an input distance function. The first partial
derivative of the input distance function with respect to the \( i \)-th output is negative. This indicates that the addition of an extra unit of output, with all other variables held constant, reduces the amount needed to put the observation onto the efficient frontier by deflating the input vector. A positive second cross partial derivative is evidence of economies of scope, which exist between output \( i \) and \( j \) if:

\[
\frac{\partial^2 D}{\partial y_i \partial y_j} < 0, \quad i \neq j, \quad i, j = 1, 2\ldots N
\]

Conversely, a negative second cross partial derivative of the input distance function indicates diseconomies of scope.

3.2 Empirical Model

3.2.1 Trans-Log Input Distance Function

Following Coelli and Perelman (1996), assuming we have cross sectional data on \( I \) firms with \( N \) inputs and \( M \) outputs, an input distance function takes the form:

\[
d^I_{it} = d^I_i(x_{i1}, x_{i2} \ldots x_{Ni}, q_{i1}, q_{i2} \ldots q_{Mi})
\]

Where \( x_{ni} \) is the \( N_i - \text{th} \) input of the firm \( i \); \( q_{mi} \) is the \( M_i - \text{th} \) output; and \( d_{it}^I \geq 1 \) is the amount by which inputs vectors can be reduced without affecting output vectors. Important properties of Equation (9) are that it is; is non-decreasing, linear homogenous and concave in inputs; and non-increasing and quasi concave in outputs.

The model structure of Coelli and Perelman (1996) and Kumbhakar et al. (2003) used to define the trans-log stochastic input distance function, and used in this analysis is:

\[
\ln d^I_{it} = \beta_0 + \sum_{i=1}^{N} \beta_i \ln X_i + \sum_{i=1}^{N} \alpha_i \ln Y_i + \lambda_i t + 0.5 \sum_{i=1}^{N} \sum_{j=1}^{N} \beta_{ij} \ln X_i \ln X_j + 0.5 \sum_{i=1}^{M} \sum_{j=1}^{M} \alpha_{ij} \ln Y_i \ln Y_j + 0.5 \lambda_{ii} t^2 + \sum_{i=1}^{M} \sum_{j=1}^{M} \omega_{ij} \ln X_i \ln Y_j + v_i - u_i
\]

Where \( i = 1, 2\ldots N \); \( d^I_{it} \) is the input distance for the \( i \)-th districts in time period \( t \); \( X_i \) and \( Y_i \) are the input and output vectors of the \( i \)-th district, respectively; and the Greek letters are parameters to be estimated. To obtain the frontier, the distance function must be valid, and specification must satisfy the following homogeneity restrictions:

\[
\sum_{i=1}^{N} \beta_i = 1, \quad \sum_{j=1}^{N} \beta_{ij} = 0, i = 1, 2\ldots N, \quad \sum_{j=1}^{M} \alpha_{ij} = 0, i = 1, 2\ldots N
\]

In addition to homogeneity restrictions, it must also satisfy the following symmetry restrictions due to Young’s theorem (Coelli et al., 2006).

\[
\beta_{ik} = \beta_{ki}, \quad \text{and} \quad \alpha_{ij} = \alpha_{ji}, \quad \text{for all} \quad i, k, j \quad \text{and} \quad l.
\]
The \( v_i \) factor represents random errors assumed to be independently and identically distributed with zero mean and variance \( \sigma_v^2, \text{N}(0, \sigma_v^2) \), while the \( u_i \) factor indicates the technical inefficiency effects and are assumed to be half normal and independently distributed such that \( u_i \) is defined by the truncation at zero of the normal distribution with known variance \( \sigma_u^2 \). The variance parameters \( \sigma_v^2 \) and \( \sigma_u^2 \) are replaced by \( \gamma = \sigma_u^2 / (\sigma_v^2 + \sigma_u^2) \) and \( \sigma_s^2 = (\sigma_v^2 + \sigma_u^2) \) (Coelli et al., 1998).

The technical efficiency of each firm can be predicted using the conditional expectation of \( \exp(-u_{it}) \), given the value of \( e_{it} = v_{it} - u_{it} \). Since \( u_{it} \) is the non-negative random variable, these technical efficiency predictions are between zero and 1, with a value of one indicating a fully efficient firm. The input distances are predicted as \( d_i = E[\exp(u)/e] \).

3.2.2 Estimation Procedures

1) Variables

The panel data used in estimating the stochastic input distance function and economies of scope (diseconomies) in 18 Botswana districts and the commercial sector includes: crops (total production of sorghum, maize and millet, and beans/pulses in tonnes); number of cattle (from sales and home slaughter); and number of goats/sheep (from sales and home slaughter)—by districts as outputs. The eight inputs in the analysis are factors of production and include: land (area planted in hectares); labour, which includes livestock labour use per average herd of animals in various years and total labour used for ploughing and planting crops by regions; cattle and goats/sheep herd size \( (10^3 \text{ heads}) \) measured on entry to the farm; rainfall recorded by districts and regions; total weight of seeds planted per hectare; total fertilizer applied by districts and regions; and the daily intake of feed by animals (tonnes). No data is available for the use of veterinary supplies, the provision of feeding facilities, the cost of weeds and pest control for crops, or the costs of disease control in livestock. However this lack of data is not considered as a drawback to this analysis, because the data used in this study cover the most important variables.

Estimation of the trans-log stochastic input distance function requires transformation of the original data and the mean correction of the variables. Prior to taking the input variable of interest (e.g., Land) to the Left-Hand Side (LHS) of the trans-log equation to be estimated, it was necessary to divide each of the remaining input variables by the Land variable. Natural logs were then derived for all the variables and the mean corrected accept time period calculated.

2) Model Specification

The multiple-input multiple-output stochastic input distance function used to calculate the technical efficiency for each sampled Botswana district in each year from 1979 to 1996, and the mean technical efficiency by year was applied using a trans-log functional form covering 340 observations. Prior to estimation, the means of the log variables were adjusted to zero so that the coefficient of the first order
term can be interpreted as elasticities, evaluated at the sample means. Following Coelli and Perelman (1996), we set: \( \ln d_{it} = v_{it} - u_{it} \), and impose the restrictions required for homogeneity of degree+1 in inputs (with 8 inputs because, in this study, the sum of 187654321 equals 8). To obtain the estimation form of the trans-log input distance function with time trend:

\[
\begin{align*}
- \ln X_1 &= \beta_0 + \sum_{k=1}^{K-1} \beta_k \ln X_{kit} + \sum_{m=1}^{M} \alpha_m \ln Y_{mit} + \lambda_i t + \frac{1}{2} \sum_{k=1}^{K-1} \sum_{k'=1}^{K-1} \beta_{kk'} \ln X_{kit} \ln X_{kit} \\
&\quad + \frac{1}{2} \sum_{m=1}^{M} \sum_{m'=1}^{M} \alpha_{mm'} (\ln Y_{mit})^2 + 0.5 \lambda_2 t^2 + \sum_{k=1}^{K-1} \sum_{i=1}^{M} \beta_{mi} \ln X_{kit} \ln X_{1} + \sum_{m=1}^{M} \sum_{s=1}^{M} \alpha_{ms} \ln Y_{mit} \ln Y_{sit} \\
&\quad + \sum_{k=1}^{K-1} \sum_{m=1}^{M} \omega_{km} \ln X_{kit} \ln X_{1} * \ln Y_{mit} + \sum_{k=1}^{K} \phi_k D_i + v_{it} - u_{it} \quad (11)
\end{align*}
\]

Where \( X_1 \) is land planted (in hectares); \( X_2 \) is total labour (number of workers) used for crops and livestock; \( X_3 \) is cattle herd entry to the farm (10^3 heads); \( X_4 \) is goats/sheep herd entry to the farm (10^3 heads); \( X_5 \) is draft power (10^3 heads); \( X_6 \) is total fertiliser applied by districts and regions (in Mt); \( X_7 \) is amount of rainfall recorded by districts and regions (in ML); \( X_8 \) is daily intake of feed by animals (in tonnes); \( Y_1 \) is total production of crops (maize, millet sorghum, beans/pulses); \( Y_2 \) is total production of cattle at sales and home slaughter; and \( Y_3 \) is total production of goats/sheep at sales and home slaughter.

The choice of which input variable to put on the Left-Hand Side (LHS) is arbitrary (in this study: Land) for enabling the model. If distance function is a function of all inputs and outputs, and -ln Land is put on the LHS of the equation while keeping the input distance on the RHS, and if \( u=0 \) for district \( i \) (no inefficiency), the LHS will be equal to zero: that is, the district on the frontier \( (d_i=1) \) as \( \exp(0)=1 \). The coefficient of the first order terms in the estimated model can be interpreted as elasticities. For example, in the case of input variables, the coefficient reflects the percentage change in the set of outputs for a 1 percent change in this particular input, and for outputs the coefficients reflect the percentage change in outputs as a result of a 1 percent change in the set of inputs. Estimates of the parameters of the trans-log input distance function were obtained using the maximum likelihood procedure, detailed by Coelli and Perelman (1996), processed by the FRONTIER 4.1 computer program. Various hypothesis tests were undertaken using likelihood ratio tests set at the 5% level of significance.

### 3.3 Data and Data Sources

Livestock production data (cattle, goats and sheep) for Botswana’s 18 districts and commercial sector used in this study were obtained from the Botswana Agricultural Census Report, prepared by the Botswana Agricultural Statistics and Central Statistics Office (CSO) from 1979 to 1996. Livestock labour used per average head of cattle and goats/sheep from Farm Management Surveys (Department of Agricultural Planning and Statistics) were also included in the analysis. The number of cattle, goats and sheep aggregated in cattle equivalents by using appropriate weights, for the districts came from the Botswana Agricultural Census Report and Botswana Agricultural Statistics (CSO, 1979-1996) (Irz & Thirtle, 2004).
The total production of crops (e.g., sorghum, maize, millet) by districts was collected from the Botswana Agricultural Census Report and Botswana Agricultural statistics, while the total labour used for ploughing and planting was collected from Botswana Agricultural Statistics (CSO, 1979-1996). The total weight of seeds planted per hectare, seed quantities, total fertilizer used, total number of animals used for draft power and area planted by districts were obtained from Botswana Agricultural Statistics (CSO, 1979-1996).

4. Results and Discussion

4.1 Hypotheses Testing

Table 1, below, represents performance statistics for the alternative specification of the model evaluated using the generalised likelihood ratio test, which compares the likelihood function under the null hypothesis and alternative hypothesis. The first test focuses on the statistical significance of the $\gamma$ (gamma) parameter ($H_0: \gamma=0$), which compares the stochastic frontier model with the mean input distance function, estimated assuming that the inefficiency term is non stochastic and is equal to zero, using a simple z-test. This implies that any deviation from the frontier of the input requirements set is explained solely by random shocks so ordinary least squares is an appropriate representation of the model. Battese and Coelli (1992) stated that the closer $\gamma$ is to 1, the more significant is the presence of technical inefficiency. This means that when $\gamma=1$, 100 percent of the variation in observed output from the frontier is due to inefficiency. The $\gamma$ parameter in Table 3 is 0.977 and the test ratio shows that this parameter is significantly different from zero that the stochastic frontier is an adequate representation of the model.

The second test concerns the separability of the inputs and outputs in the input distance function. This hypothesis is defined by mathematically equating all cross terms between inputs and outputs to zero ($\omega_{ij}=0$) (Irz & Thirtle, 2004). The likelihood ratio test computed is greater than the critical value reported in Table 1, therefore this restriction is rejected, which means that it is not possible to aggregate consistently the two outputs into a single index. This result suggests the necessity of using the input distance function rather than a stochastic frontier production function, which requires output aggregation before estimation. Another test performed compares the trans-log functional form with the null hypothesis that the Cobb-Douglas function form is an adequate representation of the data. Table 1 reports the likelihood ratio test computed using the maximised likelihood ratio values obtained from Table 2 (Cobb-Douglas estimates) and Table 4 (Trans-log estimates). This null hypothesis is rejected, implying that restrictions imposed by the Cobb-Douglas functional form are inappropriate.
Table 1. Tests of Hypotheses

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Parameter Restrictions</th>
<th>Test Statistic</th>
<th>$\chi^2$ 0.95 Value</th>
<th>Decision</th>
<th>Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>No inefficiency effect</td>
<td>$\gamma = 0$</td>
<td>42.87</td>
<td>3.84</td>
<td>Reject $H_0$</td>
<td>Stochastic</td>
</tr>
<tr>
<td>Input-output Separability</td>
<td>$\omega_{ij} = 0$ all i, all j</td>
<td>217.78</td>
<td>11.07</td>
<td>Reject $H_0$</td>
<td>No output aggregation</td>
</tr>
<tr>
<td>Cobb-Douglas</td>
<td>$\beta_{ij} = \alpha_{ij} = \omega_{ij} = \phi_i$</td>
<td>518.84</td>
<td>41.34</td>
<td>Reject $H_0$</td>
<td>Trans-log</td>
</tr>
</tbody>
</table>

Altogether, the results of these specification tests show the complexity of the technological relationships in Botswana agriculture production. Technical inefficiencies are significant, input and outputs are not separable, and the trans-log function is an adequate representation of the data. Irz and Thirtle (2004) also found these results.

Table 2. Maximum Likelihood Estimates of the Cobb-Douglas Function

<table>
<thead>
<tr>
<th>Variables</th>
<th>Parameters</th>
<th>Coefficient</th>
<th>Standard-error</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$\beta_0$</td>
<td>-8.732</td>
<td>0.085</td>
<td>-103.183</td>
</tr>
<tr>
<td>$\ln LA^*$ (labour)</td>
<td>$\beta_1$</td>
<td>0.133</td>
<td>0.024</td>
<td>5.477</td>
</tr>
<tr>
<td>$\ln HC^*$ (cattle hd size)</td>
<td>$\beta_2$</td>
<td>0.126</td>
<td>0.027</td>
<td>4.610</td>
</tr>
<tr>
<td>$\ln HGS^*$ (goats+sheep hd sz)</td>
<td>$\beta_3$</td>
<td>0.166</td>
<td>0.024</td>
<td>6.948</td>
</tr>
<tr>
<td>$\ln DP^*$ (draft power)</td>
<td>$\beta_4$</td>
<td>0.058</td>
<td>0.026</td>
<td>2.197</td>
</tr>
<tr>
<td>$\ln FRT^*$ (fertilier)</td>
<td>$\beta_5$</td>
<td>0.033</td>
<td>0.014</td>
<td>2.348</td>
</tr>
<tr>
<td>$\ln RF^*$ (rainfall)</td>
<td>$\beta_6$</td>
<td>0.269</td>
<td>0.031</td>
<td>8.603</td>
</tr>
<tr>
<td>$\ln FD^*$ (feeds)</td>
<td>$\beta_7$</td>
<td>0.236</td>
<td>0.020</td>
<td>11.920</td>
</tr>
<tr>
<td>$\ln YCR$ (crops output)</td>
<td>$\alpha_1$</td>
<td>-0.021</td>
<td>0.004</td>
<td>-5.422</td>
</tr>
<tr>
<td>$\ln YCA$ (cattle output)</td>
<td>$\alpha_2$</td>
<td>-0.047</td>
<td>0.010</td>
<td>-4.514</td>
</tr>
<tr>
<td>$\ln YGS$ (goats+sheep output)</td>
<td>$\alpha_3$</td>
<td>-0.098</td>
<td>0.014</td>
<td>-7.223</td>
</tr>
<tr>
<td>$t$</td>
<td>$\lambda_1$</td>
<td>0.018</td>
<td>0.004</td>
<td>4.589</td>
</tr>
<tr>
<td>sigma-squard</td>
<td>$\sigma^2$</td>
<td>0.054</td>
<td>0.005</td>
<td>11.615</td>
</tr>
<tr>
<td>gamma</td>
<td>$\gamma$</td>
<td>0.669</td>
<td>0.055</td>
<td>12.125</td>
</tr>
<tr>
<td>mu</td>
<td>$u_i$</td>
<td>0.379</td>
<td>0.064</td>
<td>5.931</td>
</tr>
<tr>
<td>eta</td>
<td>$\epsilon_i$</td>
<td>0.003</td>
<td>0.007</td>
<td>0.442</td>
</tr>
<tr>
<td>log likelihood function</td>
<td></td>
<td></td>
<td></td>
<td>157.79</td>
</tr>
</tbody>
</table>
4.2 Frontier Estimates

The estimate of the partial input and output elasticities from the ordinary least squares estimation of the stochastic input distance function model are represented in Table 3 (excluding dummies, squared and cross terms). The sum of the coefficients of inputs, using ordinary least squares estimates is 1.022. Assuming homogeneity of degree+1 in inputs (that is, the partial elasticities of all input variables sum to one), the implied elasticity for land used for agricultural production, the variable to the left-hand side of the estimated model is -0.022.

Table 3. Ordinary Least Squares Estimates of Input and Output Elasticities

<table>
<thead>
<tr>
<th>Variables</th>
<th>Parameters</th>
<th>Coefficients</th>
<th>Standard Errors</th>
<th>T-ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$\beta_0$</td>
<td>-8.988</td>
<td>0.076</td>
<td>-117.874</td>
</tr>
<tr>
<td>Labour ($X_2$)</td>
<td>$\beta_1$</td>
<td>0.129</td>
<td>0.029</td>
<td>4.489</td>
</tr>
<tr>
<td>Cattle head size ($X_3$)</td>
<td>$\beta_2$</td>
<td>0.189</td>
<td>0.036</td>
<td>5.299</td>
</tr>
<tr>
<td>Goats+sheep head size ($X_4$)</td>
<td>$\beta_3$</td>
<td>0.257</td>
<td>0.031</td>
<td>8.378</td>
</tr>
<tr>
<td>Draft power ($X_5$)</td>
<td>$\beta_4$</td>
<td>0.019</td>
<td>0.028</td>
<td>0.681</td>
</tr>
<tr>
<td>Fertiliser ($X_6$)</td>
<td>$\beta_5$</td>
<td>0.030</td>
<td>0.017</td>
<td>1.762</td>
</tr>
<tr>
<td>Rainfall ($X_7$)</td>
<td>$\beta_6$</td>
<td>0.179</td>
<td>0.028</td>
<td>6.482</td>
</tr>
<tr>
<td>Feeds ($X_8$)</td>
<td>$\beta_7$</td>
<td>0.219</td>
<td>0.024</td>
<td>9.216</td>
</tr>
<tr>
<td>Crops output ($Y_1$)</td>
<td>$\alpha_1$</td>
<td>-0.029</td>
<td>0.006</td>
<td>-4.652</td>
</tr>
<tr>
<td>Cattle output ($Y_2$)</td>
<td>$\alpha_2$</td>
<td>-0.076</td>
<td>0.021</td>
<td>-3.579</td>
</tr>
<tr>
<td>Goats+sheep output ($Y_3$)</td>
<td>$\alpha_3$</td>
<td>-0.166</td>
<td>0.023</td>
<td>-7.102</td>
</tr>
<tr>
<td>T</td>
<td>$\lambda$</td>
<td>0.040</td>
<td>0.009</td>
<td>4.448</td>
</tr>
<tr>
<td>sigma-squared</td>
<td>$\sigma^2$</td>
<td>0.008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log likelihood function</td>
<td></td>
<td>395.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR test of the one-sided error</td>
<td></td>
<td>83.51</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Estimates of the maximum likelihood estimation of the stochastic input distance function model are presented in Table 4 below.

Table 4. Maximum Likelihood Estimates of the Input and Output Elasticities

<table>
<thead>
<tr>
<th>Variables</th>
<th>Parameter</th>
<th>Coefficient</th>
<th>Standard Errors</th>
<th>T-ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$\beta_0$</td>
<td>-8.855</td>
<td>0.070</td>
<td>-127.254</td>
</tr>
<tr>
<td>Labour ($X_2$)</td>
<td>$\beta_1$</td>
<td>0.157</td>
<td>0.026</td>
<td>5.967</td>
</tr>
<tr>
<td>Cattle head size ($X_3$)</td>
<td>$\beta_2$</td>
<td>0.229</td>
<td>0.030</td>
<td>7.640</td>
</tr>
<tr>
<td>Goats+sheep head size ($X_4$)</td>
<td>$\beta_3$</td>
<td>0.222</td>
<td>0.026</td>
<td>8.465</td>
</tr>
<tr>
<td>Draft power ($X_5$)</td>
<td>$\beta_4$</td>
<td>-0.012</td>
<td>0.025</td>
<td>-0.491</td>
</tr>
<tr>
<td>---------------------</td>
<td>-----------</td>
<td>--------</td>
<td>-------</td>
<td>--------</td>
</tr>
<tr>
<td>Fertiliser ($X_6$)</td>
<td>$\beta_5$</td>
<td>0.014</td>
<td>0.014</td>
<td>1.046</td>
</tr>
<tr>
<td>Rainfall ($X_7$)</td>
<td>$\beta_6$</td>
<td>0.128</td>
<td>0.027</td>
<td>4.710</td>
</tr>
<tr>
<td>Feeds ($X_8$)</td>
<td>$\beta_7$</td>
<td>0.266</td>
<td>0.027</td>
<td>9.895</td>
</tr>
<tr>
<td>Crops output ($Y_1$)</td>
<td>$\alpha_1$</td>
<td>-0.028</td>
<td>0.005</td>
<td>-5.246</td>
</tr>
<tr>
<td>Cattle output ($Y_2$)</td>
<td>$\alpha_2$</td>
<td>-0.067</td>
<td>0.018</td>
<td>-3.672</td>
</tr>
<tr>
<td>Goats+sheep output ($Y_3$)</td>
<td>$\alpha_3$</td>
<td>-0.168</td>
<td>0.020</td>
<td>-8.379</td>
</tr>
<tr>
<td>Time-trend (t)</td>
<td>$\lambda_1$</td>
<td>0.038</td>
<td>0.008</td>
<td>4.890</td>
</tr>
<tr>
<td>$D_1$</td>
<td>$\phi_1$</td>
<td>-0.321</td>
<td>0.051</td>
<td>-6.322</td>
</tr>
<tr>
<td>$D_2$</td>
<td>$\phi_2$</td>
<td>-0.684</td>
<td>0.067</td>
<td>-10.279</td>
</tr>
<tr>
<td>$D_3$</td>
<td>$\phi_3$</td>
<td>-0.122</td>
<td>0.057</td>
<td>-2.158</td>
</tr>
<tr>
<td>$D_4$</td>
<td>$\phi_4$</td>
<td>-0.313</td>
<td>0.057</td>
<td>-5.494</td>
</tr>
<tr>
<td>$D_5$</td>
<td>$\phi_5$</td>
<td>-0.402</td>
<td>0.071</td>
<td>-5.677</td>
</tr>
<tr>
<td>$D_6$</td>
<td>$\phi_6$</td>
<td>-0.553</td>
<td>0.072</td>
<td>-7.674</td>
</tr>
<tr>
<td>$D_7$</td>
<td>$\phi_7$</td>
<td>-0.491</td>
<td>0.069</td>
<td>-7.065</td>
</tr>
<tr>
<td>$D_8$</td>
<td>$\phi_8$</td>
<td>-0.754</td>
<td>0.075</td>
<td>-10.058</td>
</tr>
<tr>
<td>$D_9$</td>
<td>$\phi_9$</td>
<td>-0.770</td>
<td>0.068</td>
<td>-11.279</td>
</tr>
<tr>
<td>$D_{10}$</td>
<td>$\phi_{10}$</td>
<td>-0.597</td>
<td>0.070</td>
<td>-8.591</td>
</tr>
<tr>
<td>$D_{11}$</td>
<td>$\phi_{11}$</td>
<td>-0.674</td>
<td>0.067</td>
<td>-10.085</td>
</tr>
<tr>
<td>$D_{12}$</td>
<td>$\phi_{12}$</td>
<td>-0.712</td>
<td>0.061</td>
<td>-11.684</td>
</tr>
<tr>
<td>$D_{13}$</td>
<td>$\phi_{13}$</td>
<td>-0.527</td>
<td>0.063</td>
<td>-8.409</td>
</tr>
<tr>
<td>$D_{14}$</td>
<td>$\phi_{14}$</td>
<td>-0.579</td>
<td>0.076</td>
<td>-7.611</td>
</tr>
<tr>
<td>$D_{15}$</td>
<td>$\phi_{15}$</td>
<td>0.116</td>
<td>0.076</td>
<td>1.527</td>
</tr>
<tr>
<td>$D_{16}$</td>
<td>$\phi_{16}$</td>
<td>-0.252</td>
<td>0.084</td>
<td>-3.006</td>
</tr>
<tr>
<td>$D_{17}$</td>
<td>$\phi_{17}$</td>
<td>-0.047</td>
<td>0.099</td>
<td>-0.472</td>
</tr>
<tr>
<td>$D_{18}$</td>
<td>$\phi_{18}$</td>
<td>-0.206</td>
<td>0.143</td>
<td>-1.438</td>
</tr>
<tr>
<td>$Z$</td>
<td>$\delta_0$</td>
<td>-0.031</td>
<td>0.078</td>
<td>-0.390</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>$\delta_1$</td>
<td>-0.019</td>
<td>0.116</td>
<td>-0.161</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>$\delta_2$</td>
<td>0.059</td>
<td>0.131</td>
<td>0.453</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>$\delta_3$</td>
<td>-0.054</td>
<td>0.186</td>
<td>-0.288</td>
</tr>
<tr>
<td>$Z_4$</td>
<td>$\delta_4$</td>
<td>0.061</td>
<td>0.107</td>
<td>0.573</td>
</tr>
<tr>
<td>$Z_5$</td>
<td>$\delta_5$</td>
<td>0.188</td>
<td>0.117</td>
<td>1.606</td>
</tr>
<tr>
<td>$Z_6$</td>
<td>$\delta_6$</td>
<td>0.160</td>
<td>0.117</td>
<td>1.368</td>
</tr>
<tr>
<td>$Z_7$</td>
<td>$\delta_7$</td>
<td>0.116</td>
<td>0.100</td>
<td>1.158</td>
</tr>
<tr>
<td>$Z_8$</td>
<td>$\delta_8$</td>
<td>0.017</td>
<td>0.092</td>
<td>0.186</td>
</tr>
</tbody>
</table>
The sum of the frontier coefficients of the input variables is approximately 1.004, and the implied elasticity for land used is -0.004.

When comparing the ordinary least squares with the frontier, there is a strong consistency for all the results. The only differences are slightly higher elasticities of labour, cattle head size, and livestock feeds on the frontier. The higher values for these three variables explain some of the increased output for the highest performing district. The consistency among other results reflects the fact that, although technically inefficient, the average district is not too far from the frontier. The time trend coefficient of the ordinary least squares is 0.04 and the maximum likelihood estimate is 0.038, which implies that the average farmer and a farmer on the frontier have almost the same growth rate. From Table 4, it is estimated that output has been increasing at the rate of 3.8% per annum due to technical change, and that this estimate is significantly different from zero at the 5% significance level.

In overview, Table 4 shows that of the 103 of the estimates, only 54 are statistically significant at the 5% level of significance. The several significant cross products and squared terms indicate that trans-log is an appropriate representation of the model. As mentioned earlier, prior to estimation all variables were mean corrected (except time trend) so that the parameter estimates can be interpreted as elasticities of the distance function with respect to input quantities and output quantities.

Focusing solely on the variables themselves (not including the squared terms, cross products, dummy variables, and inefficiency effects), the coefficients of the maximum likelihood presented in Table 4 are estimated to be significantly different from zero at the 5% level, except draft power and fertiliser. The elasticities of the distance function with respect to input quantities are equal to the cost shares and
therefore, reflect the relative importance of the inputs in the production process (Irz & Thirtle, 2004).

Table 4 reveals that six of the seven elasticities are positive, as expected. The elasticity with respect to livestock feeds is the largest with a value of 0.266, which means that the cost of livestock feeds represents 26.6% of the total costs at the sample mean. Based on the importance of livestock production in Botswana, this result would be expected. Cattle head size comes next in terms of cost share with a value of 0.229, followed by the elasticity of goats/sheep with a value of 0.222, suggesting that livestock plays an important role in Botswana agriculture. Labour is one of the important factors of production in agriculture and is reflected by an elasticity of 0.157. Rainfall has an elasticity of 0.128, showing that water is crucial for agriculture production because Botswana has frequent droughts and is semi-desert country. Furthermore, fertiliser input has an elasticity of 0.014, meaning that fertiliser use represents 1.4% of the total costs at the sample mean, and this coefficient is statistically significant at the 10% level.

Elasticity of the distance function with respect to each output corresponds to a negative value of the cost elasticity of a particular output. Table 4 reports that, as expected, the three output elasticities (crops, cattle, and goats/sheep) are negative and significant. That is because increasing production of any of the three outputs results in substantial increases in costs. The estimates also indicate that the coefficients for the output variables crops, cattle and goats/sheep are -0.028, -0.067, and -0.168, respectively, reflecting the positive effects of all inputs on these three outputs. This means that a 10% increase in all inputs will increase crop production by 0.28%, cattle production by 0.67% and goats/sheep by 1.68%. These estimates indicate the dominance of livestock production in Botswana agriculture.

4.3 Evidence of Economies of Scope

The coefficient estimates reported in Table 4 were used to calculate the measure of economies of scope, defined in Equation (8), for each pair of outputs at the means of sample data. A negative first partial derivative of the input distance function with respect to the $n^{th}$ output implies that additional output, assuming other variables are constant, reduces the amount needed to put the observation onto the efficient frontier by deflating the input vector. A positive second partial derivative indicates evidence of economies of scope, while a negative second partial derivative signifies diseconomies of scope. The estimated coefficients of economies of scope for the Botswana agricultural production system are reported in Table 5 below. In order to test the hypothesis that there are no economies of scope in the production system, standard errors for these measures of economies of scope were calculated using the SHAZAM econometric software (White, 1993). Standard errors were calculated using a Taylor series expansion with the matrix manipulation option in SHAZAM.
The measure of scope economies between crops and cattle (evaluated at the sample means) was calculated to be 0.0113 with a standard error of 0.00651, as reported in Table 5. Assessing the first hypothesis, defined in the Introduction, the result suggests the significance of scope for Botswana farmers to grow crops and raise cattle, the coefficient being significant at the 5% level using the two-tailed test. Due to a relatively high standard error for crops and goats/sheep, the second hypothesis of diseconomies of scope (or economies of scope) between these two outputs cannot be rejected. For crops and goats/sheep, scope economies are not significant, so farmers producing this combination of output will experience declining average scope efficiency gains. Regarding the third hypothesis the economies of scope estimate between cattle and goats/sheep is 0.0868 with a standard error of 0.0171, indicating significance in economies of scope between these two outputs, at the 1% level. The estimated t-value for the parameter involving a combination of cattle and goats/sheep is high, revealing economies of scope. This means that the production possibility frontier between these outputs is bounded outwards.

The existence of economies of scope, observed above, indicates that farmers have had most success when they diversified into the production of cattle and goats/sheep by commercialising their operations through adaptive strategies for these combined outputs. This lead to increased productivity given that the farming system under observation relies heavily on the farm inputs of livestock feeds, labour, cattle and goats/sheep herd size, and land resources. The ability of best-practice farmers or farming districts to reduce the use of livestock feeds and make efficient use of labour is crucial in the production of cattle and goats/sheep. Overall, the production of this type of output generates efficiency gains due to economies of scope because farmers have the opportunity to choose activities that complement each other, given the seasonal nature of their labour demands to utilise labour inputs throughout the year. This demonstrates the importance of economies of scope in the production of cattle and goats/sheep to enable farmers to realize higher economic returns with reduced risk.

On the other hand, the observed diseconomies of scope for the production of crops and goats/sheep outputs indicate that farmers or districts experience declining average economies of scope gains, implying that they cannot achieve productivity gains when they produce these two outputs, with different and often overlapping labour demands.

Table 5. Estimates of Economies of Scope

<table>
<thead>
<tr>
<th>Output Combination</th>
<th>Scope economies coefficient</th>
<th>Standard Error</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crops and Cattle</td>
<td>0.011</td>
<td>0.0065</td>
<td>1.732</td>
</tr>
<tr>
<td>Crops and goats/sheep</td>
<td>0.008</td>
<td>0.0077</td>
<td>1.044</td>
</tr>
<tr>
<td>Cattle and goats/sheep</td>
<td>0.087</td>
<td>0.0171</td>
<td>5.073</td>
</tr>
</tbody>
</table>
4.4 Efficiencies

The null hypothesis of no inefficiency was performed using the LR test. To illustrate, the input distance function results reported in Table 3 and Table 4 were used to compute LR

\[ 2*(395.777-417.210)=42.866 \]

This value is also reported in Table 4. The test statistics exceed the 5% significance level critical value of \( \chi^2_{0.05} = 28.869 \) so the null hypothesis of no inefficiency effects is rejected. Thus, both the z-test and the LR test suggest that an average response function is not an adequate representation of the data and so the technical inefficiency term \( (\mu_t) \) is a significant addition to the model. These results show that significant technical inefficiency exists in Botswana agriculture.

Since the mean efficiency is 88.5 percent, there is inefficiency in the production system, and it is significant. This means that the efficiency result is consistent with the previous study by Irz and Thirtle (2004), whose mean efficiency was found to be 85.0 percent. The gamma coefficient is 0.977, reflecting the fact that 97.7 percent of disturbance in the production system is due to inefficiency with one-sided error and, therefore, 2.3 percent is due to the stochastic disturbance term with two-sided error. This result is in accordance with the expectation that most of the disturbance is explained by efficiency because the majority of traditional farmers in Botswana have not adopted improved technologies such as fertiliser use, improved high quality crops, irrigation, improved animals, rotational grazing, and livestock vaccination. Irz and Thirtle (2004), report that Botswana agriculture has undergone technological regression from 1979 to 1996 at the rate of 3.1 per cent, and Seleka (1999) stated that crop yields declined by 6.1 percent from 1968 to 1990, so districts that define the frontier use more inputs per unit of output over time. This is not because technology is forgotten or deteriorating, but is the result of the long cycle of continuing drought that dominated the country since the 1990s. This loss of productivity due to continuing droughts is efficiency loss, but any frontier model will fail to discriminate between technological change and efficiency change, especially when all districts are affected (Irz & Thirtle, 2004).

Prior to estimation, 17 districts and commercial sector dummy variables were used as inefficiency variables, with Borolong district used as a base. Table 4 shows the 18 inefficiency variables, only 4 of which were observed to significantly contribute to the explanation of technical inefficiency, at the 5% significance level in the production system of Botswana agriculture, being three districts (Kgatleng, Tati, & Kgalagadi, n. d.) and the commercial sector. The districts of Kweneng South, Phalapye and Chobe contributed to the explanation of technical inefficiency at the 10% level in the production system.

The figure below shows the average technical efficiency estimates of the 18 Botswana districts and the commercial sector.
The technical efficiency of each district and commercial sector can be predicted using the conditional expectation of $\exp(-u_{ij})$, and the technical efficiency predictions are between the value of zero and one, with the value of one indicating a fully efficient district. The figure above shows that, on average, Tati has the highest average technical efficiency an estimate, indicating that in terms of efficiency Tati district is performing near its fully efficient frontier. Second is Serowe followed by Bamalete/Tlokweng, Mahalapye and Chobe districts, indicating that these districts have the same production efficiency. The commercial sector has the lowest average technical efficiency. Following Irz and Thirtle (2004) inefficiency increases over time in the commercial sector, at 0.26 percent per annum, which suggest that some producers are not assimilating the new technologies and are falling behind the advancing frontier.

5. Concluding Remarks and Policy Implication
This paper contributes to the understanding of the performance of the Botswana agricultural production system emphasising the crop and livestock sectors. A stochastic input distance function is used to estimate production technologies and obtain measures of economies of scope from an estimated model using the panel data of 340 agricultural inputs and outputs across 18 districts and the commercial sector of Botswana, from 1979 to 1996. Because prices of input data were not available for obtaining estimates of the cost function parameters, the measure of economies of scope had to be defined by the second order partial derivative of the input distance function with respect to output vectors. The aim of this study is to calculate the estimates of the production relations, estimate the technical efficiency of Botswana agriculture using input distance functions, and estimate measures of economies of scope from the estimates of parameters of the second order derivatives of the input distance function with respect to output variables.

A trans-log input distance function was estimated for the Botswana agricultural production system, and
all the estimates of the input distance function are significant at the 5% level, all having the expected
signs, except for draft power. The elasticity with respect to livestock feeds is the largest representing
26.6% of the total costs at the sample mean. Based on the importance of livestock production in
Botswana, this result would be expected. Cattle head size is next in terms of cost share with a value of
0.229, followed by the elasticity of goats/sheep with a value of 0.222, suggesting that livestock plays an
important role in Botswana agriculture. Labour is one of the important factors of production in
agriculture and is reflected by an elasticity of 0.157. Rainfall has an elasticity of 0.128, and fertiliser
input use represents 1.4% of total cost share. The elasticity with respect to outputs was significant with
expected negative signs reflecting the positive effects of all inputs on outputs. The crop, cattle, and
goats/sheep output has elasticities of -0.028, -0.067, and -0.168, respectively.

The existence of economies of scope in the production of cattle and goats/sheep was significant at 1
percent level. Economies of scope were also significant between the production of crops and cattle at 5
percent level, and there was no evidence of economies of scope between crops and goats/sheep. These
results indicate that there is potential for higher efficiency gains of integrated operations for purposes of
higher economic returns with reduced risk. In other words, farmers would have had most success when
they diversified into the production of cattle and goats/sheep by commercialising their operations
through adaptive strategies for these combined outputs. This lead to increased productivity given that
the farming system under observation relies heavily on the farm inputs of livestock feeds, labour, cattle
and goats/sheep herd size, and land resources. Therefore, policies aimed at supporting and encouraging
farmer to concentrate more in the production of livestock, more particularly diversify in the production
of cattle and goats/sheep can provide much needed boost to realise higher economic returns through
efficient use of labour because it will enable farmers to choose production activities that complement
each other, given the seasonal nature of their labour demands to utilise labour inputs throughout the
year.

This study also provides information on the technical inefficiency of the agricultural production system.
Mean technical efficiency of 88.5 percent indicates that significant technical inefficiency exists, which
implies that there may be opportunities for increasing the output of crops, cattle, and goats/sheep
without increasing the use of factor inputs or through the introduction of improved technology. The
mean efficiency result obtained in this study is consistent with the one found by Irz and Thirtle (2004),
whose mean efficiency was found to be 85.0 percent. The results of this research are of particular
interest to agricultural policies that aim to improve the livelihood of the rural population and to
encourage them to continue producing food to meet dietary demand of the nation. However, the
efficiency of these agricultural support programmes has not been successful in contributing to
economic development. With investment in research and technology, and with more innovative
production, processing and marketing of agricultural commodities, agriculture can make a significant
contribution towards building a modern Botswana economy. From a socio-economic perspective, there
is need for some form of support because some of the poorest, most marginal and vulnerable people in
society continue to rely on agriculture as the main source of income for their livelihood. Therefore the rural farming community, more especially traditional farmers need to be encouraged to adopt modern farming technologies such as fertiliser use, use of hybrid seeds, irrigation systems, rotational grazing, livestock vaccination and improved animal breeds to better their lives and be motivated to continue producing food for the nation. They should also be encouraged to practice commercial farming as opposed to subsistence farming as a means of increasing food production in Botswana.

Limitations of the research presented in this study are recognised. Perhaps most importantly, technical inefficiency estimates were based only on the dummy variables of 17 different districts and the commercial sector dummy variable. This is because the data relating to variables that could help to explain inefficiency better in the model were not available. For example, there is no data for the level of education of farmers, social and cultural obligations, or access to information about diseases such as AIDS and malaria. Therefore, the implications for further research are that using very recent data sets and appropriate technical inefficiency variables should enable better analysis of scope economies and diseconomies, as well as a more plausible comparison of productivity and technical efficiency. A suggested modification is to omit variables that did not help to explain the model. Another useful direction would be to obtain the data for priced inputs and explore estimates of the cost function for comparison with those of the input distance function.

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