

Original Paper

On Hegel's View of Dialectical Infinity

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Abstract

It is well known that the problem of finity and infinity is the basic problem of mathematics, and it is also the basic problem of Philosophy. From the perspective of philosophy and mathematics, this paper comprehensively reviews and analyzes Hegel's view of dialectical infinity, introduces Engels' discussion on infinity, deeply analyzes the characteristics of the thought of actual infinity, and points out: Hegel's thought of real infinity is completely different from the thought of actual infinity, the Being of infinity (objective infinity) is not equal to the completed infinity (subjective infinity), the mathematical limit is a real infinity, and real infinity is the inner law of infinite things and truth; the view of actual infinity views the objective material world from the viewpoint of static rather than motion, denying the contradiction between finity and infinity, so it is actually a downright idealist. In this paper, the author puts forward the Infinite Exchange Paradox, which strongly questions the idea of actual infinity in Hilbert Hotel Problem, and points out the internal irreconcilable contradiction in the idea of actual infinity. At the same time, we made a detailed comparison of Hegel's view of infinity and the view of mathematical infinity, and on this basis, the author gives a complete definition of the view of dialectical infinity: abandoning the wrong aspects of the potential infinity and actual infinity, and actively absorbing correct aspects of both, that is, not only to recognize the existence and knowability of infinite objectivity, but also to admit the incompleteness of infinite process. The reexcavation of Hegel's view of dialectical infinity and the criticism of the actual infinity thought aim to find possible philosophical solutions for Russell's Paradox and the problem of Continuum Hypothesis.

Keywords

actual infinity, real infinity (genuine infinity, true infinity), potential infinity; bad infinity (wrong infinity), the view of dialectical infinity, Infinite Exchange Paradox

It is well known that the problem of finity and infinity is the basic problem of mathematics, and it is also the basic problem of Philosophy. Many famous philosophers and mathematicians in history have put forward their own propositions on the *infinite problem*. Aristotle was the first philosopher to clearly

distinguish between the *actual infinity* and the *potential infinity*. Philosophy development to Hegel's era, the problem of finity and infinity became one of the core problems of his research in *Hegel's Science of Logic*; Based on the problem of infinity in mathematics, he put forward two kinds of infinite concepts of *bad infinity* and *real infinity*, and used the great dialectical thought to comprehensively elaborate the dialectical relationship between the two, that is, to put forward the idea of *dialectical infinity*. Engels gave a high degree of affirmation and evaluation to his thought of *dialectical infinity*, and from then on, the human understanding of infinity has entered the era of *dialectical infinity*. However, history chose to forget, and his thought of *dialectical infinity* has not been highly valued by the philosophical and mathematical circles, and the human understanding of the infinite once again fell into a state of chaos.

1. The Origin and Controversy of the Infinite Problem in Mathematics

1.1 What Are the Actual Infinity and the Potential Infinity?

As we know, people hold definitely opposite opinions about 'infinity' all the time in the history of mathematics. Some uphold the *potential infinity* concept, while others believe in the *actual infinity* concept. The view of *potential infinity* means that we cannot end an infinite process and reach an end or all of it. Conversely, the *actual infinity* viewer thinks that we can complete an infinite process and reach its destination or its all. Therefore, the contradiction between *potential infinity* and *actual infinity* lies in whether it can finish or end an infinite process. Their contradictions and the oppositions are self-evident.

For the difference between the two infinite views, in *The Logical Foundation of Mathematics and Infinity*, Mr. Zhu Wujia gives a detailed analysis from mathematics and philosophy. He pointed out in the paper: *Actual infinity* is transformed from the present progressive form (going) into a complete form (gone), and *potential infinity* is enhanced by the present tense (going) as always present progressive (going). Whether it is *potential infinity* or *actual infinity*, are non finite processes. From the two aspects of the philosophy of mathematics, the difference between the two is described as follows (Zhu Wujia, 2008, pp. 204-207):

Actual infinity: is a complete (gone). For the infinite process must reach the end of process, infinite process enumeration procedures must be able to complete;

Potential infinity: is a progressive (going). For the infinite process denial to reach the end of process, infinite process enumeration procedures must not be completed.

1.2 Two Attitudes of Mathematicians and Philosophers towards the Problem of Infinity in History

In ancient Greece, Anaximander was the first thinker to think about the infinity, and he regarded infinity as borderless and immortal, and regarded infinity as the ultimate root of all *beings*. Aristotle is the first scholar to explicitly admit only *potential infinity* against *actual infinity* in history, and Plato is the first scholar to admit the *actual infinity* and against *potential infinity*.

(1) Aristotle's view of infinity

Aristotle was the epitome of ancient Greek philosophy, Marx called him the most erudite of ancient Greek philosophers, and Engels called him ancient Hegel. His infinite thought is mainly: Infinity exists, but infinity can not exist in the way of actuality, that is, infinity can only be potential; Infinity cannot be separated from perceptual things, nor can it exist on its own.

He admitted that infinity existed. "But if there is no infinity at all, it is clear that many unexplained conclusions will result in, for example, the beginning and end of time, and the quantity cannot be divided into smaller quantities, nor will the numbers be infinite." (Aristotle, 2016a, p. 85)

He first analyzed the meaning of infinity, believing that the essence of infinity is "*besides forever*". He pointed out: "The true meaning of infinity is just the opposite of what is commonly understood. It's not that there's nothing else, but that there is *always more*. And the nothing else is complete or accomplished." (Aristotle, 2016a, p. 87) In *Metaphysics*, he believes that infinity is meant to be inexhaustible; he describes infinity as follows: "Infinity (endless) or (a) is impossible to achieve, for its nature is inexhaustible (which is similar to what is always invisible to the voice), or (b) is allowed to go on endlessly, or (c) it is difficult to get to the end, or (d) though naturally accessible, he has never been to the end. It is impossible to say that infinity is a detachable independent reality and is not visible." (Aristotle, 2016b, p. 253) In other words, in Aristotle's view, infinity is endless and can not be crossed.

He believes that infinity is not a reality, that is, there can be no actual infinity in human subjective world. He pointed out that "there is no actual infinite object" (Aristotle, 2016a, p. 85); "Things are said to exist, one to the existence of potential and the other to the existence of actuality. Now, as we have already said, the quantity is not infinite in actuality, but infinite in division (it is not difficult to refute the line that can not be divided), so there is only infinite, in potential., but infinity is not, there will be no infinity of actuality." (Aristotle, 2016a, p. 85); "So, since no perceptual quantity is infinite, it is impossible to have a quantity who is more than all the quantum." (Aristotle, 2016a, p. 89); "It is also clear that infinity cannot be regarded as an accomplished thing, an entity, or a source." (Aristotle, 2016a, p. 79)

He thinks that infinity is potential, infinity cannot be crossed, infinity cannot be separated from perceptual things, can not exist alone. In Aristotle's view, infinity is a kind of relation (attribute), is the expression form of object existence, namely quantitative existence form of object, therefore can also be said to be an attribute of object, namely attributes of the quantity of an object. Such as time, space has infinite attributes, also call time and space is infinite. He noted that: "It is impossible to say that infinity is a kind of infinity free from perceptual things." (Aristotle, 2016a, p. 79); "Generally speaking, there can be no sentimental object is infinite." (Aristotle, 2016a, p. 82); He believes that infinity is impenetrable, "Infinity one is something that cannot be passed through. The other is the thing which can be talked through, but not at an end." (Aristotle, 2016a, p. 78)

To sum up, Aristotle's thought of infinity is a simple, materialistic thought. It only analyzes infinity from the aspect of quantity, but deeply reveals the essence of bad infinity.

(2) Plato's view of infinity

The idea of *actual infinity* in mathematics actually originated from the ancient Greek philosopher Plato. Plato believed that infinity is an actual thing, and Aristotle objected that infinity is a realized thing. Aristotle said: "But this contradicts the notion that infinity is a realized thing, because the latter must be a certain quantity. Therefore, infinity belongs to entities as attributes" (Aristotle, 2016a, p. 80). In Plato's view, infinity is something that can be accomplished and realized, that is, infinite process can be accomplished. Plato believes that infinity is a reality, "Some people, such as Pythagoreans and Plato, see infinity as a free entity rather than an attribute of other things." (Aristotle, 2016a, p. 75); "But Plato argued that there were two infinities: big and small." (Aristotle, 2016a, p. 76), but Plato has never used this *infinity, infinitesimal*.

(3) Hume's view of infinity

Hume is a typical theorist of potential infinity, who believes that the legitimacy of the inductive method can never prove theoretically. He stated in *A Treatise of Human Nature*: "Thus not only our reason fails us in the discovery of the ultimate connexion of causes and effects, but even after experience has informed us of their constant conjunction, it is impossible for us to satisfy ourselves by our reason, why we should extend that experience beyond those particular instances, which have fallen under our observation. We suppose, but are never able to prove, that there must be a resemblance betwixt those objects, of which we have had experience, and those which lie beyond the reach of our discovery." (Hume, 2015, p. 105)

He explicitly opposed the idea of *actual infinity*, and believed that *actual infinity* brings obvious contradiction, arguing that the definition of mathematics is not precise. He said: "It is universally recognized that the ability of the mind is limited and can never be given a sufficient and appropriate concept of *infinity*; even if this is not recognized, it is evident enough from the most mundane observations and experiences." (Hume, p. 35); "No matter how many times it is divided, it cannot bring the imagination closer to the final division than the original idea formed by imagination." (Hume, p. 53); "This clearly proves that the concept of face, line and point is not allowed to be further divided, that is, the concept of face cannot be further divided in thickness, the concept of line cannot be further divided in width and thickness, and the concept of point in length, width, thickness cannot be divided in any aspect." (Hume, p. 54); "For, since any concept of quantity is not infinitely divisible, it is clear that an attempt to prove that quantity itself allows such a division and to prove it by some ideas directly contrary to it in this respect is the most obvious fallacy imaginable. This fallacy itself is very obvious, then any argument based on it will inevitably lead to a new fallacy and contain a clear contradiction." (Hume, p. 63)

(4) Gauss's view of infinity

The great mathematical genius Gauss clearly opposed to the *actual infinity*. He expressed his views in a very firm tone in his letter to Heinrich Schumacher, "I object to the use of an infinite quantity as a completed entity, this is absolutely not allowed in mathematics, infinity is just a way of talking about the limits".

(5) Engels's view of infinity

Engels also explicitly opposed the thought of inductive universal, that is, against the idea of *actual infinity*. He pointed out that in *Dialectics of Nature*, "To the Pan-Inductionists. With all the induction in the world we would never have got to the point of becoming clear about the process of induction. Only the analysis of this process could accomplish this. —Induction and deduction belong together as necessarily as synthesis and analysis. Instead of one-sidedly lauding one to the skies at the expense of the other, we should seek to apply each of them in its place, and that can only be done by bearing in mind that they belong together, that they supplement each other. According to the inductionists, induction is an infallible method. It is so little so that its apparently surest results are every day overthrown by new discoveries." (Engels, 1964, pp. 231-232) Then he used the phrase to emphasize: "The form of development of natural science, in so far as it thinks, is the hypothesis." (Engels, 1964, p. 244)

1.3 A Great Philosophical Discussion on the Foundation of Mathematics in Modern History

In modern history, since the founder of the set theory, Cantor founded the transfinite number theory (an *actual infinity*), it has shocked the whole mathematics circle and the whole philosophy circle. In addition to the help of the mathematical master Hilbert and the emergence of Russell's Paradox, the problem of finity and infinity has once again become the focus of attention in the mathematical and philosophical circles. Since then, the study of mathematical philosophy has entered a prosperous era when one hundred flowers bloom and one hundred schools of thought contend. It mainly includes the following typical representatives:

(1) Poincaré's view of infinity

Henri Poincaré a great French mathematician, an anthropologist and a philosopher of science, also explicitly rejected the *actual infinity*. He had written in his book *The Foundations of Science*, and he pointed out that: "... There is no *actual infinity* (given complete). Cantor doctrine, forget this, they fell into contradiction..... Like Cantor doctrine, logistic also forget that, and encountered the same difficulties. The question, however, is to know whether they have taken this path by accident or by necessity. In my opinion, this problem is above suspicion; in Russell's logic, the assurance of actual infinity is the basic thing. It is this that distinguishes it from Hilbert's logic....." (Poincaré 2008, p. 148) Mr. Poincaré made a profound analysis of the thought of Cantor and Russell.

In *The Final Meditation*, he makes a profound exposition on the contradiction between finity and infinity and the relationship between subject and object: "There is a known reality that is outside of us and does not depend on us, but everything we know about it depends on us, so it is nothing more than

creation, a hierarchy of successive gains. The rest is real, but never knowable.” (Poincaré 2015, pp. 96-97) The above discussion shows that Poincaré like Aristotle, believes that the infinite existed, but that the human understanding of it can only be a way of *potential infinity*.

Poincaré opposed seeing *infinity* as a fixed quantity. In *The Foundations of Science*, he states that: “The concept of infinity has been introduced into mathematics long ago; but this infinity is what philosophers call becoming. Mathematical infinity is simply a quantity that can increase beyond all its limits: it is a variable, and it cannot be said to exceed all limits, but only that it can exceed them.” (Poincaré 2008, p. 106)

(2) The infinite view of Intuitionist School

The intuitionism represented by Brouwer is a firm opponent of *actual infinity*, and they believe that there is only *potential infinity* and no objective actual infinity. Its main point of view: insist on the potential infinity but reject the actual infinity, “existence must be constructed” is its slogan, think that the natural number set N is always in the structure, that the actual infinity object is unconstructible; it does not recognize the existence of infinity as an object, so its infinite view is typical potential infinity, and its philosophical view belongs to typical idealism. Its typical representative figures are Brouwer and Poincaré. Poincaré advocates the most basic intuitionism and can be regarded as credible without further analysis. He denounces the concept of actual infinite set many times and advocates the concept of potential infinity. Brouwer put forward the idea that existence must be constructed and became the founder of intuitionism.

(3) The infinite view of Formalism School

The essence of Formalism School is to formalize and axiom the system, think mathematics is a pure symbol game, think that the truth of mathematics lies in the non-contradiction of formal system, they do not care about the objective reality of mathematics.

The formalism, represented by Hilbert, is contradictory in itself on “*problem of infinity*”. On the one hand, they admit the theory of infinite set, thus they admit the reality of infinity, so they are typical *actual infinity* theorists, but on the other hand, they insist on “*the principle of limited doctrine*” in the concrete application, and worry about the use of the concept and method of *actual infinity*, almost as much as the intuitionists believe that credibility can only exist in the finity. They believe that infinity objects are beyond intuition and untrustworthy, so they are also *potential infinity* theorists. Hilbert said in a famous speech, “There is no problem as infinity, has deeply touched many people’s emotions; no idea as infinity as ever, so very fruitful to inspire the mind of man; nor is there any concept as it is infinity, so desperately need to be clarified.” (Zhu Wujia, 1987, p. 296) It can be seen that his research on infinity has a great passion and expectation. So Mr. Zhu Wujia called him more vividly as an “*actual infinity theorists in the front*” and a “*finity theorists behind*”. (Zhu Wujia, 2008, p. 146)

This fully illustrates the contradiction of Hilbert on *the problem of infinity*, the fundamental reason is that he does not realize the dialectics of *the problem of infinity*, does not see that infinity is the contradictory unity of *real infinity and bad infinity*, and does not see that the *being* of infinity and the

completeness of processes are completely different concepts. On the *being* of infinity object, the Formalism School is consistent with the Logicism School and opposite to the Intuitionism School. Hilbert insists on the principle of finiteness in the concrete mathematical reasoning method, which stands on the same line as the Intuitionist School. Therefore, Hilbert is a less determined theorist of *actual infinity*, a contradictory theorist of *actual infinity*, whose infinite thought is closer to the view of *dialectical infinity*.

(4) The infinite view of Logicism School

Russell as the representative of the school of logic, believed that the whole theory of mathematics can be classified into logic, and recognized the theory of infinite set under the view of *actual infinity*, confirmed the rationality of the *actual infinite* research object in the field of Mathematics. Therefore, as far as the view of infinity is concerned, they are the typical theorists of *actual infinity* (Zhu Wujia, 1987, p. 271). It is generally believed that Russell and his followers clearly acknowledge the *being* of infinite objects. However, Russell developed his *ramified type theory* in order to eliminate the paradox of set theory, therefore, the *infinite object* in the Russell system is represented as a hierarchical structure in different classes and levels. This is a disguised return to the view of *potential infinity*.

(5) Wittgenstein's view of infinity

Wittgenstein is one of the most influential philosophers in the 20th century, and especially his thought of philosophy of mathematics attracted a long debate. His infinite thought mainly is: he opposes the *actual infinity*, the objective being of infinity, thinks that the infinity is a kind of infinite possibility expressed by the law, but not the reality; he is opposed to the one-to-one correspondence of an infinite set to his own subset and to the use of Cantor Diagonal Argument, so he is a typical theorist of *potential infinity*.

He denied the infinite reality. He believed that infinite reality could not be proved, and that symbols could not express infinite reality. As he said in *Research in Basic Mathematics*, "it says that *actual infinity* cannot be grasped by mathematical symbolic systems at all, so it can only be described and not shown. This description may have been grasped in a way similar to the following: a large number of things that cannot be held in hand by people are lifted by packing them in boxes." (Wittgenstein, 2013, p. 210)

He opposes the use of Cantor Diagonal Method, arguing that the infinite cannot be exhausted by finity. "For we have a legitimate feeling that where we can talk about the last thing, there can be no '*no last thing at all*'." (Wittgenstein, 2013, p. 207); "Please don't forget: mathematicians' thoughts on infinity are, after all, finite ones. Here's what I want to say: they all have an end." (Wittgenstein, 2013, p. 228)

He holds that infinity and finity are completely different categories, and infinity is an inherent stipulation. He thinks that infinity is not a number, it is not the size of a quantity competing with a finity, but an inherent stipulation. He pointed out that, "Infinite sets and Finite sets are two different logical categories. Something that can be meaningfully expressed to one category cannot be meaningfully expressed to another category." (Wittgenstein, 2013, p. 206) Taking the number π as an example, the

number π expresses an infinite law which is accompanied by the actual observation, that is, the number π is a rule. In essence, this is Hegel's thought of real infinity.

He thinks infinity is a possibility. In his opinion, the difference between finity and infinity is not a difference in quantity, but a logical difference; infinity is not a quantity or an extension; infinity is an infinite possibility expressed by law, and infinity itself is not comparable in size. So what he thinks of as infinite possibility is actually a variable, a process (*bad infinity*) rather than an outcome. Thus, Wittgenstein was again a theorist of *potential infinity*.

Therefore, Wittgenstein's thought of infinite basically obeys Hegel's view of dialectical infinite. His only deficiency is his denial of infinite objectivity, and he was with the intuitionists but overtook them. However, his infinite thought failed to rise to the level of Hegel's view of dialectical infinite, and failed to grasp the philosophical significance of infinity as free self infinity - *real infinity*.

(6) Robinson's view of infinity

That is, the founder of modern non-standard analysis Robinson also opposed to *actual infinity*. In his view, no matter in the real experience or rational thinking, there is no *actual infinity*, but in order to carry out the work of mathematics, mathematicians have to regard the things that do not exist as if they were there.

2. The Infinite Exchange Paradox reveals the Internal Irreconcilable Contradictions of the View of Actual Infinity

The so-called *Infinite Exchange Paradox* refers to the idea that we use the thought of *actual infinity*-----that is, the idea that the infinite process can be accomplished, and we can transform the two equivalent infinities (with *one-to-one correspondence*) into mutually nonequivalent infinities. This profoundly exposes the inherent defects of *the actual infinite thought*, it moves the contradiction to infinity, but the contradiction never disappears. In other words, the infinite process is impossible to complete, thus further supporting *the infinite view of dialectical materialism* (to be specified in a later chapter).

The paradox is as follows:

As we known one-to-one mapping $f : N \rightarrow W$, Where N is the set of natural numbers, $W = \{a_1, a_2, a_3, \dots\}$ is a countable infinite set. $\forall n \in N$, we have $f(n) = a_n \in W$.

Here are a series of successive transformations, and we think that the infinite process can be completed and ended. Let T represent the transformation, T (n) for the n-th transformation:

T(1): Exchanging the *elephants* of the natural numbers 1 and 2, so that 1 corresponds to a_2 , and 2 corresponds to a_1 ;

After T (1) is completed, make the transformation T (2): that is, exchanging the elephants of 2 and 3, so that 2 corresponds to a_3 , and 3 corresponds to a_1 ;.....;

When T (n) is transformed, it occurs after the transformation T (n-1), that is, exchanging the elephants of n and n+1, so that n corresponds to a_{n+1} , and n+1 corresponds to a_1 . And so on, until infinity, we think this infinite process can be done.

At the end of this infinite continuous transformation, we will find the following contradiction: we can no longer find a natural number that corresponds to the element a_1 of W . This is the *Infinite Exchange Paradox*.

3. Hegel's View of Dialectical Infinity

The problem of Finiteness and Infinity is one of the core problems in *Hegel's Science of Logic*. He put forward two kinds of concepts of infinity, "bad infinite" and "real infinite", based on the *infinite problem* in mathematics, and used the great dialectics thought to comprehensively elaborate the dialectical relationship between the two, that is, put forward the idea of *dialectical infinity*. In Hegel, the essence of infinity is change and transfer; based on the theory of the negation of negation, he comprehensively elaborated the inner process of speculation and transcendence from *bad infinity* to *real infinity*, and strongly refuted the dualism of finite and infinite opposition (that is, the idea of "actual infinity"). Hegel's *real infinity* is law, truth, and the connection and unity of the inner matter of infinite things. Truth is concrete, and the negation and sublation (beyond) of *bad infinity*, we reach the *real infinity*, that is, the truth, and reach the height of understanding of the essence of infinite things. The transition from *bad infinity* to *real infinity* marks the deepening of human cognition, reflecting the development process of human understanding of the infinite from possibility to reality, from abstract to concrete, reflects the subjective initiative of human beings in understanding the problem of infinity.

3.1 Specific Analysis of Hegel's thought of Dialectical Infinity

Hegel, the great philosopher, made a deep and detailed study of the infinite and finite things from philosophy and Mathematics. Its core idea is to admit the *being of bad infinity*, and think that *the bad infinity* is not the true infinity; that we should give up and go beyond the *infinite progress*, and then realize the essence of infinity, by sublating *bad infinity* to understand *the real infinity*. At the same time, he also believes that these two kinds of infinity are mutually transformed, with unity, that infinity has two kinds of infinite attributes. However, there is an essential difference between Hegel's *real infinity* and *actual infinity* in Mathematics. Therefore, Hegel's view of infinity is essentially a kind of dialectical view of infinity.

(1) What is bad infinity?

Hegel analyzes the infinity and infinite process as below, "This infinity is the wrong or negative infinity: it is only a negation of a finite: but the finite rises again the same as ever, and is never got rid of and absorbed. In other words, this infinite only expresses that there *ought to be* an elimination of the

finite. The progression into the infinite never gets further than a statement of the contradiction involved in the finite, viz. that it is somewhat as well as somewhat else. It only publishes again and again the alternation between these two terms, each of which calls up the other.” (Hegel, 2015, p. 149)

On the attribute of *bad infinity*, he described in *Hegel's Science of Logic*: “This spurious infinity is in itself the same thing as the perennial ought; it is the negation of the finite it is true, but it cannot in truth free itself therefrom. The finite reappears in the infinite itself as its other, because it is only in its connection with its other, the finite, that the infinite is. The progress to infinity is, consequently, only the perpetual repetition of one and the same content, one and the same tedious alternation of this finite and infinite.” (Hegel, 1969, p. 142)

Hegel speaks thoroughly about this monotonous repetition of *bad infinity*, “Here then we meet, in the first place, that continual expansion of quantity, and especially of number, beyond itself, which Kant describes as awful. The only really awful thing about it is the awful wearisomeness of ever fixing, and anon unfixing a limit, without advancing a single step.” (Hegel, 2015, p. 168)

(2) What is real infinity?

Above, Hegel analyzes the *infinite progress*, and thinks that it is necessary to sublimate the *bad infinity* and then to understand the true infinity, which is of great significance.

Hegel thinks finite things always have to sublimate themselves, “But the true view of the matter is, that life, as life, involves the germ of death, and that the finite, being at war within itself, causes its own dissolution.” (Hegel, 2015, p. 126); “We are aware that everything finite, instead of being inflexible and ultimate, is rather changeable and transient; and this is exactly what we mean by that Dialectic of the finite, by which the finite, as implicitly other than what it is, is forced to surrender its own immediate or natural being, and to turn suddenly into its opposite.” (Hegel, 2015, p. 128)

About what is *real infinity*, he further explained, “But such a progression into the infinite is not the real infinite. That consists in being at home with itself in its antithesis, or, if enunciated as a process, in coming to itself in its other. Much depends on a right estimate of the notion of infinity, as distinguished from the wrong infinity of endless progression, with which we are too apt to rest satisfied.” (Hegel, 2015, p. 150)

(3) How do we transition from *bad infinity* to *real infinity*

Hegel first pointed out that infinite progress is *bad infinity*, not *real infinity*. He described that, “As we then said, it is not the expression of a true, but of a wrong infinity, which is never more than a bare ‘ought’, and thus really remains within the limits of finitude.” (Hegel, 2015, p. 168)

He points out that there is an inner connection between real infinity and bad infinity (infinite progress), not two things that are completely unrelated to each other. He noted that, “Which means, that the true infinite is more than a mere world beyond the finite, and that we, in order to become conscious of it, must relinquish that *progressus in infinitum*.” (Hegel, 2015, p. 168) That is to say, in order to realize the *real infinity*, we must discard, surmount and abandon the *bad infinity*. However, this is not to say that Hegel wants to deny the existence of *bad infinity*, but not to end and to complete an infinite

progress. The above-mentioned evidence shows that Hegel in the history of philosophy for the first time to promote the transition from the *bad infinity* to the *real infinity*, which is a huge progress in human understanding.

He pointed out that people cannot escape from the bad infinity. He notes that in *The Logic of Hegel*, “To suppose that by stepping out into that infinity we release ourselves from the finite, is in truth but to seek the release which comes by flight. But the man who flees is not yet free: in fleeing he is still conditioned by that from which he flees. If it be also said, that the infinite is unattainable, the statement is true, only because the idea of infinity has been burdened with the circumstance of being simply and solely negative.” (Hegel, 2015, p. 150)

How to achieve real infinity, Hegel believes the negation of negation of *being*, *Being-in-itself* changes to *Being-for-self*, we reach real infinity. So then Hegel went on analyzing: “In its relation to an other, somewhat is virtually an other, as compared with that other: and since what is passed into is quite the same as what passes over, since both have one and the same attribute, viz. to be an other, it follows that something in its passage into other only joins with itself. This reference binding it to itself, in the passage, and in the other, is the genuine Infinity. Or, under a negative aspect: what becomes changed is the other, it becomes the other of the other. Thus we find ourselves once more with Being, but as negation of the negation, as Being-for-self.” (Hegel, 2015, p. 151) The above shows that Hegel clearly defines the *real infinity*. What is the *real infinity*? The *real infinity* is self-related in the other, which is the common nature of the infinite things, the mutual stipulation, the internal connection or the universality, is a kind of affirmation (negation of negation), and is a kind of *Being-for-self*. Hegel believes that the concept of limit in calculus is this really infinite good example of this, and this is essentially consistent with Marx’s understanding of the limit. Therefore, Hegel’s infinite view is closer to Marx’s dialectical infinite view. Such as the infinite set $\left\{\frac{1}{n}\right\}$, when n tends to infinity (n is a natural number), its limit is 0: 0 here, that is, the common essence between the infinite elements of the set, the mutual stipulation, and the inner relation, that is to say, 0 is the common nature or attribute of all elements. That is, limit 0 is a real infinity, and $\left\{\frac{1}{n}\right\}$ is an infinite process, a bad infinity. However, 0 is

not an element of the set $\left\{\frac{1}{n}\right\}$, it is a thing that $\left\{\frac{1}{n}\right\}$ cannot reach in an infinite process. Obviously, the

thought of limit and the thought of *actual infinity* are two completely different concepts.

How to go from *bad infinity* into *real infinity*? In *Hegel’s Science of Logic*, Hegel said, “As has already been shown, finitude is only as a transcending of itself; it therefore contains infinity, the other of itself. Similarly, infinity is only as a transcending of the finite; it therefore essentially contains its other and is, consequently, in its own self the other of itself. The finite is not sublated by the infinite as by a power existing outside it; on the contrary, its infinity consists in sublating its own self.” (Hegel, 1969, pp. 145-146) That is to say, the finite thing is the self sublation of the *infinite process*, and then it is transformed into *infinite thing* and becomes *real infinity*.

(4) The unity of bad infinity and real infinity; refute the dualism of the opposition between finity and infinity

Hegel, however, did not completely oppose the *bad infinity* and the *real infinity*, he thinks they are contradicted each other, mutual transformation, mutual connection, with unity; they are the two aspects of the opposition and unity of infinity. In fact, this is to say that the *real infinity* which has sublated *infinite progress* is the finite thing on another level. In the section of introduction named “infinity” in *Hegel’s Science of Logic*, he analyzed the infinity from three aspects: “The infinite is: (a) in its *simple determination*, affirmative as negation of the *finite*; (b) but thus it is in *alternating determination* with the finite, and is the abstract, *one-sided* infinite; (c) the self-sublation of this infinite and of the finite, as a *single process*—this is the *true or genuine infinite*.” (Hegel, 1969, p. 137)

It can be seen that Hegel’s “infinite object” is the unity of *bad infinity and real infinity*. Then, he discusses the transformation from finite to infinite, “In the first place, the negation of the finite and infinite which is posited in the infinite progress can be taken as simple, hence as separate and merely successive. But in addition there is also present in the infinite progress their *connexion*. First there is the finite, then this is transcended and this negative or beyond of the finite is the infinite, and then this negation is again transcended, so that there arises a new limit, a *finite* again. This is the complete, self-closing movement which has arrived at that which constituted the beginning; what arises is the *same* as that from which the movement *began*, that is, the finite is restored; it has therefore united *with itself*, has in its beyond only found *itself* again.” (Hegel, 1969, pp. 146-147) In this thesis, Hegel dialectically analyzes the relationship between finity and infinity, and thinks that *finity* is transformed into *infinity*, and *infinity* is transformed into *finity* on another level. For example, the natural number set N contains natural numbers of *bad infinity*, but once it is formed, it becomes a finite in the set level.

More vividly, Hegel respectively used the *line*, *circle* to describe *bad infinity* and *real infinity*. He wrote, “The image of the progress to infinity is the *straight line*, at the two limits of which alone the infinite is, and always only is where the line -which is determinate being- is not, and which goes *out beyond* to this negation of its determinate being, that is, to the indeterminate; the image of true infinity, bent back into itself, becomes the *circle*, the line which has reached itself, which is closed and wholly present, without *beginning* and *end*.” (Hegel, 1969, p. 149)

In order to further explain the unity of *bad infinity* and *real infinity*, Hegel strongly refuted the dualism about the opposition between finity and infinity; and the concept of *actual infinity* in mathematics is the contrary dualism. He pointed out that, “The dualism, which puts an insuperable opposition between finite and infinite, fails to note the simple circumstance that the infinite is thereby only one of two, and is reduced to a particular, to which the finite forms the other particular. Such an infinite, which is only a particular, is co-terminous with the finite, which makes for it a limit and a barrier: it is not what it ought to be and means to be, that is, the infinite, but only finite.....The Being of the finite is made an absolute Being, and by this dualism gets a fixed ground of its own. Touched, so to speak, by the infinite, it would be annihilated. But it must not be touched by the infinite. There must be an abyss, an

impassable gulf between the two, with the infinite abiding on yonder side and the finite steadfast on this". (Hegel, 2015, pp. 151-152) The dualism regarded the finite as an absolute being, but did not see it will be sublated. In Hegel's view, the contrary infinite is not really infinite (*The Actual Infinity* in mathematics is the opposite infinity), it is purely on the other side, there is no inherent relationship with infinite progress (*bad infinity*), and in fact is only a finite. Therefore, in Hegel's view, *actual infinity* is not really infinity, but only a finite, a finite infinity.

Then, Hegel discusses the dual meaning of the finite and infinite: "The finite has the double meaning of being first, only the finite *over against* the infinite which stands opposed to it, and secondly, of being the finite and *at the same time* the infinite opposed to it. The infinite, too, has the double meaning of being *one* of these two moments—as such it is the spurious infinite—and also the infinite in which both, the infinite and its other, are only moments. The infinite, therefore, as now before us is, in fact, the process in which it is deposed to being only *one* of its determinations, the opposite of the finite, and so to being itself only one of the finites, and then raising this its difference from itself into the affirmation of itself and through this mediation becoming the *true* infinite." (Hegel, 1969, p. 148) In the above description, Hegel believes that the infinite thing has a double meaning, not only confirms that the *bad infinity* is a basic form of infinity, but also criticizes the one sidedness, which only talks about the *bad infinity* and does not pay attention to the *real infinity*. Hegel holds that the infinite is the unity of *bad infinity* and *real infinity*, "it is on its own account just as much finite as infinite" (Hegel, 1969, p. 153), and the *real infinity* is the essence and the higher form of infinite. From the cognitive process, it can be said that the *bad infinity* is the result of people's understanding of the infinite things from outside, from the phenomenon, while the *real infinity* is the product of people's understanding of infinity, which is based on the fact that people go deep into the interior of things, from the general connection of things, in essence. The transition from *bad infinity* to *real infinity* marks the deepening of human cognition, reflecting the development process of human understanding of the infinite from possibility to reality, from abstract to concrete, reflects the subjective initiative of human beings in understanding the problem of infinity. Thus it can be seen that Hegel's view of infinity is completely in accordance with the dialectical materialism of Marx, and it is the view of dialectical infinity.

(5) Analyze and explain the *bad infinity*, *real infinity* and their dialectical relationship from the perspective of mathematical infinity

In order to strengthen the analysis of the *bad infinity* and *real infinity* and the relationship between them, Hegel further analyzed in detail from the infinite mathematical angle, pointed out the mathematical infinite series is a *bad infinity*, limit and the finite form of expression is *real infinity* (because the real infinity is also a finite object), and analyzed the dialectic relationship between the two. He gave an

example to, "The fraction $\frac{2}{7}$ can be expressed as $0.285714\dots\dots$, $\frac{1}{1-a}$ as $1 + a + a^2 + a^3 + \dots\dots$ etc. As so expressed it is an infinite series; the fraction itself is called the sum, or *finite expression* of it." (Hegel, 1969, pp. 246-247) Above, on the right is an infinite series, and

the left is the finite representation of the series. In his view, “Now the nature of this infinity of the series is self-evident; it is the spurious infinity of the progression.” (Hegel, 1969, p. 247) That is, the infinite series on the right belongs to *bad infinity*. At the same time, he believes that the finite form is a

real infinity, he pointed out, “Thus the usually so-called ordinary *sum*, the $\frac{2}{7}$ or $\frac{1}{1-a}$ is in fact a ratio; and this so-called *finite expression* is the truly *infinite expression*”. (Hegel, 1969, p. 248)

Hegel’s analysis of the dialectical relationship between the *bad infinity* and the *real infinity* is also very thorough. First, he points out the difference between the two, “In this infinite series, this inexactitude is actually present, whereas in the genuine mathematical infinite there is only an appearance of inexactitude. These two kinds of mathematical infinite are as little to be confounded as are the two kinds of philosophical infinite. In representing the genuine mathematical infinite, *the form of series* was used originally and it has recently again been invoked; but this form is not necessary for it. On the contrary, the infinite of the infinite series is essentially different from the genuine infinite as the sequel will show. Indeed the form of infinite series is even inferior to the fractional expression.” (Hegel, 1969, pp. 247-248) Next, he thinks that the infinite series is not complete, and the finite form of expression is complete. He said, “For the infinite series contains the spurious infinity, because what the series is meant to express remains an *ought-to-be* and what it does express is burdened with a beyond which does not vanish and *differs* from what was meant to be expressed. It is infinite not because of the terms actually expressed but because they are incomplete, because the other which essentially belongs to them is beyond them; what is really present in the series, no matter how many terms there may be, is only something finite, in the proper meaning of that word, posited as finite, i.e., as something *which is not what it ought to be*. But on the other hand, what is called the *finite expression* or the *sum* of such a series lacks nothing; it contains that complete value which the series only seeks; the *beyond* is recalled from its flight; what it is and what it ought to be are not separate but the same.” (Hegel, 1969, p. 248)

Hegel went on to explain the qualitative difference and unity of the two, and thought that the infinite series is also a finite form of expression, the *real infinity* also contains the regulations of infinity. He pointed out, “What distinguishes these two is more precisely this, that in the infinite series the *negative* is *outside* its terms which are present only *qua* parts of the *amount*. On the other hand, in the finite expression which is a ratio, the *negative* is immanent as the *reciprocal* determining of the sides of the ratio and this is an accomplished return-into-self, a self-related unity as a negation of the negation (*both sides of the ratio are only moments*), and consequently has within it the determination of infinity. Thus the usually so-called *sum*, the $\frac{2}{7}$ or $\frac{1}{1-a}$ is in fact a ratio; and this so-called *finite expression* is the

truly *infinite expression*. The infinite *series*, on the other hand, is in truth a *sum*; its purpose is to represent in the form of a sum what is in itself a ratio, and the existing terms of the series are not terms of a ratio but of an aggregate. Furthermore, the series is in fact the *finite expression*; for it is the incomplete aggregate and remains essentially deficient.” (Hegel, 1969, p. 248)

(6) *Bad infinity cannot be completed, the real infinity is the concrete, certain, completed Being-for-self, is the result of our sublation, beyond the bad infinity*

However, there is an essential difference between Hegel's *real infinity* and the *actual infinity* in Mathematics. Hegel believes that the infinite can be completed and at the same time can not be completed (that is, *real infinity* can be completed and *bad infinity* can not be completed), manifested in his dialectical understanding of the infinity. He thinks that the completion of *real infinity* and the non-completion of *bad infinity* (inexhaustible) are completely different.

From first to last Hegel did not acknowledge the *infinite progress* to finish to end, but proposed that we should go beyond this *infinite progress* to understand the essence of infinity, the inner connection and mutual stipulation of the infinite things, that is to recognize the genuine infinity (*real infinity*), that is to say, *real infinity* is the result of our sublation and transcendence of bad infinity. On the one hand, he believes that the *bad infinity* is essentially deficient, is the incomplete aggregate, so it is not complete; on the other hand, he pointed out that the *real infinity* has nothing to do with *exhausting infinity*, that is, it has nothing to do with the *actual infinity*. In *Hegel's Science of Logic*, he said that, "He starts by defining the infinite as the absolute affirmation of any kind of natural existence, the finite on the contrary as a determinateness, as a negation. That is to say, the absolute affirmation of an existence is to be taken as its relation to itself, its not being dependent on an *other*; the finite, on the other hand, is negation, a ceasing-to-be in the form of a *relation* to an *other* which begins *outside* it. Now the absolute affirmation of an existence does not, it is true, exhaust the notion of infinity; this implies that infinity is an affirmation, not as immediate, but only as restored by the reflection of the other into itself, or as negation of the negative." (Hegel, 1969, pp. 249-250)

In Hegel's view, the *real infinity* is an affirmation (negation of negation), certainly is a finite, so it is completed, present, concrete, and known. The *real infinity* is *self-related in the other*, is the sublation and negation of the *bad infinity*, which is the common essence, the mutual stipulation, the internal connection or the universality of the infinite things, is a kind of *Being-for-self*, and therefore is a more advanced finity. In *Hegel's Science of Logic*, he continued to analyze that, "Spinoza calls the infinite of a series the infinite of the imagination; on the other hand, the infinite as self-relation he calls the infinite of thought, or *infinitum actu*. It is, namely, *actu, actually* infinite because it is complete and present within itself. Thus the series $0.285714\dots$ or $1 + a + a^2 + a^3 + \dots$ is the infinite merely of imagination or supposition; for it has no actuality, it definitely lacks something; on the other hand $\frac{2}{7}$ or $\frac{1}{1-a}$ is *actually* not only what the series is in its developed terms, but is, in addition, what the series lacks, what it only *ought to be*." (Hegel, 1969, pp. 250-251) Here, "*the infinite of the imagination*" is the *bad infinity*, and the "*infinitum actu*" is the *real infinity*.

Real infinity as the negation of *bad infinity*, as a negation of negation, is an affirmation, but also concrete. In *The Logic of Hegel* he noted, "(1) The result of Dialectic is positive, because its own content was specific, or because its result, instead of being an empty and abstract nothing, is rather the

negation of certain specific terms: which terms are contained in the result, for the very reason that it is a result and not an immediate nothing. (2) It follows from this that the rational stage, though it be an abstraction of thought, is still concrete, being not a plain formal unity, but a unity of distinct terms of thought.” (Hegel, 2015, p. 130) The limit in mathematics is a real infinity, as a negation of negation, is a concrete affirmation.

3.2 Summary of Hegel's View of Dialectical Infinity

The brief introduction is as follows: Hegel believes that the infinite thing has a double meaning, infinite thing is the unity of *bad infinite* and *real infinite*; not only confirms that the *bad infinity* is a basic form of infinity, but also criticizes the one sidedness, which only talks about the *bad infinity* and does not pay attention to the *real infinity*. So infinity is the unity of *bad infinity* and *real infinity*, it is a *free self Being*; *Real infinity* cannot be separated from *bad infinity*, *Being-for-self* is inseparable from *Being-in-itself*, *Bad infinity* is the carrier of *real infinity*, *real infinity* is the purpose and direction of *bad infinity*. Human understanding of the infinity, from possible to reality, from the abstract to the concrete, has completed the transformation from *bad infinity* to *real infinity*. From the cognitive process, it can be said that the *bad infinity* is the result of people's understanding of the infinite things from outside, from the phenomenon, while the *real infinity* is the product of people's understanding of infinity, which is based on the fact that people go deep into the interior of things, from the general connection of things, in essence. The transition from *bad infinity* to *real infinity* marks the deepening of human cognition, reflecting the development process of human understanding of the infinite from possibility to reality, from abstract to concrete, reflects the subjective initiative of human beings in understanding the problem of infinity.

The *real infinity* is present, concrete, completed infinity, is *Being-for-self* and rational *Being*, is the completed quality; and the *bad infinity* is possible, abstract, uncompleted infinite, is *Being-in-itself* and intellectual *Being*. The difference between *real infinity* and *bad infinity* reflects the opposition between dialectical understanding and metaphysical understanding. Hegel concludes in *The Logic of Hegel* that: “But by Dialectic is meant an indwelling tendency outwards and beyond; by which the one-sidedness and limitation of the formulae of understanding is seen in its true light, and shown to be the negation of these formulae. Things are finite, just because they involve their own dissolution. Thus understood, Dialectic is discovered to be the life and soul of scientific progress, the dynamic which alone gives an immanent connexion and necessity to the subject-matter of science; and, in a word, is seen to constitute the real and true, as opposed to the external, exaltation above the finite.” (Hegel, 2015, p. 126) Therefore, from *bad infinity* to *real infinity* is an inner transcendence, a dialectical process. *Real infinity* and *bad infinity* are the basic forms of infinity. Hegel put forward the profound dialectical conclusion that *real infinity* contains and subtracts *bad infinity*, and tries to grasp infinity concretely and realistically, and opposes abstract inferences about it. The transformation from *bad infinity* to *real infinity* is the transformation from understanding to reason, is a great leap in human understanding of infinity, and is the highest task of Hegel's philosophy.

The View of Dialectical Infinity holds that *real infinity* (or true infinity) is the connection and unity of the inner quality of *bad infinity*, so *real infinity* represents and reflects the quality of *bad infinity*; bad (or evil) infinity represents quantity of infinity (motion), *real infinity* represents quality (law, commonness or connection) of infinity. The View of Dialectical Infinity holds that the *actual infinity* separates the finite from the infinite, and looks at things from the point of view of stillness rather than movement, which has an internal irreconcilable contradiction, and is a metaphysical idealistic infinite view. The concept of *actual infinity* is exactly the same thing as Transcendentalism, which in essence believes that the development of the world, the movement will have an end. However, Hegel's view of infinity is not only to see *the universal contact of the objective world (real infinity)*, but also the objective reality of "*the infinite process cannot be completed (bad infinity)*", so it is a scientific view of infinity, but also a view of dialectical infinity.

Thus it can be seen that the concept of *actual infinity* in mathematics holds that the *infinite progress* can be completed, which is completely contrary to Hegel's thought of dialectical infinity.

4. Engels's Dialectical Analysis of Infinity and the Negation of the Actual Infinity

Engels paid special attention to the correct understanding of the infinite, and carried out a profound analysis and thinking of the infinite in the two books of *Dialectics of Nature* and *Anti-Dihring*. Mainly in the following areas:

4.1 Engels Thought That the Theory of Mathematics Must Obey the Reality, and the Understanding of the Infinite Should Be Explained from the Objective Reality.

Engels said: "Mathematical infinity is taken from reality, although unconsciously, and therefore can only be explained from reality and not from itself, from mathematical abstraction." (Engels, 1964, p. 276)

In *Anti-Dihring*, Engels reiterated the idea, "Because in mathematics it is necessary to start from definite, finite terms in order to reach the indefinite, the infinite, all mathematical series, positive or negative, must start from 1, or they cannot be used for calculation. The abstract requirement of a mathematician is, however, far from being a compulsory law for the world of reality." (Engels, 1947, p. 74)

4.2 Engels Thought That Infinity Was a Kind of Objective Being, Affirmed Hegel's Dialectical Infinite View.

In *Dialectics of Nature*, Engels pointed out: "When we say that matter and motion are not created and are indestructible, we are saying that the world exists as infinite progress, i.e., in the form of *bad infinity*, and thereby we have understood all of this process that is to be understood." (Engels, 1964, p. 240) The actual is to recognize the objective existence of the infinity.

Then, Engels evaluated and affirmed the infinite view of Hegel, he pointed out: "*Bad infinity. True Infinity* was already correctly put by Hegel in filled space and time, in the process of nature and in history. The whole of nature also is now merged in the history, and history is only differentiated from

natural history as the evolutionary process of self-conscious organisms. This infinite complexity of nature and history has within it the infinity of space and time—*bad infinity*—only as a sublated factor, essential but not predominant.” (Engels, 1964, p. 240)

4.3 Engels Thinks That Infinity Is Purely Composed of Finites, and That This Contradiction Will Not Be Destroyed, That Is, the Infinite Process Can Not Be Completed. This Is a Direct Negation of the Actual Infinity, But Also a Direct Affirmation of Hegel’s View of Bad Infinity.

On the essence of infinity, in *Anti-Dühring*, Engels sharply pointed out: “For that matter, Herr Dühring will never succeed in conceiving real infinity without contradiction. Infinity is a contradiction, and is full of contradictions. From the outset it is a contradiction that infinity is composed of nothing but finites, and yet this is the case. The limitedness of the material world leads no less to contradictions than its unlimitedness, and every attempt to get over these contradictions leads, as we have seen, to new and worse contradictions. It is just *because* infinity is a contradiction that it is an infinite process, unrolling endlessly in time and in space. The removal of the contradiction would be the end of infinity. Hegel saw this quite correctly, and for that reason treated with well-merited contempt the gentlemen who subtilised over this contradiction.” (Engels, 1947, pp. 74-75) In the above description, we know that in Hegel’s and Engels’s view *the contradiction between finity and infinity* can not be eliminated, that is, the *infinite process* can not be terminated.

On the completion of the *infinite process*, Engels denied directly in *Anti-Dühring*. He said, “It is just the same with eternal truths. If mankind ever reached the stage at which it should work only with eternal truths, with results of thought which possess sovereign validity and an unconditional claim to truth, it would then have reached the point where the infinity of the intellectual world both in its actuality and in its potentiality had been exhausted, and thus the famous miracle of the counted uncountable would have been performed.” (Engels, 1947, p. 129)

In order to explain the thought of *bad infinity*, Engels thought that we can not use the identity of the thinking and the objective world to prove the reality of the product of human thinking, and criticized the mysterious Hegel’s thought - *absolute spirit*. He said, “To attempt to prove the reality of any product of thought by the identity of thinking and being was indeed one of the most absurd delirious fantasies of—a Hegel” (Engels, 1947, p. 61). “But in human history infinite progress is recognized by Hegel as the sole true form of existence of “*spirit*”, except that fantastically this development is assumed to have an end—in the production of the Hegelian Philosophy.” (Engels, 1964, p241) The above discussion shows that, we can not prove the reality of our thinking products—*the infinite process can be completed*, because of the identity of the finite subjective world and the objective material world. This actually pointed out that the view of *actual infinity* is fatally flawed. That is to say, *the infinite transcendence of thinking* does not mean *the completion of the objective infinite process*, and *the infinite transcendence of thinking* has never really been done.

4.4 Engels Thought That Infinite Things Can Be Known, But Also Can Not Be Known, Which Is Consistent with the Essence of Thought of Hegel's Bad Infinity and Real Infinity. That Is, The Real Infinity (As The Universal Connection Between Infinite Things, the Inner Connection) Can Be Known and Completed, and the Bad Infinity Can Not Be Known, Can Not Be Completed.

First of all, Engels believes that the infinity can be known, that is, Hegel's *real infinity* can be known. In *Dialectics of Nature*, he said, "This is quite correct in so far as only finite objects enter the sphere of our knowledge. But the proposition needs to be supplemented by this: '*fundamentally we can know only the infinite.*' In fact all real, exhaustive knowledge consists solely in raising the individual thing in thought from individuality into particularity and from this into universality, in seeking and establishing the infinite in the finite, the eternal in the transitory. The form of universality, however, is the form of self-completeness, hence of infinity; it is the comprehension of the many finites in the infinite. The form of universality in nature is *law*, and no one talks more of the eternal character of the laws of nature than the natural scientists..... All true knowledge of nature is knowledge of the eternal, the infinite, and hence essentially absolute." (Engels, 1964, pp. 237-238) Here, "the form of universality is law" refers to Hegel's *real infinity*, which is the form of self-completeness, is a completed, known *Being-for-self*, so it is a kind of affirmation, a more advanced finity.

On the other hand, Engels also pointed out that the infinity is not known, that is to say, Hegel's *bad infinity* can not be known. He said, "But this absolute knowledge has an important drawback. Just as the infinity of knowable matter is composed of the purely finite things, so the infinity of thought which knows the absolute is composed of an infinite number of finite human minds, working side by side and successively at this infinite knowledge, committing practical and theoretical blunders, setting out from erroneous, one-sided, and false premises, pursuing false, tortuous, and uncertain paths, and often not even finding what is right when they run their noses against it (Priestley). The cognition of the infinite is therefore beset with double difficulty and from its very nature can only take place in an infinite asymptotic progress. And that fully suffices us in order to be able to say: the infinite is just as much knowable as unknowable, and that is all that we need". (Engels, 1947, p. 238)

4.5 Engels' Criticism of Mathematical Chaos

Adhering to the *actual Infinity*, abandoning the *potential infinity*, the development of mathematics fell into a vicious circle. Engels gave a harsh criticism of the gentlemen who caused the confusion of mathematics, and points out that only by insisting on materialist dialectics can we solve the above confusion.

The three crises in the history of mathematical development have already explained the above conclusion, and a series of paradoxes in the development of set theory have proved the above arguments again. For example, the Paradox of Cantor explains that there is no collection of all sets. In other words, the sets are always in constant growth, which is also a *potential infinity* concept. Drop problem (Extension of the Zeno Paradox) illustrates that the throwing process is impossible to complete. Based on the *actual infinity* ideas, the *Infinite Exchange Paradox* overthrew the possibilities of Hilbert

Hotel Problem. The great logician Kurt Gödel used the Incompleteness Theorem to completely negate the Hilbert's program of *actual infinity*, it shows that the development of things is a kind of non *actual infinite* form, that is, the understanding of things is in a state of continuous progress but not once and for all.

Mr. Ling Esheng pointed out in the article *The Three Mathematical Crises Caused by Bad Infinity*: "It is a sensible way to avoid conflict, as people try to push the contradiction to infinity". However, the result of the logic analysis is that the infinite distance, which is supposed to be not contradictory, is also a trap. Hilbert still made a mistake about infinity (Ling Esheng, 1992, p. 71). That is to say, Hilbert want to abandon the *bad infinity*, but was eventually trapped by *bad infinity*. The logic school represented by Russell, to develop the *ramified type theory* in order to solve the paradox in set theory, to seize the *vicious circle principle*, is actually a return to the "*potential infinity*" this way up (Xu Lizhi, 2000, pp. 156-157).

The above shows that people in the development of mathematical theory with *actual infinite* ideas, there has been one after another contradiction, had to go back to deal with a new, greater *potential infinity*.

Hegel made it very clear, "Here then we meet, in the first place, that continual expansion of quantity, and especially of number, beyond itself, which Kant describes as awful. The only really awful thing about it is the awful wearisomeness of ever fixing, and anon unfixing a limit, without advancing a single step." (Hegel, 2015, p. 168). This is no doubt a new prescription for our mathematicians.

Engels gave a clear criticism of the development of Higher Mathematics. In his correspondence with Marx, Engels criticized the stubborn ideas that still exist in the minds of some mathematicians today. He said: "Why do mathematicians stubbornly make it mysterious? But this is the one sidedness of the thought and method of those gentlemen" (Marx, 1975, p. 211). This is a warning to a large group of people in today's mathematics! Einstein also gave a similar criticism, in the article *On Science* he pointed out, "Most of the errors in philosophy and logic are due to the tendency of human reason to regard symbols as something real."

How to solve the confusion in the study of mathematics, so that the development of the road to return to health is to return to the path of development of dialectical materialism?

Engels pointed out that the theory and principle, first of all, should be suitable for objective reality. He pointed out, "it is not nature and the realm of man which conform to these principles, but the principles are only valid in so far as they are in conformity with nature and history." (Engels, 1947, p. 50) As of the infinite views in mathematics, only suitable for the nature, have vitality.

Then, Engels pointed out that only with the materialist dialectics as the guiding ideology of scientific research, to solve the confusion. In the chapter of *Anti-Dühring* old Preface in the book of *Dialectics of Nature*, he pointed out that, "One can scarcely pick up a theoretical book on natural science without getting the impression that natural scientists themselves feel how much they are dominated by this incoherence and confusion, and that the so-called philosophy now current offers them absolutely no

way out. And here there really is no other way out, no possibility of achieving clarity, than by a return, in one form or another, from metaphysical to dialectical thinking.” (Engels, 1964, p. 46)

In the section *natural science and philosophy*, he said again: “The rigid categories disappeared here; mathematics arrived at a field where even such simple relations as those of mere abstract quantity, *bad infinity*, assumed a completely dialectical form and compelled the mathematicians to become dialectical, unconsciously and against their will. There is nothing more comical than the twistings, subterfuges, and expedients employed by the mathematicians to solve this contradiction, to reconcile higher and lower mathematics, to make clear to their understanding that what they had arrived at as an undeniable result is not sheer nonsense, and in general rationally to explain the starting-point, method, and result of the mathematics of the infinite.” (Engels, 1964, p. 206) This shows that in order to solve the confusion of mathematical science, we must regard the materialist dialectics as its guiding ideology.

5. The Difference Analysis between Hegel’s View of Infinity and the Two Views of Infinity in Mathematics

From the above two aspects of mathematics and philosophy, we can see that there are essential differences in the concept of all kinds of infinity view. Mainly summarized as follows:

5.1 The Main Features and Relations of Hegel’s Bad Infinity and Real Infinity:

- The *bad infinity* and the *real infinity* are two aspects of the same infinity, two attributes, who also cannot do without whom. *Bad infinity* is the carrier of *real infinity*, *bad infinity* contains *real infinity*, *real infinity* is the intrinsic essence of infinity, and *bad infinity* is the external appearance of infinity.
- The infinite is the objective being, this is self-evident. As long as we adhere to the materialist dialectics, we naturally come to the same conclusion. The objective material world in infinite space, time is a typical infinite object.
- *Bad infinity*: admitting infinite objective existence, but that the *infinite process* is not to be completed, so it is a kind of *bad infinity*.
- *Real Infinity*: The view that infinity is *self-related in the other*, is the sublation of the *bad infinity*, is the common nature, the inner connection or universality of the infinite things. Therefore, it is considered that the infinity is knowable and can be accomplished. It is the result of a change in quality, is the sublation and negation of the *bad infinity*, is the other side of *bad infinity*, and thus is a must, is a more advanced finity.

Combined with the characteristics of *bad infinity* and *real infinity*, we can see Hegel’s infinite view is: the infinite is the objective existence, the infinite can be known, the *infinite progress* can not be completed. In fact, this is the infinite view of dialectical materialism. Of course, there is a difference between Hegel’s infinite view and the infinite view of dialectical materialism. Marx and Engels, after excluding the mysterious elements of Hegel’s thinking, developed into the infinite view of dialectical materialism, namely the infinite view of Marxism.

5.2 The Main Difference between Bad Infinity and Potential Infinity:

Although people often regard the two as the same thing, but in philosophy, the two have a fundamental qualitative difference.

- The *potential infinity* view holds that infinity is not an objective being, but a potential, because it cannot be completed;
- The *bad infinity* view holds that infinity is an objective being, but at the same time, it can not be completed;
- Their similarities:
The *infinite progress* can not be completed;
Relatively finite human consciousness, it is potential. This reflects a profound contradiction between the subjective and objective world. Therefore, *potential infinity* can be regarded as a fragment of *bad infinity*, a segment of continuous development and change, a quantity of motion change, and a quantity that continually manifests bad infinity.
- Because of the great similarities between the two, people tend to regard the two as the same concept.

5.3 The Main Difference between Real infinity and Actual infinity:

After careful study of Hegel's infinite theory, it is obvious that the concept of Hegel's *real infinity* is completely different from that of *actual infinity* in Mathematics.

- *Actual infinity* view in mathematics: The infinite is an objective being, the *infinite progress* can be completed, and this is directly opposed to the *bad infinity* in philosophy, to admit *actual infinity* is a direct negation of *bad infinity*;
- Hegel's *real infinity*: refers to the internal relations, common essence and regularity of infinite things, and not whether an *infinite progress* can be completed. Since *real infinity* and *bad infinity* are dialectical and unified, it is necessary to admit that *real infinity* must admit the being of infinite objectivity, admit *bad infinity* and admit that it is impossible to complete the infinite.

In order to admit the *real infinity*, we must first admit the *bad infinity*, because the *real infinity* is the sublation of the *bad infinity*, rather than the abandonment of the *bad infinity*. The *bad infinity* and the *real infinity* are the different properties of the infinite thing, which is a unity, and it is indispensable, to admit one is to admit the other.

- The relationship between the two and the *bad infinity*: *The actual infinity* is directly opposed to the *bad infinity*, there is no dialectical unity, and the *real infinity* is a dialectical unity with the *bad infinity*, which can not be separated from the *bad infinity*.
- Common of the two: They all recognize the objective existence of infinite objects; they are a kind of finite (*the actual infinity* is a kind of finity, see behind the discussion)

- The difference between the two: *The actual infinity* thinks that *infinite progress* can be completed, and the *real infinity* thinks *infinite progress* impossible. The former is an idealist Epistemology, and the latter is a dialectical materialist Epistemology.

It can be seen that the concept of *actual infinity* runs counter to Hegel's thought of *bad infinity* and *real infinity*.

5.4 The Similarities and Differences between Potential Infinity and Actual Infinity:

Combined with the mathematical and philosophical analysis of Zhu Wujia's views on these two kinds of infinity, the following points are summarized:

- On the objective existence of infinite: The *actual infinity* holds that the infinite objective exists, and the *potential infinity* thinks that infinity is not a kind of reality, and denies the existence of infinite objective;
- About the infinite whether can be accomplished: The *actual infinity* thinks that infinity can be done, and on the other hand, the *potential infinity* holds that infinite affirmatively cannot be completed.

In the viewer of *potential infinity*, infinity is a kind of potential, which is in constant construction, so it is not a kind of reality, this is obviously denying the objective existence of infinity with the impossibility of the subjective world's understanding of infinity. The infinite object as the objective thing is true, not with the will of human beings. Thus the concept of *potential infinity* has made a subjective mistake, so it is an infinite view of idealism.

To sum up, *actual infinity* and *potential infinity* are all to make the finity and infinity opposition, but failed to unify them, in fact, this is the continuation of Kant's transcendental thought of antinomy. While Hegel's view of infinity is the dialectical unity of finity and infinity, combining quantity of the infinity and quality of the infinity to examine the infinity, thus forming a dialectical view of infinity, which is *the view of dialectical infinity*.

6. The Contradiction between Finitude and Infinity Is the Concrete Embodiment of the Relationship between Subject and Object

The contradiction between the subjective world and the objective material world is *a contradiction between finity and infinity*. The problem of the relationship between Thinking and Existence is also a problem of finity and infinity. *The contradiction between finity and infinity* is the concrete embodiment of the relationship between subject and object.

Before we analyze, we must first clarify a fundamental proposition: the subjective world is a finite world, the objective world is an infinite world (the infinity of time and space is a typical representative).

6.1 The Subjective World Is a Finite World, and Finitude Is a Natural Attribute of the Subjective World.

The finite nature of human thinking, Engels made a brilliant exposition in two in *Anti Dühring*, that person's thinking is limited, is finite, is a process of progress and development, that is to say the

subjective world is a finite world. He said in the article of *transcendentalism*, “Mankind therefore finds itself faced with a contradiction: on the one hand, it has to gain an exhaustive knowledge of the world system in all its interrelations; and on the other hand, because of the nature both of men and of the world system, this task can never be completely fulfilled. But this contradiction lies not only in the nature of the two factors -the world, and man- it is also the main lever of all intellectual advance, and finds its solution continuously, day by day, in the endless progressive development of humanity, just as for example mathematical problems find their solution in an infinite series or continued fractions. Each mental image of the world system is and remains in actual fact limited, objectively by the historical conditions and subjectively by the physical and mental constitution of its originator.” (Engels, 1947, pp. 53-54) In the article of *eternal truth*, he went on to state, “Here once again we find the same contradiction as we found above, between the character of human thought, necessarily conceived as absolute, and its reality in individual human beings all of whom think only limitedly. This is a contradiction which can be resolved only in the course of infinite progress, in what is -at least practically for us- an endless succession of generations of mankind. In this sense human thought is just as much sovereign as not sovereign, and its capacity for knowledge just as much unlimited as limited. It is sovereign and unlimited in its disposition, its vocation, its possibilities and its historical ultimate goal; it is not sovereign and it is limited in its individual realization and in reality at any particular moment.” (Engels, 1947, p. 129) As a result, the subjective world is a constantly evolving and growing finite world, while the objective world is an absolute infinite world; and *the contradiction between finity and infinity* is a constant conflict between the subjective world and the objective world, but it can not be ended.

6.2 *The Objective Material World Is an Infinite World.*

Two typical representatives of the objective world: one is time and the other is space. As the basic common sense of dialectical materialism philosophy, we state as follows: objective matter is inseparable from movement, matter is the carrier of motion, movement is the attribute of matter, and there is no matter without movement, which shows that movement is universal, eternal and unconditional, and therefore absolute. The moving matter takes time and space as its own form of being. Time and space, as the form of the being of matter, their objective reality is unchangeable, unconditional, and thus is absolute. Matter is indestructible, infinite, and in perpetual motion, so the space and time as a form of material movement are infinite. The material world is infinite in time and space, while the time and space of concrete things are limited, temporary in time and bounded in space. Therefore, the objective material world is an infinite world, and it exists infinitely. Infinity is an essential attribute of the objective material world.

6.3 The Contradiction between Finitude and Infinity Is a Contradiction That Human Beings Face when They Understand the Objective Material World. It is That the Human Subjective World (Finite) at This Moment can not Fully Understand and Grasp the Endless Objective Material World (Infinite).

It reflects that the human subjective world in the understanding of the objective material world encountered a difficulty, a contradiction, and this difficult or contradiction is long-term, eternal, can not be solved in the end, is an insurmountable gully when people understand the infinite world. This is the negative, impotent side of humanity in the understanding of the objective material world.

If we use the finitude on behalf of the cognitive ability of human beings, with the infinity on behalf of the objective world, then *the contradiction between finitude and infinity* is equivalent to the contradiction between the limited cognitive ability of human beings and the infinite objective material world. The human understanding of the material world is to know the world in a certain point (or a certain nature) of the infinite material world, rather than to know all of the objective material world, that is to say, human beings know a part of the infinite objective material world, a finite part, because *infinity* is made up of *finitude*. In this way, *the contradiction between finitude and infinity* is bound to become an eternal contradiction between the subjective limited world and the objective infinite world, which cannot be resolved.

On the contradiction between the subjective world and the objective world (that is, the relationship between subject and object), Einstein also said, "There is a huge world outside us, it leaves us human and exists independently, it is like a great and eternal mystery to us, but at least in part is what our observation and thought can do." Infinity, although it is an objective existence, but for the limited cognitive ability of human beings, it is a kind of recessive existence; Such as the set of natural numbers N has infinitely many elements, is a kind of objective existence, but for the limited human, it is a kind of recessive existence. Therefore, the infinity is not only the unity of *real infinity* and *bad infinity*, but also the unity of the objective existence and the recessive existence, which embodies *the contradiction between finitude and infinity*.

Therefore, from the point of view of finitude as an integral part of infinity, we can say that human beings can not only know the objective world, but also can not know the objective world, and this is *the contradiction between finitude and infinity*, which is the natural dialectics of Marx's theory on the relationship between finitude and infinity.

7. Actual Infinity Does not Reflect the Essence of Infinite Things

The view of *actual infinity* abandons *the contradiction between finitude and infinity*, which is an idealism view of infinity out and out, which is essentially a view of finitude. Infinity exists and cannot be completed, incompleteness is the fundamental attribute of infinity. This section is based on Hegel's thought of dialectical infinity, comprehensively analyzes the error of *actual infinity*, deeply expounds the concept of limit in mathematics, and points out that *mathematical limit* is a real infinity.

7.1 The Actual Infinity and the Potential Infinity (Bad Infinity) Are A Kind of Opposite Relation, and There Is No Dialectical Unity Relation, So the Actual Infinity Is a False Infinity.

The view of *actual infinity* is a direct negation of *potential infinity*, and there is no certainty. In this way, *the contradiction between finity and infinity* (infinite process) disappears in it, and the contradiction is eliminated, and that is the end of infinity. Hegel has already given a clear explanation of *the contradiction between finity and infinity* from the dialectical point of view: “Now *the infinite progress* is only the *expression* of this contradiction, not its *resolution*” (Hegel, 1969, p. 227). Hegel also strongly refuted the dualism about the opposition between finity and infinity, and argues that the opposite infinity is not real infinity, but only finity, and the concept of *actual infinity* in mathematics is the contrary dualism. Therefore, *actual infinity* is not real infinity, but only a finite one.

We can't find any shadow of the *potential infinity* (*infinite progress* or *bad infinity*) in the *actual infinity*, but only the finite. This infinity, which has completely abandoned *the infinite process*, is not so much an infinite, as it is a real finite. Because it abandoned the *infinite process*, it also abandoned *the contradiction between finity and infinity*, so that they become a completely finity.

7.2 There Is No Qualitative Connection between Actual Infinity and Infinite Process (Potential Infinity)

Hegel emphasizes the unity of finity and infinity, and *the view of actual infinity* does not combine the infinite quantity with the infinite quality. It is in the negative of the quantity at the same time did not find the quality of things from the negative, but the quality of things with a negative, or even to the opposite of finite being, become an abstraction, artificially cut off contact between the infinity and finity, finally let oneself fall into nihilism, let oneself become an other that has no inner relation with the infinite process, that is to say, *actual infinity* and *infinite process* are entirely different things.

Dialectical negation is the link of connection. In Hegel's view, the *real infinity*, is *self-related in the other*; which is the common essence of the infinite things, the mutual stipulation, the internal connection or the universality, is negation of negation, but rather a kind of affirmation, is a kind of *Being-for-self*. At the same time, *finite being* is provided with *determinateness of quality*, is finite and variable, both positive and negative contains, and this kind of *determinateness of quality* (i.e., mutual regulation, common essence) is the link of finite and infinite, namely the link of *bad infinity* and *real infinity*. In the problem of Hilbert Hotel, *the view of actual infinity* in the negative of *infinite progress* at the same time denies the mutual stipulation (the inner relation) of the infinite things, the essence of *no spare room*, and concludes that *there is a spare room* in this completely opposite and contrary conclusion. This is completely contrary to the unity of finite and infinite, contrary to the unity of *bad infinity* and *real infinity*. The view of *actual infinity* believes such links do not exist, to deny its own regression (i.e., total negation rather than dialectical negation), see themselves as something that is purely on the other side of the world, no internal connection with the *infinite progress* (*bad infinity*), thus artificially cut off the relation between finite and infinite, eventually led to the no unified quality (there is no unity between the quality of the *actual infinity* and the quality of *infinite progress*), lead to the quantity of their own and their own quality is not uniform, and let the *actual infinity* become a thing

that has nothing to do with the *infinite progress* and the *potential infinity*, and becomes an abstract thing, an other. Hegel used *the line*, *the circle* to describe the *bad infinity*, *real infinity*, and *the actual infinity* is neither the *straight line*, not *the circle*, but it means that there is an *ultimate goal* (or *absolute truth*), and that the *ultimate goal* is a goal that can be reached after experiencing and completing an *infinite progress*. Therefore, it is not a dialectical sublation of *infinite progress* (i.e., *bad infinite*), but rather a mechanical, metaphysical and artificial abandonment, artificially cut off contact with *infinite progress*, so as to allow themselves to become something that is purely on the other side of the world. The qualitative change of things does not mean that things and the original things absolutely have no connection, completely cut off. On the contrary, the dialectical negation is the link of connection, the qualitative change with no connection does not exist. The reachability of finite space is often used to illustrate the completeness of the infinite process. For example, by using 1 of the coordinate axis X up to 0, people think that the infinite process of $\left\{ \frac{1}{n} \right\}$ can be completed. The essence is to discard the

infinite progress and replace the infinity with the finite, treat the infinity simply as a finite, impose the product of human thinking on the objective material, artificially create irreconcilable oppositions, while the infinite progress $\left\{ \frac{1}{n} \right\}$ still exists independently. This kind of imaginary infinity (that is, *actual*

infinity) is not infinite progress itself, but has no internal connection with infinite progress; it is just a spectator and other thing.

Hegel wants to combine the quantity of infinity with the quality of infinity, and holds that the quantitative infinity is not the infinite progress of simple quantity, but the stipulation of its own quality. (The so-called *quantum* is the quantity that has the stipulation, has the limit, this stipulation is the boundary.) But this kind of *actual infinite* in mathematics does not combine the quantity and the quality, therefore, the thought is followed by Kant's transcendental, subjective, metaphysical infinite thought, not Hegel's dialectical thought of infinite. Therefore, Hegel strongly criticized this kind of *actual infinity*: he considers that this kind of *actual infinity* is a nihilistic abstract, and it mistakenly regards the objective existence of infinity as the completed, already known infinity, treats the infinite simply as finite. In the analysis of Kant's thought of the antinomy he pointed out the essence of this kind of *actual infinity*, "Such intellect commits the error of holding such mental fictions, such abstractions, as an infinite number of parts, to be something true and actual; but this sensuous consciousness does not let itself be brought beyond the empirical element to thought. The Kantian solution of the antinomy likewise consists solely in the supposition that reason should not soar beyond sensuous perception and should take the world of appearance, the phenomenal world, as it is. This solution leaves the content of the antinomy itself on one side; it does not attain to the nature of the *Notion* of its determinations, each of which, isolated on its own, is null and is in its own self only the transition into its other, the unity of both being *quantity* in which they have their truth." (Hegel, 1969, p. 199). As a result, the *actual infinity* becomes an abstraction, it is not only beyond *the infinite progress (bad infinity)*, also crossed

the real infinity, so as to let itself become a real *nothing*; because in Hegel's view, over the limit will be no.

Hegel and Kant object to regard infinity as a pure quantum (bounded quantity), as a maximum, as the completed quantity of a certain unit, that is, as an *actual infinity*. The essence of *actual infinity* is to treat infinity as a quantum, a limit, not a variable. Hegel pointed out, "Now since the infinitely great or small is that which cannot be increased or diminished, it is in fact no longer a quantum as such." (Hegel, 1969, p. 243), and "Kant objects to infinite wholes being regarded as a maximum, as a completed amount of a given unit." (Hegel, 1969, p. 243), that is, Hegel and Kant think infinity and infinitesimal are one variable, which is completely consistent with *the limit theory* of modern mathematics. Infinity as a process is a variable, and the essence of infinity is movement and change. Therefore, Hegel's philosophy is essentially a philosophy of motion, which fully reflects the bad infinity of the infinite. Infinity and infinitesimal are both one variable, one infinite progress, and Hegel explains: "The infinite quantum as infinitely great or infinitely small is itself implicitly the infinite progress; as great or small it is a quantum and at the same time it is the non-being of quantum." (Hegel, 1969, p. 238), "The continuity of quantum with its other produces the conjunction of both in the expression of an infinitely great or infinitely small. Since both still bear the character of quantum they remain alterable, and the absolute determinateness which would be a being-for-self is, therefore, not attained." (Hegel, 1969, p. 227). The infinitesimal quantity in the limit is a variable, neither a very small quantity nor an arbitrary small quantity, but a variable with zero as the limit, so it is only a disappearing quantity, which is completely consistent with the limit theory of modern mathematics. The reason why the dialectical essence of infinitesimal concept is not clear for a long time lies in the inability to correctly understand the qualitative *moment* (or the qualitative stipulation) appearing in the infinite quantity. Hegel's amazing feat on infinity is the outstanding embodiment of his dialectic thought.

The limit in mathematics is a real infinity. Hegel holds that *infinite quantum* includes not only the quantitative determinateness (that is, infinite progress, *bad infinity*), but also the qualitative determinateness (that is, *real infinity*). Real infinity is a qualitative stipulation; the reason why dx/dy belongs to real infinity is that it is no longer purely quantitative. Hegel pointed out, "It is a quantitative determinateness in qualitative form; its infinity consists in its being a qualitative determinateness." (Hegel, 1969, p. 245), the mathematical infinity here is *real infinity*. The so-called finite form of expression (fraction itself) is precisely the form of expression of *real infinity*, while the infinite of series is truly the form of finite expression, because it obviously belongs to the bad infinity such as infinite progress. ".....in general, with the *functions of variable magnitudes*. This infinite is the *genuine mathematical* qualitative infinite which Spinoza also had in mind." (Hegel, 1969, p. 251), so dx/dy is a real infinity. Bad Infinity is only a kind of quantitative determinateness, which shows the contradictory movement and infinite progress between pure quantum and quantum. The real infinity is the quantitative determinateness of quality, which forms the contradictory movement of quantity and quality; because this contradiction is intrinsic and inevitable, it achieves the real unity of finite and

infinite (He Jiannan, 1983). So Hegel concludes: “The infinite quantum as the unity of both moments, of the quantitative and qualitative determinateness, is in the first instance a ratio.” (Hegel, 1969, p. 314) This ratio is real infinity, and the limit concept is the *infinite quantum* here.

Infinitesimal is a disappearing quantity (variable), neither *being* (any quantity) nor nothing (pure zero), but the unity of being and nothing. Dx , dy , as an infinitesimal quantity, are no longer quantum; their meaning is only in relation, only means the *moment*, that is, the determinations of the differential co-efficient dx/dy ; So they are both zero and not zero, not zero in the process of change, but the trend is zero. So Hegel stated, “ Dx , dy , are no longer quanta, nor are they supposed to signify quanta; it is solely in their relation to each other that they have any meaning, a *meaning merely* as *moments*. They are no longer *something* (something taken as a quantum), not finite differences; but neither are they *nothing*; not empty nullities. Apart from their relation they are pure nullities, but they are intended to be taken only as moments of the relation, as determinations of the differential co-efficient dx/dy .” (Hegel, 1969, p. 253)

Hegel holds that mathematical infinity is *real infinity* (referring to limit or dx/dy), and severely criticizes metaphysical infinity (refers to *bad infinity*). Hegel believes that the usual metaphysical infinity is *bad infinity*, and he points out: “while the ordinary metaphysical—by which is understood the abstract, spurious infinite” (Hegel, 1969, p. 249). Hegel puts forward that the infinite in the function of variables is a kind of real infinity of quality, while the bad infinity is pure negation, lacking the meaning of positive quality. Hegel’s idea of real infinity helps to understand the dialectical relationship between result and process, infinite and finite, static and motion, quantitative change and qualitative change in limit theory. In essence, the limit theory of modern calculus embodies the interrelation and transformation between finite and infinite, the unity of infinite quantity and infinite quality, the unity of quantity and variable, and the dialectical unity of real infinity and bad infinity. With regard to this contradiction in mathematics, Hegel made a clear distinction between mathematical infinity (real infinity) and metaphysical infinity (bad infinity), he pointed out that: “But in a philosophical respect the mathematical infinite is important because underlying it, in fact, is the notion of the genuine infinite and it is far superior to the ordinary so-called *metaphysical infinite* on which are based the objections to the mathematical infinite. Often, the science of mathematics can only defend itself against these objections by denying the competence of metaphysics, asserting that it has nothing to do with that science and does not have to trouble itself about metaphysical concepts so long as it operates consistently within its own sphere. Mathematics has to consider not what is true in itself but what is true in its own domain. Metaphysics, though disagreeing with the use of the mathematical infinite, cannot deny or invalidate the brilliant results obtained from it, and mathematics cannot reach clearness about the metaphysics of its own concept or, therefore, about the derivation of the modes of procedure necessitated by the use of the infinite.” (Hegel, 1969, p. 241) Engels, in *Dialectics of Nature*, also expounds the profound role of mathematical infinity (that is, real infinity), “The differential calculus

for the first time makes it possible for natural science to represent mathematically *processes* and not only *states*: motion.” (Engels, 1964, p. 277)

On the size problem between infinity Hegel gave a clear argument. Hegel holds that infinity is comparable and can be compared only in the sense of *real infinity*, that is, in the sense of the qualitative determinateness of infinity. What is in *ratio* is no longer quantitative, but qualitative, Hegel pointed out that: “on the other hand the qualitative is what it is only in its distinction from an other. The said infinite magnitudes, therefore, are not merely comparable, but they exist only as moments of comparison, i.e. of the ratio.” (Hegel, 1969, p. 255) The two infinities in the *ratio* are two infinite advances, and their rate of change is a *real infinity*, reflecting the size of a qualitative determinateness.

Hegel thought that the infinite of mathematics is important in philosophy, because it is based on the foundation of *real infinity*, much better than metaphysical infinity. The infinity of mathematics and the infinity of metaphysics are important problems in the theory of infinity. The true mathematical infinity is *real infinity*, and the infinite of metaphysics is *bad infinity*. The infinite of metaphysics insists on the external negativity (*bad infinity*), and the boundary is never sublated; but *real infinity* insists on the intrinsic negation, and this kind of negation is the return to itself, the unity of the quantity and the quality of itself. The limit and the disappearance of the quantitative relationship do not mean that the disappearance of the qualitative provisions. Hegel emphasized the ratio of quality, because it is the basis for the conversion of a finite quantity to an infinite quantity. Therefore, it is very important for the healthy development of mathematical science to take seriously *the view of actual infinity* and *the view of potential infinity* in mathematics.

7.3 The View of Actual Infinity Confuses Thinking and Being, Imposes Human Subjective Thinking on the Objective Infinite Things Itself, and Wrongly Understands the Objective Being of Infinite Things As the Completion of Infinite Process (the Process of Human Beings Understanding Infinite Things), So as to Believe That the Development and Movement of the World Will Certainly Have An End.

It completely replaces the objective material world itself with the subjective world product, or completely equates and imposes the subjective world product on the objective material world, which fundamentally negates the difference and contradiction between the subjective world and the objective world. The *actual infinity* abandoned *the contradiction between finity and infinity*, also abandoned *the real infinity*; it applies the product of human thought to the infinite world of objective material, and is opposed to the objective material world, thus the concept of *actual infinity* made the same mistake with Hegel, it is an out and out of idealism view of infinity.

“*Infinity can be done*”, this is a kind of understanding of the subjective world, but the view of *actual infinity* holds that it is the objective material world itself. Engels early has given a relentless criticism to this idealistic understanding. He said, “To attempt to prove the reality of any product of thought by the identity of thinking and being was indeed one of the most absurd delirious fantasies of—a Hegel” (Engels, 1947, p. 61). The above discussion shows that, we can not prove the reality of our thinking products—*the infinite process can be completed*, because of the identity of the finite subjective world

and the objective material world. This actually pointed out the fatal defect of the view of *actual infinity*. That is to say, *the infinite transcendence of thinking* does not mean *the completion of the objective infinite process*, and *the infinite transcendence of thinking* has never really been done. The *existent infinity* is not equal to the *known infinity*.

Infinity as infinite object itself, that is, as an *infinite process*, as a *bad infinity*, is impossible to complete, impossible to end. However, this does not mean that the infinite object (including both the completed finity and the unfinished infinity) can not be regarded as a whole, a collection, and in fact, the human body always consciously, unconsciously put the infinite object as a whole, a collection. And this is the side of the transcendence of infinity, but also the natural, inevitable result and product of thought when the infinity is in the human brain, is the representation and reflection of the objective infinite world in the subjective finite world, and real infinity is the result of this transcendence. It is just that infinity can not be fully expressed in the finite human consciousness, and it is because of the inability to fully express, this is the fundamental idea of *bad infinity* or *potential infinity*.

The view of *actual infinity* always confuses the two concepts—infinite process (that is, *the bad infinity*) and global infinity (which is a kind of *real infinity*), confuses the finite and the infinite, confuses *bad infinity* and *real infinity*, and replaces the *infinite process* (which is the objective process of understanding the infinite material world) with the concept of global infinity (that is, the infinite transcendence of thinking, which is the concept of subjective understanding of the objective infinite world), that is, to replace *bad infinity* with *real infinity*, regards the transcendental cognition of the subjective finite world to the infinite object as the infinite process itself, regards the objective being of global infinity as the completion of the *infinite process*, regards the product of thinking as the process of human beings understanding the objective infinite world itself, replaces objective things themselves with subjective cognition, so that *the contradiction between finity and infinity* is completely denied, completely abandoned, completely eliminated. In the view of *actual infinity*, *the contradiction between finity and infinity* is eliminated, so that it is possible to complete and end an *infinite process*; but *the contradiction between finity and infinity* has never been eliminated, and it exists forever. As Engels in *Anti-Dühring* sharply pointed out, “the removal of the contradiction would be the end of infinity. Hegel saw this quite correctly, and for that reason treated with well-merited contempt the gentlemen who subtilised over this contradiction.” (Engels, 1947, p. 48) As a result, the concept of *actual infinity* is always confronted with a fatal blow, and constantly encounters new contradictions and new paradoxes, which is exactly the error of the infinite view of Cantor.

Just based on this wrong idealism recognition, in the view of *actual infinity*, when the infinite is a collection of objects, that is, as a whole, it can be done, because at this time the essence of *infinite* (that is, *the infinite process*, *the bad infinity*) has been completely abandoned, so it can be completed (that is, the infinite transcendence of thinking). The concept of *actual infinity* regards the completion as the “*infinite process can be completed*”, and the infinite objective being is equivalent to the *infinite process*, regards the completion of the *real infinity* as the completion of the *bad infinity*, with *real*

infinity instead of *bad infinity*, thus completely abandon *the contradiction between finity and infinity*, this is the essence of the error of the *actual infinite* view. *Being* is a kind of objective, *completion* is a kind of subjective, and the *actual infinite* view confused the two, which is bound to be the opposite of materialism, will inevitably lead to insurmountable contradictions. Let us further illustrate: “a line” as a *whole infinite* is a kind of objective being, so it is a must, is *Being-for-self*, is the completed quality, is a finite (that is, a real infinity), but the infinite extension of the line is a *bad infinity*, which is abstract, can not be completed, so is *Being-in-itself* and *intellectual being*; the *actual infinity* equated the former to the latter, and used the former to replace the latter, used the *Being* to replace the *completion*, thereby allowing itself to suffer a fatal blow.

This infinite concept, abandoned the *infinite process*, imposed the product of thinking on the objective material world, must be an infinite view anti dialectical materialism, is a real idealist view. However, Hegel’s view of infinity is not only to see “*the universal contact of the objective world (real infinity)*”, see the objective reality of “*the infinite process can not be completed (bad infinity)*”, so it is a scientific concept of infinite, is a dialectical view of infinity, and therefore is an infinite view of dialectical materialism. Therefore, the infinite as an *infinite process* is eternal, is absolute, and can not be completed; and when the infinite as a whole, as a *real infinite*, that is, to abandon the *infinite process*, this infinity can be completed, because at this time, this infinite is not so much an infinite, as it is a *real finity*. Such as the concept of natural numbers set N , is an infinite object, but it abandoned the *infinite process* between elements (*bad infinity*). The infinite object, which has left the *infinite process*, has become a mere concept or name, and has become a finite thing, just like a finite object in a meta-language. Thus, Russell is so sure to say, “We have to admit that there are real infinite sets,” and in fact, our mathematicians do, and do well. We consider the existence of the natural number set as the actual needs of people, which is the concrete embodiment of the dialectical infinite view.

In *Anti Dühring*, Engels fully criticized the absurd idea that “*the product of thought is imposed on the external objective world*” as Mr. Dühring, which effectively reveals the essence of mathematics, and holds that mathematics must reflect the objective material world correctly. He said, “But it is not at all true that in pure mathematics the mind deals only with its own creations and imaginations. The concepts of number and figure have not been derived from any source other than the world of reality. Like the idea of number, so the idea of figure is borrowed exclusively from the external world, and does not arise in the mind out of pure thought. Like all other sciences, mathematics arose out of the needs of men..... But, as in every department of thought, at a certain stage of development the laws, which were abstracted from the real world, become divorced from the real world, and are set up against it as something independent, as laws coming from outside, to which the world has to conform.” (Engels, 1947, pp. 55-56)

Engels also gave a strong criticism on this idealism of Hegel’s *absolute spirit*. He said, “The Hegelian system, in itself, was a colossal miscarriage—but it was also the last of its kind. It was suffering, in fact, from an internal and incurable contradiction. Upon the one hand, its essential proposition was the

conception that human history is a process of evolution, which, by its very nature, cannot find its intellectual final term in the discovery of any so-called absolute truth. But, on the other hand, it laid claim to being the very essence of this absolute truth. A system of natural and historical knowledge, embracing everything, and final for all time, is a contradiction to the fundamental laws of dialectic reasoning. This law, indeed, by no means excludes, but, on the contrary, includes the idea that the systematic knowledge of the external universe can make giant strides from age to age.” (Engels, 1947, p. 37) Here, Engels deeply criticized Hegel’s idea of *absolute truth*, which is of *actual infinity*.

What we are here to oppose is the *final truth* of Mr. Dühring’s Transcendentalism, which is criticized by Mr. Engels in *Anti-Dühring*; the view of *actual infinity* is exactly the same thing as Transcendentalism, which in essence believes that the development of the world, the movement will have an end. While the essence of infinity essentially means the *inexhaustible attribute of infinity*, “inexhaustible” is “infinity”, “infinity” is “inexhaustible”. The view of *actual infinity* holds that the infinity can be completed, which is directly opposed to the objective material world.

In *Anti Dühring*, Engels further criticized this idea, “If at any time in the development of mankind such a final, conclusive system of the interconnections within the world—physical as well as mental and historical—were brought about, this would mean that human knowledge had reached its limit, and, from the moment when society had been brought into accord with that system, further historical development would be cut short—which would be an absurd idea, sheer nonsense.” (Engels, 1947, p. 53) “Mankind therefore finds itself faced with a contradiction: on the one hand, it has to gain an exhaustive knowledge of the world system in all its interrelations; and on the other hand, because of the nature both of men and of the world system, this task can never be completely fulfilled.” (Engels, 1947, p. 53) “Each mental image of the world system is and remains in actual fact limited, objectively by the historical conditions and subjectively by the physical and mental constitution of its originator. But Herr Dühring explains in advance that his mode of reasoning is such that it excludes any tendency to a subjectively limited conception of the world. We saw above that he was omnipresent—on all possible celestial bodies. We now see that he is also omniscient. He has solved the ultimate problems of science and thus nailed boards across the future of all science.” (Engels, 1947, p. 54)

The *existent* infinity is not equal to the *known infinity*, and the objective infinity is not equal to the subjective infinity, and the *actual infinity* is neither *bad infinity* nor *real infinity*. We can make a very vivid metaphor for the *real infinity* and the *bad infinity*: *the bad infinite* as a devil trapped in a closed box, if we don’t open this box, the devil will never come out (i.e. infinite progress can never be completed). In this analogy, “*the devil can never come out*” is a kind of *bad infinity*, and the *real infinity* refers to the intrinsic quality of the event—“*There’s a devil in the closed box*”. Combining the two, the statements of the whole event is that: “*there is a devil in the closed box, but the devil can never come out.*” The former indicates the nature of the event, the latter indicates that the event movement, the possibility of development, both interdependent but not the same. More vivid metaphor, we can take *the straight line* which Hegel used to describe the *bad infinity* to specify: *a point on a*

straight line moves freely in the straight line. In this event, the “*point in a straight line*” represents the essence of the event, the inherent qualitative stipulation, reflecting the relationship between the point and the line, is a *real infinity*. And *the free movement of points in the straight line* represents the *bad infinity*, indicating that the *point* can reach any position on the straight line, but in any case the freedom movement can not get rid of this line, are unlikely to change the intrinsic nature of the point in the line. If the expression of these events is still a bit fuzzy, then let us change the “*straight line*” into a closed *circle* to describe the event, the event will be transformed into such an event: A point on a closed circle moves freely on the circle. In the same way, “*the point in the closed circle*” represents the internal stipulation of the event, which reflects the relationship between the *point* and *circle*, represents the topological property of the event, and thus is a *real infinity*. And the “*free movement of a point on the circle*” represents the *bad infinity*, indicating that the *point* can move freely on the circle, but in any case it is impossible to get rid of the circle. That is to say, the result of *bad infinity* is *real infinity*, not *actual infinity*; in this case, we cannot find any shadow of *actual infinity*. *Actual infinity* thinks that it has already got rid of the *bad infinity*, but still in the *infinite progress*. It is not from the *quality* to consider the infinite, to sublimate the *infinite progress*, but subjective, wishful thinking to interrupt the *infinite progress*, with subjective instead of objective, so it is a self-deceiving, idealist epistemology, is a form of antinomy.

We can see everywhere Mr Dühring’s view of “*being is to be known*” which Engels criticized and opposed. If you say *existent infinity* once be thought, it is thought of as the *unified thing—accomplished infinity (actual infinity)*, it is too funny. In *Anti Dühring*, Engels continues to discuss, “This is indeed system-creating! Within the space of the next six lines Herr Dühring has transformed the *oneness* of being, by means of our unified thought, into its *unit*. As the essence of all thought consists in bringing things together into a unity, so being, as soon as it is conceived, is conceived as unified, and the idea of the world as indivisible; and because *conceived* being, the *idea of the world*, is unified, therefore real being, the real world, is also an indivisible unity.” (Engels, 1947, p. 60) “To attempt to prove the reality of any product of thought by the identity of thinking and being was indeed one of the most absurd delirious fantasies of—a Hegel” (Engels, 1947, p. 61). This has been explained that: the *existent infinity* once we are thinking, we can’t get the reality of this thinking product—*accomplished infinity (actual infinity)*. Here, Engels ruthlessly refuted this idealism thought of *actual infinity* which escapes from *the contradiction between finity and infinity*.

The view of *actual infinity* regards itself as the *real objective infinite world*, so it must be the opposite of dialectical materialism. Therefore, *the view of actual infinity* is essentially a view of finity, which abandons the difference between the objective world and the subjective world, and denies the unity of opposites between the subjective world and the objective world—*the contradiction between finity and infinity*, thus negated the continuous progress of the subjective world.

8. The Definition of the View of Dialectical Infinity and Its Philosophical Significance

Adhere to the dialectical understanding of infinity. Redefine, recognize the *actual infinity* and the *potential infinity*, fully absorb Hegel's thought of dialectical infinity, return to the scientific concept of infinity, that is, infinity is an objective *Being*, infinity can be known, but the infinite process cannot be completed. And this is the infinite view of dialectical materialism.

According to the above, although *the potential infinite view* denied the objective being of infinity because of the "*infinite process can not be completed*", this is the wrong side. However, due to the fact that it does not escape from *the contradiction between finity and infinity*, it objectively acknowledges the difficulty of human epistemology, the insurmountable contradiction; it acknowledges that we cannot end an infinite process and reach a certain destination, which does not consider the *objective existent infinity* as *accomplished infinity*. Therefore, the concept of *potential infinity* in mathematics has its right side, and this side has completely adhered to the materialist dialectics.

But the view of *actual infinity*, admits on the surface that the infinitude of the objective material world, but believes that human beings can fully grasp, understand the infinite material world, complete an *infinite progression*, thus avoiding the objective, epistemological *contradiction between finity and infinity*. Therefore, in essence, the concept of *actual infinity* negates the infinitude of the objective material world (*the bad infinity*), but only acknowledges the finiteness of the objective material world. This determines that the concept of *actual infinity* always deals with, treats a variety of infinite objects with the finite method, which brings one after another bigger and worse contradictions. We believe that the essence of the Third Mathematical Crisis is the crisis caused by the view of *actual infinity*—exhausted an infinite (*exhausted an inexhaustible thing*). The Max Ordinal Paradox and the Max Cardinality Paradox all embody the fundamental error of *actual infinity*.

We have carried out a comprehensive analysis of the concept of *potential infinity* and *actual infinity*, and we have comprehensively discussed the philosophical errors of *potential infinity* and *actual infinity*. The conclusion is that we must admit the objective being of the *contradiction between finity and infinity* (i.e., the infinite process can not be completed), and admit the objective reality of the infinite object; we must admit the *bad infinity*, but also admit the *real infinity*; we must admit that the *bad infinity* can not be completed, but also admit the *real infinity* can be known, can be completed. Therefore, from a mathematical point of view, it is to admit that all natural numbers into a set N , recognized the objective existence of N as an infinite object.

Recognizing that N is an infinite set is not a denial of the *bad infinity*, not denying that the *infinite process* cannot be completed. From the perspective of philosophy, the being of the infinite object is not transferred by human will, also recognized the rationality of N as an infinite object, this is the one side of infinity that can be transcended (real infinity is a result of surpassing infinity, sublation of infinity); but the human cognitive ability is limited, which determines the objective reality of the *infinite process*, also is to admit *infinite process* can not be completed. This is the dialectical interpretation of infinity in Marxist philosophy. Marx's profound analysis of the concept of limit undoubtedly shows that he is a

staunch opponent of actual infinity. As the true saying of Marx: “The question whether objective truth can be attributed to human thinking is not a question of theory but is a practical question. Man must prove the truth, i.e., the reality and power, the this-sidedness [Diesseitigkeit] of his thinking, in practice. The dispute over the reality or non-reality of thinking which is isolated from practice is a purely scholastic question.” The practice of mathematical science makes us recognize the essence of actual infinity.

To sum up, we comprehensively introduce Hegel’s dialectical infinity, and also introduce Engels’s critical inheritance and development of Hegel’s infinite view, it also introduces the differences between the four kinds of infinite views, and analyzes the errors of *actual infinity*, so now we can naturally sum up *the view of dialectical infinity*: infinite objective exists, infinity can be known, but *infinite process* can not be completed. The essence of the view of dialectical infinity is movement and change. Concretely speaking, any infinity is the dialectical unity of *bad infinity* and *real infinity*. It is an objective existence, and infinity itself contains *the contradiction between finity and infinity*, so the objective existence of infinity does not mean that the infinite process can end, complete. *Real infinity* is the inherent qualitative stipulation of infinite things, that is, the inner connection, law and truth, while *bad infinity* is infinite progress, no terminating repetition and alternating, it deeply embodies *the contradiction between finity and infinity*. *Real infinity* can be recognized and completed, while *bad infinity* cannot be recognized and can not be completed; *bad infinity* (infinite process) is the concrete manifestation of *the contradiction between finity and infinity* rather than the solution of such contradiction, and this determines that *the contradiction between finity and infinity* will never die out. *Real infinity* represents infinite quality (essence), while *bad infinity* represents infinite quantity (movement and change). *Real infinity* is inseparable from *bad infinity*, *bad infinity* is the carrier of *real infinity*, and *real infinity* is the goal and direction of *bad infinity*. The *real infinity* is present, concrete, positive, rational, completed infinity, is *Being-for-self* and rational *Being*, is the completed quality; and the *bad infinity* is possible, abstract, negative, uncompleted infinite, is *Being-in-itself* and *intellectual Being*.

Infinity exists and cannot be traversed, completed; infinity is a black hole, but it can be sublimated and transcended. Process (*bad infinity*) forever, law (*real infinity*) eternal; process is a variable, law is a constant; Infinity cannot be crossed, but can be transcended, the result of transcendence is a real infinity. The existence of infinity and the incompleteness of process are two completely different concepts, they are two aspects of contradiction and cannot be replaced by each other; it is precisely because of their existence that there is the existence of *the contradiction between finity and infinity*. Infinity, like a black hole, can come in without going out, endless, never over. The infinite view of dialectical materialism insists on the inextinguishment of this kind of contradiction, and holds that infinity exists objectively and can be recognized, but the infinite process cannot be completed, that is, the infinite contradiction is eternal; On the other hand, the view of *actual infinity* regards the objective existence of infinity as the completion of the infinite process, replacing the subjective with the objective, replacing the *bad infinity*

with the *real infinity*, abandoning *the contradiction between finity and infinity* completely, and thinking that the contradiction can be ended and solved. Therefore, this thought followed Kant's transcendental, subjective, metaphysical infinite thought, not Hegel's dialectical infinite thought.

Therefore, to correctly understand and grasp *the contradiction between finity and infinity* and correctly understand *real infinity* and *bad infinity* is the essence of solving the third Mathematical Crisis in the field of mathematics and philosophy, and is of great guiding significance to completely solve Russell's Paradox and the problem of Continuum Hypothesis.

In summary, we believe that there are insurmountable internal contradictions in the view of *actual infinity* that *infinite process* can be completed. At the level of philosophy and mathematics, Hegel puts forward the concept of *real infinity* in accordance with the dialectic thought, which embodies the essence and relation of infinite object. This is something that the thought of *actual infinity* (this lower-level, subjective idealistic view of infinity) can not be compared. We believe that adhering to *the view of dialectical infinity* will bring a brighter future to the study of basic mathematics.

References

- Aristotle. (2016a). *Physics*. The Commercial Press (China).
- Aristotle. (2016b). *Metaphysics*. The Commercial Press (China).
- Engels. (1947). *Anti-Dihring* (translated by Emile Burns). LeoPard Books India.
- Engels. (1964). *Dialectics of Nature* (translated by Clemens Dutt). Wellred 2012.
- He, J. N. (1983). *Hegel's Idea of Infinity in Mathematics and Its Significance*. Foreign Philosophy (part 4) (China).
- Hegel. (1969). *Hegel's Science of Logic* (translated by A.V. Miller). Humanity Books.
- Hegel. (2015). *The Logic of Hegel* (translated by William Wallace, M.A.). Forgotten Books.
- Hume. (2015). *A Treatise of Human Nature*. The Commercial Press (China).
- Ling Esheng. (1992). The Three Mathematical Crises Caused by Bad Infinity. *Journal of East China Jiaotong University (China)*, 9(1).
- Marx. (1975). *Mathematical Manuscripts*. People's Publishing House (China).
- Poincaré (2008). *The Foundations of Science*. The Commercial Press (China).
- Poincaré (2015). *The Final Meditation*. The Commercial Press (China).
- Robinson, A. (1980). *Non-standard Analysis*. Science Press (China).
- Wen, C. R. (2003). Hegel's Thought about Quantity and Mathematical Infinity. *Journal of Anhui University (China)*, 3.
- Wittgenstein. (2013). *Research in Basic Mathematics*. The Commercial Press (China).
- Xu, L. Z., & Hsu, L.C. (2000). *Topics in Mathematical Methodology* (3rd ed.). Huazhong University of Science and Technology Press (China)
- Zhu, W. J. (1987). *Foundations of Geometry and Mathematics*. Liaoning Education Press (China).

Zhu, W. J. (2008). *The Logical Foundation of Mathematics and Infinity*. Dalian University of Technology Press (China)

Zhu, W. J. (2010). Analysis and Research of Cantor-Hilbert Diagonal Proof Argument. *Journal of Nanjing Xiaozhuang University (China)*, 3.