

Original Paper

Developing Student-Teachers' Understanding of Geometrical Figures/Objects Using a Bicycle Rubber Tube

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Abstract

This paper addresses the ways that a bicycle rubber tube can be used to develop learners' understanding of geometrical figures/objects. The use of a bicycle rubber tube is important because student-teachers and in-service teachers can use the material to teach learners in schools to understand various geometrical figures/objects. In doing so, geometrical figures and objects, metric, and metric space are understood relationally. Using a bicycle rubber tube, it was found that various geometrical figures and objects were formed, including rectangle, triangle, square, and pentagon. The finding has implications in teaching geometrical figures/objects, including teachers can use a bicycle rubber tube to develop learners' understanding of a circle, rectangle, triangle, square, pentagon and hexagon.

Keywords

geometrical figures/shapes, metric, topology, bicycle rubber tube

1. Introduction

This paper presents the learned activities on how an elastic rubber tube was used for forming various geometrical figures/objects. In forming various geometrical figures, the concepts of metric and topological spaces were found useful. This paper contributes to the use of real objects in forming various geometrical shapes and figures, in this case a bicycle rubber tube. This is important for active engagement of student-teachers with mathematical concepts and support intended learning.

This paper raises the questions in what ways is it different to teach and learn with a moldable shape that returns to the original shape, compared to, say, a series of separate, rigid objects? As a student watches a pentagon returns to a circular form, what learning affordances or enjoyments are there?

This paper is influenced by the work of Karssenbergh (2014) on “learning geometry by designing persian mosaics” (p. 43) on developing and implementing lesson plan in university mathematics classrooms. Karssenbergh designed mathematics lessons that students learned by acting while focusing on their origin cultures in mathematics classrooms, which played a significant role in stimulating their interest in learning mathematics as well as learning achievements. However, the lessons based on a European context and focused on students in secondary schools. This paper presents the lesson focused on African context, in particular post-colonial context such as in Tanzania while using a bicycle rubber tube. The university student-teachers learned by doing in designing and forming geometrical shapes/figures using real objects available in local environment. In doing so, Zaslavsky (1999) argues that “students become aware of the role of mathematics in all societies. They realize that mathematical practices arose out of people’s real needs and interests” (p. 318).

The design and formation of geometrical figures/shapes are originated in the history of mathematics (Gerdes, 1998, 2010; Kartz, 1998). As such, the formation and design of geometrical figures/shapes depend on three stages. First stage involves creativity by realizing the specific patterns that are designed (Hogendijk, 2012). Second stage involves drawing the geometrical shapes/figures by presenting the instructions showing how drawing process took place (Necipoglu, 1995). The last stage involves scaling the constructed patterns of a drawing (Necipoglu, 1995).

2. The Didactical Approach: Learning by Doing

The design and formation of geometrical shapes and figures are based on the conceptual framework of activity (Cole & Engestrom, 1993). Cole and Engestrom suggest using a mediational triangle by expanding it to model any activity. In our lesson with student-teachers, a rubber tube was formulated in the shape of a triangle and then was expanded to form other geometrical shapes including a circle, triangle and pentagon.

3. Learning Mathematics by Doing in Forming and Designing Geometrical Figures/Shapes Using a Bicycle Rubber Tube

In this example, learning by doing is used in a way that student-teachers focused on giving an interesting project followed by presentations and discussions. The lesson was stimulating by making a colorful geometrical shapes and figures using bicycle rubber tube. Student-teachers worked together in a small group which reduced time needed to work on the activity per teacher.

A mediational triangle for student-teachers who participated in the lesson for forming geometrical shapes/figures using a bicycle rubber tube involved the following aspects:

Table 1. A Mediation Triangle for Forming Geometrical Figures/Shapes Using Bicycle Rubber Tube

Aspects	Active Process
Subject	The student-teachers' participation.
Tools	An elastic rubber tube (a bicycle rubber tube) of circumference 194 cm for forming various geometrical shapes and figures, rubber bands for fixing points, and tape measure for measuring various sides of the geometrical shapes resulting from the bicycle rubber tube (circular elastic rubber).
Objects	To gain knowledge and skills in forming and designing geometrical figures using tools available in their daily environment. To collectively form and design geometrical figures by giving mathematical analysis.
Extra rules	The teacher educator divides a class into small groups. The educator gives instructions on how the project and presentations are assessed.
Communities	The group of geometrical formulators and designers presents (3 student-teachers). Other student teachers become audiences for the presentations.
Division of labour	The teacher educator gives a project task to formulate and design geometrical figures/shapes using a bicycle rubber tube followed by small group presentations. The educator has power to assess the group presentations while student-teachers own the class (power in practice).
Outcomes	Presentations and mathematical analysis of the formed and designed geometrical shapes/ figures using a bicycle rubber tube. These demonstrate the knowledge and skills gained by students on geometry, including geometrical objects/figures, and construction.

4. Mathematical Tools

Before the student-teachers could form and design geometrical figures/shapes using a bicycle rubber tube, they would need to become aware of metric and metric space, topology and topological space, and geometrical shapes/figure. This is important because formation of geometrical shapes and figures involves measurements and understanding the shapes of the materials so that we can form various shapes from a bicycle rubber tube by stretching the material without destroying it.

4.1 Metric and Metric Space

Metric space refers to the space occupied by the distance function between two points in a given set (Copson, 1968; Jain & Ahmad, 1999; Korner, 2015). The space occupied by the length of a ruler is an example. Also, Singh (2019) defines metric space as let \overline{Y} be a non-empty set. A metric on \overline{Y} is a

function $d: Y \times Y \rightarrow \mathbb{R}$ such that the following conditions are satisfied for all $a, b, c \in Y$:

- $d(a, b) \geq 0$
- $d(a, b) = 0$ if and only if $a = b \forall a, b \in Y$ (positivity)
- $d(a, b) = d(b, a) \quad \forall a, b \in Y$ (symmetry)
- $d(a, c) \leq d(a, b) + d(b, c), \forall a, b, c \in Y$ (triangle inequality)

The set Y together with a metric d is known as a metric space, the elements of Y are called points. The value $d(a, b)$ on a pair of points $a, b \in Y$ refers to the distance between a and b .

Metric space is used in measuring the distance between two points. For example, the distance between two points on a ruler, say P and Q is given by $d(p, q) = |P - Q|$ which is always positive. The distance between the two points on the ruler is symmetry. For instance, the distance from points P to Q on the ruler is equal to the distance from Q to P . That is, $d(p, q) = d(q, p)$ (Korner, 2015; Singh, 2019; Sutherland, 2009).

Metric space can be used in choosing the shortest distance between two points. For instance, when transforming a bicycle rubber tube into a triangle, let say PQR , it is found that the distance of one side is always less or equal to the sum of two other sides. This demonstrates the triangular equality in the triangle PQR such that

$d(p, q) \leq d(p, r) + d(r, q)$. Suppose that $d(p, q) = 5 \text{ cm}$, $d(p, r) = 3 \text{ cm}$ and $d(r, q) = 4 \text{ cm}$, then $5 \text{ cm} \leq 3 \text{ cm} + 4 \text{ cm}$. In general, for

all $a, b, c \in Y, d(a, c) \leq d(a, b) + d(b, c)$. Geometrically, this representation can be addressed as: In any right-angled triangle, the length of a side is less than or equal to the sum of the length of the other two sides (Singh, 2019; Sutherland, 2009).

4.2 Topology and Topological Space

A topology is a geometrical perspective of an object. It can be known as the deformation of a material into different homeomorphic shapes without destroying the physical structure (or nature) of the material. Such material can be bent, crumpled, stretched or even pulled (Kelly, 1995; Korner, 2015; Munkres, 2000; Singh, 2019). In this context, we are going to employ an elastic material to demonstrate topology.

The space occupied by a geometrical perspective of an object is called a topological space.

(Singh, 2019) defined a topological structure or simply a topology on set Y as a collection of W ; a subset of Y such that:

- The intersection of two members of W is in W
- The union of any collection of members of W is in W
- The empty set \emptyset and entire set Y are in W

Topological spaces can be used to form different structures or shapes which are homeomorphic to each other and can be used in different purposes (Bryant, 1994; Korner, 2015). For instance, a bicycle rubber tube can be used to form various geometrical shapes without deforming it. The shapes that can be formed

from the rubber band include a circle, rectangle, pentagon, and hexagon. The geometrical shapes formed are topologically equivalent. Using the concept of topological space saves time, space as well as other resources such as materials. For example, a carpenter may decide to design a table which can be used as both a chair and bed whereby three items are homeomorphic to each other. Therefore, the concept of topological spaces can be used in daily life to save time, cost and proper utilization of materials and spaces.

Mathematically, we can define a homeomorphism between the two spaces Y and Z as a bijective function $f: Y \rightarrow Z$ such that both f and f^{-1} are continuous. In this case, two spaces Y and Z are said to be homeomorphic, denoted by $Y \approx Z$, if there is a homeomorphism $f: Y \rightarrow Z$ (Singh, 2019).

5. The Student-Teachers Start Formulating and Designing Geometrical Figures

Two techniques were used in forming different geometrical figures and shapes using a bicycle rubber tube.

- Measurements: A tape measure to measure the lengths of geometrical shapes such as a circle, triangle, rectangle and pentagon as a result of stretching a bicycle rubber tube.
- Interpolation; we used variations of measurements to transform a circular rubber into different shapes step by step which were homeomorphic to each other or simply topologically equivalent.

Various geometrical shapes were formed by using a bicycle rubber tube including a circle, triangle, rectangle and pentagon. The new shapes so formed were measured their sides using a tape measure (see Figure 1).



Figure 1. Measurements of the Geometrical Shapes Formed from A Circular Bicycle Tube

The following steps were followed while transforming a circular bicycle tube into different geometrical shapes.

Step 1: A bicycle rubber tube was stretched to form a triangle. Then, the rubber tube was transformed into triangle by fixing it at two distinct points and stretching another point without cutting, tearing or gluing it.

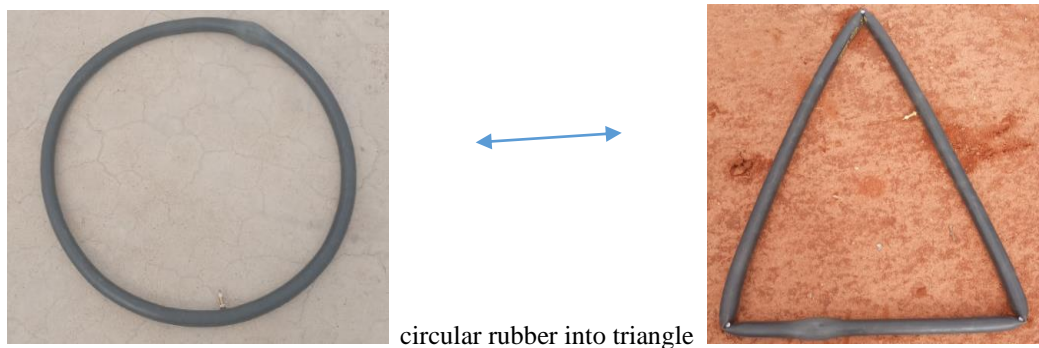


Figure 2. Transforming a Circular Bicycle Tube into A Triangular Shape

Student-teachers analyzed that a circular rubber tube of length 194 cm was transformed into a triangular shape of length 196 cm. The difference in lengths of the two geometrical shapes are a result of stretching a bicycle rubber tube in the process of forming a triangular shape. Also, circular and triangular rubber tubes so formed were homeomorphic to each other. These means the two geometrical shapes were topologically equivalent.

Step 2: A formed triangle was stretched into a rectangle: A triangular rubber tube was fixed at two distinct points and also stretched at two distinct points without cutting, tearing or gluing it.

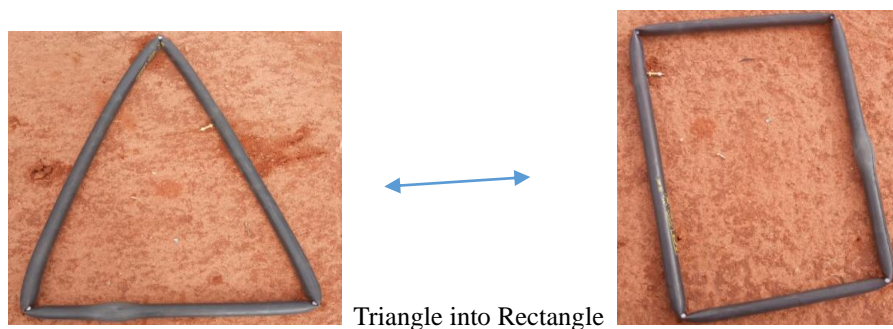


Figure 3. Transforming a Triangular Figure into a Rectangular Figure

Student-teacher analyzed that a triangular rubber tube of length 196 cm was transformed into a rectangle of length 210 cm. The difference in lengths of the two geometrical shapes are a result of stretching a bicycle rubber tube in the process of forming a rectangular shape. The shapes were topologically equivalent.

Step 3: A formed rectangle was also stretched into a pentagon. A rectangular rubber tube obtained from

step 2 was interpolated by fixing it at four distinct points and extended at one point which was different from the four points without cutting, tearing or gluing it.

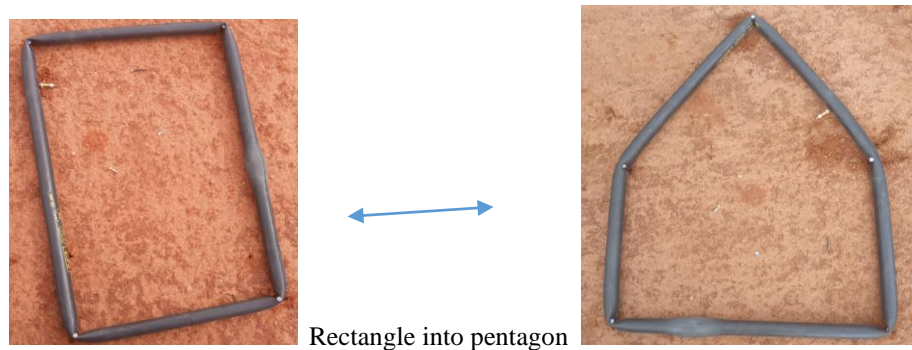


Figure 4. Transforming A Rectangular Figure into A Pentagon

Student-teachers analyzed that a rectangular rubber tube of length 210 cm obtained from step 2 was interpolated by fixing it at four distinct points and extended at one point which was different from the four points without cutting, tearing or gluing it to form a pentagon of length 212 cm. The difference in lengths of the two geometrical shapes are a result of stretching a bicycle rubber tube in the process of forming the shape of a pentagon. It was found that the shape of the rectangular rubber tube and that of the pentagon rubber tube were topologically equivalent.

Step 4: A formed pentagon was stretched into a circle. We needed to make a back change of drawing a pentagon into a circle. Since the geometrical topological shapes were formed by fixing and stretching an elastic bicycle rubber tube, then, on release the elastic material drew itself into a circle which we had as our original material.

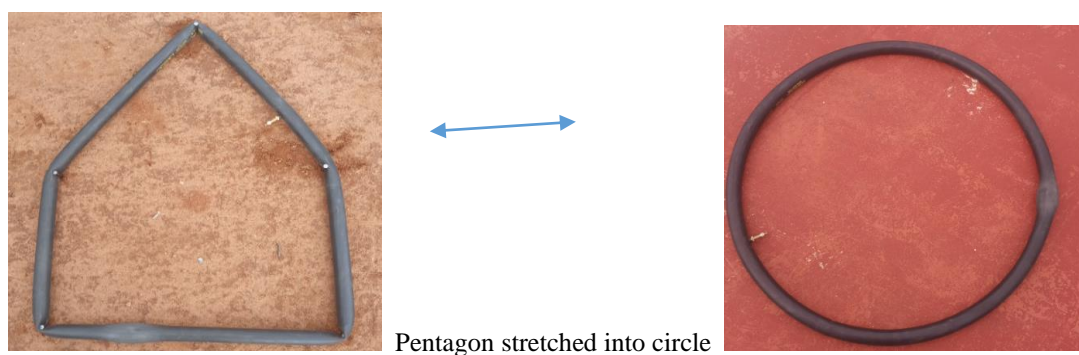


Figure 5. Transforming A Pentagon into A Circle

Student-teachers analyzed the process by focusing on the importance of transforming the shape of a pentagon formed from the rectangular rubber tube (see step 3) into a circle. This transformation was important to make a back change of drawing a pentagon of length 212cm into a circle of length 194cm.

Since the topological shapes were operated by simply being fixed and stretched, then on release the elastic material drew itself into a circle (original material). Also, the shape of a pentagon and that of a circle were topologically equivalent.

6. Reflection on the Results

Reflection is presented based on the practical activities/ operational issues involved from the results obtained in steps 1, 2, 3 and 4 during small group presentations. We noticed that all geometrical shapes from steps 1 to 4 are homeomorphic figures describing the shapes of a circle, triangle, rectangle and pentagon, are all made from circular rubber tube. This is to say, we can further manipulate the circular rubber tube to form other geometries (shapes) which are also topological equivalent. Other geometrical shapes include square, hexagon and octagon.

We also learned that the geometrical shapes formed from circular rubber tube after stretching and transforming it, were topological equivalent. However, there were a slight difference in their metrics due to stretching and compressing the rubber tube while forming various geometrical shapes.

These learning activities for student-teachers are quite different from what usually take place in their university mathematics classes. These teachers usually work on group assignment on the activities. However, the activities are not practical based. Also, these teachers work on the assigned activities while they are outside the class. Because of this, teacher educator cannot know how the learning process proceeds in small group discussions.

7. Concluding Thoughts

The use of a bicycle rubber tube in forming and designing geometrical figures/shapes in this way in this paper is critical: student-teachers were motivated in learning and created interest of using the material in teaching and learning situations to dilute the scarcity of teaching and learning aids/materials in mathematics classrooms. For instance, a mathematics teacher while teaching geometry can use a bicycle rubber tube in demonstrating the shapes of various polygons and how they are formed, instead of drawing various polygon figures on the chalkboard/white board. In doing so, it helps in managing time and spaces during teaching and learning process. Also, students learn by doing in small groups followed by class discussions on how a bicycle rubber tube can be used in forming various geometrical shapes. This identifies ways of reducing tensions among students while working on geometrical items.

We conclude with the issue of mathematics education by relating with cultural context. Our lesson was potential for giving student-teachers power in practice while in mathematics classrooms. A mathematics educator followed Karssenbergh (2014)' approach on non-western cultures in producing mathematics and profiting everyone. In our class with student-teachers, it gave positive impacts in teaching and learning geometrical figures/objects while using a moldable shape that returns to its original shape.

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