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Original Paper

How to Obtain Valid Generalized Modal Syllogisms from Valid Generalized Syllogisms

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Abstract

Making full use of the truth value definitions of sentences with quantification, possible world semantics and/or fuzzy logic, one can prove the validity of generalized modal syllogisms. This paper shows that the proof of the validity of a generalized modal syllogism can be transformed into that of its corresponding generalized syllogism, and that the generalized syllogism obtained by removing all modalities in any valid generalized modal syllogism is still valid. Therefore, the simplest way to screen out valid generalized modal syllogisms is to add modalities to valid generalized syllogisms, and then to delete all invalid syllogisms by means of the basic rules with which valid generalized modal syllogisms should meet. And then the remainders are valid. This paper illustrates how to obtain 12 valid generalized modal syllogisms by adding necessary modalities and/or possible modalities to any valid generalized syllogism. The two kinds of syllogisms discussed in this paper are composed of sentences with quantification which is the largest number of sentences in natural language. Hence, this innovative research can provide theoretical support for linguistics, logic, artificial intelligence, and among other fields.

Keywords

generalized modal syllogisms, generalized syllogisms, validity, truth value definition

1. Introduction

There are many kinds of syllogisms in natural language, such as Aristotelian syllogisms (Hui, 2023), generalized syllogisms (Xiaojun, 2020a), Aristotelian modal syllogisms (Johnson, 2004), generalized modal syllogisms, and so on. This paper shall restrict attention entirely to how to obtain valid generalized modal syllogisms from valid generalized syllogisms.

There are some studies on generalized syllogisms, such as Peterson (2000), Novák (2008a, 2008b), Moss (2010), Novák (2012), Endullis and Moss (2015), Xiaojun (2016), Cheng (2023), and so on. Nevertheless, up to now, there are no relevant literature of generalized modal syllogisms found at home or abroad. Therefore, this study is a pioneering research.

The largest number of sentences in natural language is sentences with quantification (Cheng, 2022). The two kinds of syllogisms discussed in this paper are composed of sentences with quantification (Long, 2023). And they play an important role in natural language information processing, which involves interdisciplinary research fields such as linguistics, logic, artificial intelligence, and among other fields. It is hoped that this study can provide theoretical support for these fields.

2. Preliminaries

In the paper, let *S*, *M* and *P* be the lexical variables in sentences with quantification, and *D* the domain of lexical variables. Syllogisms often contain sentences in the following forms: All *S*s are *P*, No *S*s are *P*, Some *S*s are *P*, Not all *S*s are *P*. The four sentences can be symbolized as *all*(*S*, *P*), *no*(*S*, *P*), *some*(*S*, *P*) and *not all*(*S*, *P*), and abbreviated as the proposition A, E, I and O, respectively. $\Box$ is the symbol of a necessary modality, $\Diamond$ is that of a possible modality. $|S|$ represents the cardinality of the set composed of the variable *S*. A generalized modal syllogism can be obtained by adding necessary modalities and/or possible modalities to a generalized syllogism.

Example 1:

Major premise: All middle-aged healthy hens are egg-laying hens.

Minor premise: Most chickens in this farm are middle-aged healthy hens.

Conclusion: Some chickens in this farm are egg-laying hens.

Let *S* be the chickens in the domain, *M* the middle-aged healthy hens in the domain, and *P* the egg-laying hens in the domain. Then the generalized syllogism in Example 1 can be formalized by $all(M, P) \wedge most(S, M) \Rightarrow some(S, P)$. The following generalized modal syllogism in Example 2 can be obtained by adding modalities to Example 1.

Example 2:

Major premise: All middle-aged healthy hens are necessarily egg-laying hens.

Minor premise: Most chickens in this farm are middle-aged healthy hens.

Conclusion: Some chickens in this farm are possibly egg-laying hens.

Similarly to Example 1, Example 2 can be formalized by $\Box all(M, P) \wedge most(S, M) \Rightarrow \Diamond some(S, P)$. In fact, a sentence with quantification discussed in this paper can be formalized into a tripartite structure like $Q(S, P)$. Let Q_1, Q_2, Q_3 be generalized quantifiers, and take the first figure syllogism as an example. It can be formalized as $Q_1(M, P) \wedge Q_2(S, M) \Rightarrow Q_3(S, P)$. There are four situations as follows: (1) If Q_1, Q_2 and Q_3 only range over the four Aristotelian quantifiers (that is, *all*, *some*, *no* and *not all*), one can obtain Aristotelian syllogisms (Long, 2023). (2) If Q_1, Q_2 and Q_3 range over all the generalized quantifiers including Aristotelian quantifiers, one can obtain generalized syllogisms (Xiaojun, 2021). (3)

If Q_1, Q_2 and Q_3 range over the following 12 Aristotelian modal quantifiers: *all*, *some*, *no*, *not all*, ◇*all*, ◇*some*, ◇*no*, ◇*not all*, □*all*, □*some*, □*no* and □*not all*, one can obtain Aristotelian modal syllogisms (Cheng, 2023). (4) If Q_1, Q_2 and Q_3 range over all generalized modal quantifiers including the above 12 Aristotelian modal quantifiers, one can obtain generalized modal syllogisms.

Definition 1 (truth value definitions of non-modal sentences with quantification):

(1.1) $all(S, P)\Leftrightarrow S\subseteq P$; (1.2) $no(S, P)\Leftrightarrow S\cap P=\varnothing$;

(1.3) $some(S, P)\Leftrightarrow S\cap P\neq\varnothing$; (1.4) $not\ all(S, P)\Leftrightarrow S\nsubseteq P$;

(1.5) $most(S, P)\Leftrightarrow |S\cap P|\geq 0.6|S|$.

The truth value definitions of modal sentences with quantification can be given as follows in line with Definition 1 and possible world semantics (Chellas, 1980).

Definition 2 (truth value definitions of modal sentences with quantification):

(2.1) $\Box all(S, P)$ is true if and only if $S\subseteq P$ is true in any possible world α;

(2.2) $\Diamond all(S, P)$ is true if and only if $S\subseteq P$ is true in at least one possible world α;

(2.3) $\Box no(S, P)$ is true if and only if $S\cap P=\varnothing$ is true in any possible world α;

(2.4) $\Diamond no(S, P)$ is true if and only if $S\cap P=\varnothing$ is true in at least one possible world α;

(2.5) $\Box some(S, P)$ is true if and only if $S\cap P\neq\varnothing$ is true in any possible world α;

(2.6) $\Diamond some(S, P)$ is true if and only if $S\cap P\neq\varnothing$ is true in at least one possible world α;

(2.7) $\Box not\ all(S, P)$ is true if and only if $S\nsubseteq P$ is true in any possible world α;

(2.8) $\Diamond not\ all(S, P)$ is true if and only if $S\nsubseteq P$ is true in at least one possible world α;

(2.9) $\Box most(S, P)$ is true if and only if $|S\cap P|\geq 0.6|S|$ is true in any possible world α;

(2.10) $\Diamond most(S, P)$ is true if and only if $|S\cap P|\geq 0.6|S|$ is true in at least one possible world α.

Fact 1 (a necessary proposition implies an assertoric proposition):

(1.1) $\Box all(S, P)\Rightarrow all(S, P)$; (1.2) $\Box no(S, P)\Rightarrow no(S, P)$;

(1.3) $\Box some(S, P)\Rightarrow some(S, P)$; (1.4) $\Box not\ all(S, P)\Rightarrow not\ all(S, P)$;

(1.5) $\Box most(S, P)\Rightarrow most(S, P)$.

Fact 2 (an assertoric proposition implies a possible proposition):

(2.1) $all(S, P)\Rightarrow\Diamond all(S, P)$; (2.2) $no(S, P)\Rightarrow\Diamond no(S, P)$;

(2.3) $some(S, P)\Rightarrow\Diamond some(S, P)$; (2.4) $not\ all(S, P)\Rightarrow\Diamond not\ all(S, P)$;

(2.5) $most(S, P)\Rightarrow\Diamond most(S, P)$.

Fact 3 (a necessary proposition implies an possible proposition):

(3.1) $\Box all(S, P)\Rightarrow\Diamond all(S, P)$; (3.2) $\Box no(S, P)\Rightarrow\Diamond no(S, P)$;

(3.3) $\Box some(S, P)\Rightarrow\Diamond some(S, P)$; (3.4) $\Box not\ all(S, P)\Rightarrow\Diamond not\ all(S, P)$;

(3.5) $\Box most(S, P)\Rightarrow\Diamond most(S, P)$.

The above facts are the basic knowledge of generalized quantifier theory (Peters and Westerståhl, 2006) or classical modal logic (Chagrov & Zakharyaschev, 1997). Generalized modal syllogistic is an extension of classical first-order logic, so the basic rules of the latter also hold in the former. For instance, Let p, q, r and s be propositions, if $\vdash(p\wedge q\rightarrow r)$ and $\vdash(r\rightarrow s)$, then $\vdash(p\wedge q\rightarrow s)$.

3. How to Screen out Valid Generalized Modal Syllogisms

The study of generalized modal syllogisms in this paper is inspired by that of Aristotelian modal syllogisms. It is shown that the Aristotelian syllogism obtained by removing all modalities in any valid Aristotelian modal syllogism is still valid (Xiaojun, 2020b), and the latter can be obtained by adding modalities to the former. Therefore, all valid Aristotelian modal syllogisms can be screened out from all this kind of syllogisms in the light of some basic rules with which valid Aristotelian modal syllogisms should meet (Xiaojun, 2020c).

Generalized syllogisms are extensions of Aristotelian syllogisms, and generalized modal syllogisms are extensions of Aristotelian modal syllogisms. It is found that the study of generalized modal syllogistic is along similar lines to that of Aristotelian modal syllogistic. In other words, it can be shown that the generalized syllogism obtained by removing all modalities in any valid generalized modal syllogism is still valid. Therefore, the simplest way to screen out valid generalized modal syllogisms is to add modalities to valid generalized syllogisms, and then to delete all invalid syllogisms by means of the basic rules with which valid generalized modal syllogisms should meet, and then all the rest syllogisms are valid.

For every valid generalized modal syllogism, the assertive proposition of the conclusion cannot be stronger than that of two premises. Specifically, if the weakest proposition in two premises is a possible one, then the conclusion can only be a possible one; similarly, if the weakest proposition in two premises is an assertoric one, then the conclusion can only be an assertoric one or a possible one.

Theorem 1: Let Q_1, Q_2 and Q_3 be generalized quantifiers. If the generalized syllogism $Q_1(M, P)\wedge Q_2(S, M)\Rightarrow Q_3(S, P)$ is valid, then the following 12 valid generalized modal syllogisms can be obtained by adding modalities to the generalized syllogism:

(1) $\Box Q_1(M, P)\wedge\Box Q_2(S, M)\Rightarrow\Box Q_3(S, P)$;

(2) $\Box Q_1(M, P)\wedge\Box Q_2(S, M)\Rightarrow Q_3(S, P)$;

(3) $\Box Q_1(M, P)\wedge\Box Q_2(S, M)\Rightarrow\Diamond Q_3(S, P)$;

(4) $\Box Q_1(M, P)\wedge Q_2(S, M)\Rightarrow Q_3(S, P)$;

(5) $\Box Q_1(M, P)\wedge Q_2(S, M)\Rightarrow\Diamond Q_3(S, P)$;

(6) $Q_1(M, P)\wedge\Box Q_2(S, M)\Rightarrow Q_3(S, P)$;

(7) $Q_1(M, P)\wedge\Box Q_2(S, M)\Rightarrow\Diamond Q_3(S, P)$;

(8) $\Box Q_1(M, P)\wedge\Diamond Q_2(S, M)\Rightarrow\Diamond Q_3(S, P)$;

(9) $\Diamond Q_1(M, P)\wedge\Box Q_2(S, M)\Rightarrow\Diamond Q_3(S, P)$;

(10) $Q_1(M, P)\wedge\Diamond Q_2(S, M)\Rightarrow\Diamond Q_3(S, P)$;

(11) $\Diamond Q_1(M, P)\wedge Q_2(S, M)\Rightarrow\Diamond Q_3(S, P)$;

(12) $Q_1(M, P)\wedge Q_2(S, M)\Rightarrow\Diamond Q_3(S, P)$;

In terms of Theorem 1, the following will use several examples to illustrate how to obtain the corresponding 12 valid generalized modal syllogisms from a valid generalized syllogism. Before doing this, it is necessary to prove the validity of the following generalized syllogism.

Theorem 2: The generalized syllogism *all*(M, P)$\wedge$*most*(S, M)$\Rightarrow$*some*(S, P) is valid.

Proof: Suppose that *all*(M, P) and *most*(S, M) are true, then *all*(M, P)$\Leftrightarrow M\subseteq P$ and *most*(S, M)$\Leftrightarrow$ $|S\cap M|\geq 0.6|S|$ are true in the light of the clause (1.1) and (1.5) in Definition 1, respectively. It is easily seen that $M\subseteq P$ and $|S\cap M|\geq 0.6|S|$. It follows that $|S\cap P|\geq 0.6|S|$. And it is clear that $S\cap P\neq\varnothing$. Thus, *some*(S, P) is true in line with the clause (1.3) in Definition 1, just as desired.

According to Theorem 1 and Theorem 2, one can obtain the following Theorem 3:

Theorem 3: The following 12 generalized modal syllogisms are valid:

(3.1) □*all*(M, P)$\wedge$□*most*(S, M)$\Rightarrow$□*some*(S, P);

(3.2) □*all*(M, P)$\wedge$□*most*(S, M)$\Rightarrow$*some*(S, P);

(3.3) □*all*(M, P)$\wedge$□*most*(S, M)$\Rightarrow$◇*some*(S, P);

(3.4) □*all*(M, P)$\wedge$*most*(S, M)$\Rightarrow$*some*(S, P);

(3.5) □*all*(M, P)$\wedge$*most*(S, M)$\Rightarrow$◇*some*(S, P);

(3.6) *all*(M, P)$\wedge$□*most*(S, M)$\Rightarrow$*some*(S, P);

(3.7) *all*(M, P)$\wedge$□*most*(S, M)$\Rightarrow$◇*some*(S, P);

(3.8) □*all*(M, P)$\wedge$◇*most*(S, M)$\Rightarrow$◇*some*(S, P);

(3.9) ◇*all*(M, P)$\wedge$□*most*(S, M)$\Rightarrow$◇*some*(S, P);

(3.10) *all*(M, P)$\wedge$◇*most*(S, M)$\Rightarrow$◇*some*(S, P);

(3.11) ◇*all*(M, P)$\wedge$*most*(S, M)$\Rightarrow$◇*some*(S, P);

(3.12) *all*(M, P)$\wedge$*most*(S, M)$\Rightarrow$◇*some*(S, P).

Proof: The proof of the validity of a generalized modal syllogism can be transformed into that of its corresponding generalized syllogism. For (3.1). Suppose that □*all*(M, P) and □*most*(S, M) are true, in line with the clause (2.1) in Definition 2, □*all*(M, P) is true if and only if $M\subseteq P$ is true in any possible world α. Similarly, with the help of the clause (2.9) in Definition 2, □*most*(S, M) is true if and only if $|S\cap M|\geq 0.6|S|$ is true in any possible world α. Thus it follows that $M\subseteq P$ and $|S\cap M|\geq 0.6|S|$ are true in any possible world α. It follows that $|S\cap P|\geq 0.6|S|$. And it can be seen that $S\cap P\neq\varnothing$ is true in any possible world α. Therefore, □*some*(S, P) is true in accordance with the clause (2.5) in Definition 2. The proof of (2.1) has been completed. The others can be similarly proved on the basis of the above theorems, facts and rules.

Theorem 4: The generalized syllogism *no*(P, M)$\wedge$*most*(S, M)$\Rightarrow$*not all*(S, P) is valid.

Proof: Suppose that *no*(P, M) and *most*(S, M) are true, according to the clause (1.2) and (1.5) in Definition 1, one can obtain that *no*(P, M)$\Leftrightarrow P\cap M=\varnothing$ and *most*(S, M)$\Leftrightarrow|S\cap M|\geq 0.6|S|$ are true. It can be seen that $P\cap M=\varnothing$ and $|S\cap M|\geq 0.6|S|$. Therefore, $S\not\subseteq P$. This can be prove by *reductio ad absurdum*. Suppose that $S\not\subseteq P$ is not true. It is clear that $S\subseteq P$ is true, and it has been proved that $P\cap M=\varnothing$. Thus, it follows that $S\cap M=\varnothing$ which contradicts $|S\cap M|\geq 0.6|S|$. So $S\subseteq P$ is not true. In other words, $S\not\subseteq P$ is true. According to definition (1.4), *not all*(S, P) is true, as required.

According to Theorem 1 and Theorem 4, one can obtain the following Theorem 5:

Theorem 5: The following 12 generalized modal syllogisms are valid:

(5.1) $\Box no(P, M) \wedge \Box most(S, M) \Rightarrow \Box not\ all(S, P)$;

(5.2) $\Box no(P, M) \wedge \Box most(S, M) \Rightarrow not\ all(S, P)$;

(5.3) $\Box no(P, M) \wedge \Box most(S, M) \Rightarrow \Diamond not\ all(S, P)$;

(5.4) $\Box no(P, M) \wedge most(S, M) \Rightarrow not\ all(S, P)$;

(5.5) $\Box no(P, M) \wedge most(S, M) \Rightarrow \Diamond not\ all(S, P)$;

(5.6) $no(P, M) \wedge \Box most(S, M) \Rightarrow not\ all(S, P)$;

(5.7) $no(P, M) \wedge \Box most(S, M) \Rightarrow \Diamond not\ all(S, P)$;

(5.8) $\Box no(P, M) \wedge \Diamond most(S, M) \Rightarrow \Diamond not\ all(S, P)$;

(5.9) $\Diamond no(P, M) \wedge \Box most(S, M) \Rightarrow \Diamond not\ all(S, P)$;

(5.10) $no(P, M) \wedge \Diamond most(S, M) \Rightarrow \Diamond not\ all(S, P)$;

(5.11) $\Diamond no(P, M) \wedge most(S, M) \Rightarrow \Diamond not\ all(S, P)$;

(5.12) $no(P, M) \wedge most(S, M) \Rightarrow \Diamond not\ all(S, P)$.

The proof of Theorem 5 is similar to that of Theorem 3. That is to say that its proof can be transformed into that of Theorem 4.

4. Conclusion and Future Work

The research conclusions of this paper are as follows: (1) Making full use of the truth value definitions of sentences with quantification, possible world semantics and/or fuzzy logic, one can prove the validity of generalized modal syllogisms. (2) The proof of the validity of a generalized modal syllogism can be transformed into that of its corresponding generalized syllogism. And it can be shown that the generalized syllogism obtained by removing all modalities in any valid generalized modal syllogism is still valid. So the simplest way to screen out valid generalized modal syllogisms is to add modalities to valid generalized syllogisms, and then to delete all invalid syllogisms by means of the basic rules with which valid generalized modal syllogisms should meet. And then the remainders are valid. (3) One can obtain 12 valid generalized modal syllogisms by adding necessary modalities and/or possible modalities to any valid generalized syllogism.

This study provides an unified mathematical paradigm for discussing the other generalized modal syllogisms including different generalized quantifiers (such as *a minority of*, *few*, *at most n*, *at least half of the*, *a large part of*, etc.). There are an infinite number of instances in natural language corresponding to any valid generalized modal syllogism. Hence, this study has important theoretical value and practical significance.

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References

Chagrov, A., & Zakharyaschev, M. (1997). *Modal Logic*. Oxford: Clarendon Press.

Chellas, B. F. (1980). *Modal Logic: an Introduction*. Cambridge: Cambridge University Press. https://doi.org/10.1017/CBO9780511621192

Cheng, Z. (2022). The Remaining 23 Valid Aristotelian Syllogisms can be Deduced only from the Syllogism IAI-3. *SCIREA Journal of Computer*, *7*(5), 85-95.

Cheng, Z. (2023). How to Deduce the Other 91 Valid Aristotelian Modal Syllogisms from the Syllogism □I□A□I-3. *Applied Science and Innovative Research*, *7*(1), 46-57. https://doi.org/10.22158/asir.v7n1p46

Endrullis, J., & Moss, L. S. (2015). Syllogistic logic with 'Most'. In V. de Paiva et al. (Eds.), *Logic, Language, Information, and Computation* (pp. 124-139).

Johnson, F. (2004). Aristotle's modal syllogisms. *Handbook of the History of Logic*, *I*, 247-338. https://doi.org/10.1016/S1874-5857(04)80006-2

Li, H. (2023). Reduction between categorical syllogisms based on the syllogism EIO-2. *Applied Science and Innovative Research*, *7*(1), 30-37. https://doi.org/10.22158/asir.v7n1p30

Moss, L. S. (2010). Syllogistic Logics with Verbs. *Journal of Logic and Computation*, *20*(4), 947-967. https://doi.org/10.1093/logcom/exn086

Murinová, P., & Novák, V. (2012). A Formal Theory of Generalized Intermediate Syllogisms. *Fuzzy Sets and Systems*, *186*, 47-80. https://doi.org/10.1016/j.fss.2011.07.004

Novák, V. (2008a). A formal theory of Intermediate quantifiers. *Fuzzy Sets and Systems*, *159*(10), 1229-1246. https://doi.org/10.1016/j.fss.2007.12.008

Novák, V. (2008b). A comprehensive theory of trichotomous evaluative linguistic expressions. *Fuzzy Sets and Systems*, *159*(22), 2939-2969. https://doi.org/10.1016/j.fss.2008.02.023

Peters, S., & Westerståhl, D. (2006). *Quantifiers in Language and Logic*. Claredon Press, Oxford.

Peterson, P. (2000). *Intermediate quantifiers. Logic, Linguistics, and Aristotelian Semantics*. Ashagate: Aldershot.

Wei, L. (2023). Formal system of categorical syllogistic logic based on the syllogism AEE-4. *Open Journal of Philosophy*, (13), 97-103. https://doi.org/10.4236/ojpp.2023.131006

Xiaojun, Z. (2016). The validity of generalized syllogisms including the intermediate quantifier most. *Journal of Hunan University of Science & Technology (Social Science Edition)*, (4), 27-31. (In Chinese)

Xiaojun, Z. (2020a). *Research on Extended Syllogism for Natural Language Information Processing*. Beijing: Science Press. (In Chinese)

Xiaojun, Z. (2020b). Screening out All Valid Aristotelian Modal Syllogisms. *Applied and Computational Mathematics*, *8*(6), 95-104. https://doi.org/10.11648/j.acm.20190806.12

Xiaojun, Z. (2020c). Reducible relations between/among Aristotelian modal syllogisms. *SCIREA Journal of Computer*, *5*(1), 1-33.