Applied Science and Innovative Research
ISSN 2474-4972 (Print) ISSN 2474-4980 (Online)
Vol. 7, No. 1, 2023
www.scholink.org/ojs/index.php/asir

Original Paper

Reduction between Categorical Syllogisms Based on the Syllogism *EIO-2*

Hui Li[1]

[1] School of Philosophy, Anhui University, Hefei, China

Received: December 7, 2022 Accepted: December 19, 2022 Online Published: December 22, 2022
doi:10.22158/asir.v7n1p30 URL: http://doi.org/10.22158/asir.v7n1p30

Abstract

Syllogism reasoning is a common and important form of reasoning in human thinking from Aristotle onwards. To overcome the shortcomings of previous studies, this article makes full use of set theory and classical propositional logic, and deduces the remaining 23 valid syllogisms only on the basis of the syllogism EIO-2 from the perspective of mathematical structuralism, and then successfully establishes a concise formal axiom system for categorical syllogistic logic. More specifically, the article takes advantage of the trisection structure of categorical propositions such as Q(a, b), the transformation relations between an Aristotelian quantifier and its inner and outer negation, the symmetry of the two Aristotelian quantifier (that is, no and some), and some inference rules in classical propositional logic, and derives the remaining 23 valid syllogisms from the syllogism EIO-2, so as to realize the reduction between different valid categorical syllogisms.

Keywords

categorical syllogisms, Aristotelian quantifier, symmetry, mathematical structuralism

1. Introduction

In natural language, there are various syllogisms, such as categorical syllogisms (Łukasiewicz, 1957; Moss, 2008; Westerståhl, 1989), generalized syllogisms (Murinová, & Novák, 2012; Endrullis & Moss, 2015), modal syllogisms (Johnson, 2004; Zhang, 2020a, 2020b), relational syllogisms, syllogisms with verbs (Moss, 2010), and syllogisms with Boolean operations (Ivanov & Vakarelov, 2012), and so on. The indisputable fact is that syllogism reasoning is a common and important form of reasoning in human thinking from Aristotle onwards (Patzig, 1969). This article focuses on categorical syllogisms. Unless otherwise specified, the following syllogisms refer to categorical syllogisms.

Most logic scholars believe that categorical syllogisms are also called Aristotelian syllogisms or traditional syllogisms. This article adopts this view. There are many scholars who have studied categorical syllogistic logic, such as Łukasiewicz (1957), Kulicki (2020), Preston (2020), Pereira-Fariña et al. (2014), Tennant (2014), Beihai et al. (2018), Xiaojun (2018), and so on. In previous work, most methods to judge the validity of categorical syllogisms are informal, and generally need to use the distribution of subjects and predicates in categorical propositions. Due to the lack of formal definition of distribution, there are not only many troubles in understanding relevant knowledge, but also great difficulties for natural language information processing.

It is known that only just 24 kinds of categorical syllogisms are valid among 256 kinds of categorical syllogisms. In previous studies, at least two valid syllogisms were used as basic axioms when deriving all of the other valid syllogisms, for example by Łukasiewicz (1957), Cai (1984), Zhang and Li (2016), Zhang (2018) and Huang and Zhang (2020) and Zhou et al. (2018). Take Zhang and Li (2016) as an example. Using the transformation relations between a quantifier and its three negative quantifiers, as well as the symmetry of two Aristotelian quantifiers (that is, *some* and *no*) (Zhang & Huang, 2012), the authors derived the remaining 22 valid syllogisms on the basis of the two syllogisms *AAA-1* and *EAE-1*.

To overcome the above shortcomings, this article makes full of set theory and classical propositional logic, and deduces the remaining 23 valid syllogisms only on the basis of the syllogism *EIO-2* from the perspective of mathematical structuralism, and then successfully establishes a concise formal axiom system for categorical syllogistic logic.

2. Basic Knowledge

Logic and mathematics are inseparable. Mathematical structuralism holds that mathematics has the characteristics of structuralism (Hellman, 2001). More specifically, mathematics mainly studies the structure of objects and the relations between structures (Hao, 2013). Logic not only studies thinking forms and their laws, but also the formal structures of thinking objects and the relations between structures (Hao & Kan, 2018). In short, from the perspective of mathematical structuralism, formal logic mainly studies the mathematical characteristics of the structures of thinking objects.

As we all know, a categorical syllogism is composed of three categorical propositions. From the perspective of mathematical structuralism, any categorical proposition has a tripartite structure such as *Q(a, b)*, where *Q* represents any of the four Aristotelian quantifiers (that is, *all*, *some*, *no*, *not all*), in which *a* is the subject argument of the categorical proposition, and *b* is the predicate argument of categorical propositions. More specifically, categorical syllogisms only include the following four forms of categorical propositions: *all(a, b)*, *some(a, b)*, *no(a, b)* and *not all(a, b)*, which respectively means that all *a*s are *b*s, some *a*s are *b*s, no *a*s are *b*s and not all *a*s are *b*s. Then, let *a*, *b* and *c* be lexical variables, the syllogism *EIO-2* can be denoted by *no(c, b)→(some(a, b)→not all(a, c))*, and the other categorical syllogisms can be expressed similarly.

3. The Formal Categorical Syllogistic Logic

3.1 Primitive Symbols

(1) lexical variables: *a*, *b*, *c*

(2) unary negative connectives: $\neg$

(3) binary implication connectives: $\rightarrow$

(4) quantifiers: all, some

(5) brackets: (,)

3.2 Formation Rules

(1) If *Q* is a quantifier and *a* and *b* are lexical variables, then *Q(a, b)* is a well-formed formula.

(2) If *p* and *q* are well-formed formulas, then $\neg p$ and $p\rightarrow q$ are well-formed formulas.

(3) Only formulas obtained in accordance with (1) and (2) are well-formed formulas.

For example, *all(a, b)*, $\neg$*some(a, b)*, and *some(a, b)*$\rightarrow$*all(b, c)* are well-formed formulas that are read as 'all *a*s are *b*s', 'not some *a*s are *b*s', 'if some *a*s are *bs*, then all *b*s are *c*s', respectively. The other formulas are similar.

3.3 Related Definitions

(1) Definition of biconditional connective $\leftrightarrow$: $(p\leftrightarrow q) =_{def} (p\rightarrow q)\wedge(q\leftarrow p)$.

(2) Definition of inner negative quantifier: $Q\neg(a, b) =_{def} Q(a, D-b)$, in which *D* indicates the domain of lexical variables.

(3) Definition of outer negative quantifier: $(\neg Q)(a, b) =_{def}$ It is not that *Q(a, b)*.

3.4 Basic Axioms

(1) A0: If *p* is a valid formula in classical propositional logic, then $\vdash p$.

(2) A1: $\vdash$*some(a, a)*

(3) A2 (namely the syllogism *EIO-2*): $\vdash$*no(a, b)*$\rightarrow$*some(c, b)*$\rightarrow$*not all(c, a)*

3.5 Reasoning Rules

The following rules of inference in classical propositional logic will be used. Let *p*, *q*, *r* and *s* be well-formed formulas.

Rule 1 (Modus ponens): If $\vdash(p\rightarrow q)$ and $\vdash p$, then $\vdash q$ can be inferred.

Rule 2 (Antecedent interchange): If $\vdash(p\rightarrow(q\rightarrow r))$, then $\vdash(q\rightarrow(p\rightarrow r))$ can be inferred.

Rule 3 (Subsequent weakening): If $\vdash(p\rightarrow(q\rightarrow r))$ and $\vdash(r\rightarrow s)$, then $\vdash(p\rightarrow(q\rightarrow s))$ can be inferred.

Rule 4 (Reverse): If $\vdash(p\rightarrow q)$, then $\vdash(p\rightarrow q)\rightarrow(\neg q\rightarrow\neg p)$ can be inferred.

Rule 5 (anti-syllogism): If $\vdash(p\rightarrow(q\rightarrow r))$, then $\vdash(p\rightarrow(\neg r\rightarrow\neg q))$ can be inferred.

4. Related Relations and Related Facts

Now give transformation between Aristotelian quantifiers and related facts that will be used later.

4.1 Transformation between Aristotelian Quantifiers

There are transformable relations between an Aristotelian quantifier and its inner and outer negative quantifiers. In the four Aristotelian quantifiers, *all* and *no*, *some* and *not all* are inner negations each other, and *all* and *not all*, *some* and *no* are outer negative each other. These can be summarized in the following two tables.

Table 1. Inner Negation

	quantifiers	categorical propositions
(1)	*all=no¬*	*all(a, b)↔no¬(a, b)*
(2)	*no=all¬*	*no(a, b)↔all¬(a, b)*
(3)	*some=not all¬*	*some(a, b)↔not all¬(a, b)*
(4)	*not all=some¬*	*not all(a, b)↔some¬(a, b)*

Table 2. Outer Negation

	quantifiers	categorical propositions
(1)	*¬not all=all*	*¬not all(a, b)↔all(a, b)*
(2)	*¬all=not all*	*¬all(a, b)↔not all(a, b)*
(3)	*¬no=some*	*¬no(a, b)↔some(a, b)*
(4)	*¬some=no*	*¬some(a, b)↔no(a, b)*

4.2 Related Facts

The following two facts will be useful later, which can be proved by the above definitions, axioms and reasoning rules.

Fact 1 (symmetry of *some* and *no*): (1) ⊢*some(a, b)* ↔ *some(b, a)*; (2) ⊢*no(a, b)* ↔ *no(b, a)*.

Fact 1 is a basic fact in generalized quantifier theory (Zhang & Wu, 2021). Therefore, its proof is omitted.

Fact 2 (assertoric subalternations): (1) ⊢*all(a, b)→some(a, b)*; (2) ⊢*no(a, b)→not all(a, b)*.

Proof:

[1] ⊢*no(a, b)→(some(c, b)→not all(c, a))* (by Axioms A3)

[2] ⊢*some(c, b)→(no(a, b)→not all(c, a))* (by [1] and Rule 2)

[3] ⊢*some(c, c)→(no(a, c)→not all(c, a))* (by [2])

[4] ⊢*no(a, c)↔no(c, a)* (by (2) of Fact 1)

[5] ⊢*some(c, c)→(no(c, a)→not all(c, a))* (by [3] and[4])

[6] ⊢*some(c, c)* (by Axiom A1)

[7] ⊢*no(c, a)→not all(c, a)* (by [5], [6] and Rule 1)

[8] ⊢*no(a, b)→not all(a, b)* (by [7])

[9] ⊢*(no(a, b)→not all(a, b))→(¬not all(a, b)→¬no(a, b))* (by [8] and Rule 4)

[10] ⊢*¬not all(a, b)→¬no(a, b)* (by [8], [9] and Rule 1)

[11] ⊢*all(a, b)→some(a, b)* (by [10], (1) and (3) in Table 2)

5. The Remaining 23 Valid Syllogisms can be Deduced from the Syllogism *EIO-2*

In the following theorem, *EIO-2⇒EIO-1* indicates that the validity of syllogism *EIO-2* can be deduced from the validity of *EIO-1* syllogisms, that is, there are reducible relations between these two syllogisms. The other are similar. The key to establish the proof system of categorical syllogistic logic is the reducibility between different syllogisms.

Theorem 1 (reducible relations between different categorical syllogisms): The remaining 23 valid syllogisms are derived as follows on the basis of the syllogism *EIO-2*:

(1) *EIO-2⇒EIO-1*

(2) *EIO-2⇒EIO-4*

(3) *EIO-2⇒EIO-4⇒EIO-3*

(4) *EIO-2⇒EAE-1*

(5) *EIO-2⇒AOO-2*

(6) *EIO-2⇒IAI-3*

(7) *EIO-2⇒IAI-3⇒AII-3*

(8) *EIO-2⇒IAI-3⇒IAI-4*

(9) *EIO-2⇒IAI-3⇒AII-3⇒AII-1*

(10) *EIO-2⇒EAE-1⇒EAE-2*

(11) *EIO-2⇒EAE-1⇒AEE-4*

(12) *EIO-2⇒EAE-1⇒AEE-4⇒AEE-2*

(13) *EIO-2⇒EAE-1⇒AEE-4⇒AEE-2⇒AEO-2*

(14) *EIO-2⇒EAE-1⇒AEE-4⇒AEO-4*

(15) *EIO-2⇒EAE-1⇒AEE-4⇒AEO-4⇒EAO-4*

(16) *EIO-2⇒EAE-1⇒AEE-4⇒AEO-4⇒EAO-4⇒EAO-3*

(17) *EIO-2⇒EAE-1⇒EAO-1*

(18) *EIO-2⇒EAE-1⇒EAO-1⇒EAO-2*

(19) *EIO-2⇒EAE-1⇒EAO-1⇒EAO-2⇒AAI-3*

(20) *EIO-2⇒EAE-1⇒AAA-1*

(21) *EIO-2⇒EAE-1⇒AAA-1⇒AAI-1*

(22) *EIO-2⇒EAE-1⇒AAA-1⇒AAI-1⇒AAI-4*

(23) *EIO-2⇒EAE-1⇒AAA-1⇒OAO-3*

Proof:

[1] ⊢*no(a, b)→(some(c, b)→not all(c, a))* (i.e. *EIO-2*, by Axiom A2)

[2] ⊢*no(a, b)↔no(b, a)* (by (2) in Fact 1)

[3] ⊢*no(b, a)→(some(c, b)→not all(c, a))* (i.e. *EIO-1*, by [1] and [2])

[4] ⊢*some(c, b)↔some(b, c)* (by (1) in Fact 1)

[5] ⊢*no(a, b)→(some(b, c)→not all(c, a))* (i.e. *EIO-4*, by [1] and [4])

[6] ⊢*no(b, a)→(some(b, c)→not all(c, a))* (i.e. *EIO-3*, by [2] and [5])

[7] $\vdash no(a, b)\rightarrow(\neg not\ all(c, a)\rightarrow\neg some(c, b))$ (by [1] and Rule 5)

[8] $\vdash no(a, b)\rightarrow(all(c, a)\rightarrow no(c, b))$ (i.e. *EAE-1*, by [7], (1) and (4) in Table 2)

[9] $\vdash all\neg(a, b)\rightarrow(not\ all\neg(c, b)\rightarrow not\ all(c, a))$ (by [1], (2) and (3) in Table 1)

[10] $\vdash all(a, D-b)\rightarrow(not\ all(c, D-b)\rightarrow not\ all(c, a))$ (by [9] and (2) in Definition (3))

[11] $\vdash all(a, b)\rightarrow(not\ all(c, b)\rightarrow not\ all(c, a))$ (i.e. *AOO-2*, by [10])

[12] $\vdash some(c, b)\rightarrow(\neg not\ all(c, a)\rightarrow\neg no(a, b))$ (by [1], Rule 2 and Rule 5)

[13] $\vdash some(c, b)\rightarrow(all(c, a)\rightarrow some(a, b))$ (i.e. *IAI-3*, by [12], (1) and (3) in Table 2)

[14] $\vdash some(a, b)\leftrightarrow some(b, a)$ (by (1) in Fact 1)

[15] $\vdash all(c, a)\rightarrow(some(c, b)\rightarrow some(b, a))$ (i.e. *AII-3*, by [13], [14] and Rule 2)

[16] $\vdash some(b, c)\rightarrow(all(c, a)\rightarrow some(a, b))$ (i.e. *IAI-4*, by [13] and [4])

[17] $\vdash all(c, a)\rightarrow(some(b, c)\rightarrow some(b, a))$ (i.e. *AII-1*, by [15] and [4])

[18] $\vdash no(b, a)\rightarrow(all(c, a)\rightarrow no(c, b))$ (i.e. *EAE-2*, by [8] and [2])

[19] $\vdash no(c, b)\leftrightarrow \mathrm{no}(b, c)$ (by (2) in Fact 1)

[20] $\vdash all(c, a)\rightarrow(no(a, b)\rightarrow no(b, c))$ (i.e. *AEE-4*, by [8], [19] and Rule 2)

[21] $\vdash all(c, a)\rightarrow(no(b, a)\rightarrow no(b, c))$ (i.e. *AEE-2*, by [20] and [2])

[22] $\vdash all(c, a)\rightarrow(no(b, a)\rightarrow not\ all(b, c))$ (i.e. *AEO-2*, by [21], (2) in Fact 2 and Rule 3)

[23] $\vdash all(c, a)\rightarrow(no(a, b)\rightarrow not\ all(b, c))$ (i.e. *AEO-4*, by [20], (2) in Fact 2 and Rule 3)

[24] $\vdash no(a, b)\rightarrow(\neg not\ all(b, c)\rightarrow\neg all(c, a))$ (by [23] , Rule 2 and Rule 5)

[25] $\vdash no(a, b)\rightarrow(all(b, c)\rightarrow not\ all(c, a))$ (i.e. *EAO-4*, by [24], (1) and (2) in Table 2)

[26] $\vdash no(b, a)\rightarrow(all(b, c)\rightarrow not\ all(c, a))$ (i.e. *EAO-3*, by [25] and [2])

[27] $\vdash no(a, b)\rightarrow(all(c, a)\rightarrow not\ all(c, b))$ (i.e. *EAO-1*, by [8], (2) in Fact 2 and Rule 3)

[28] $\vdash no(b, a)\rightarrow(all(c, a)\rightarrow not\ all(c, b))$ (i.e. *EAO-2*, by [27] and [2])

[29] $\vdash all(c, a)\rightarrow(\neg not\ all(c, b)\rightarrow\neg no(b, a))$ (by [28], Rule 2 and Rule 5)

[30] $\vdash all(c, a)\rightarrow(all(c, b)\rightarrow some(b, a))$ (i.e. *AAI-3*, by [29], (1) and (3) in Table 2)

[31] $\vdash all\neg(a, b)\rightarrow(all(c, a)\rightarrow all\neg(c, b))$ (by [10], (2) and (4) in Table 1)

[32] $\vdash all(a, D-b)\rightarrow(all(c, a)\rightarrow all(c, D-b))$ (by [31] and (2) in Definition (3.3))

[33] $\vdash all(a, b)\rightarrow(all(c, a)\rightarrow all(c, b))$ (i.e. *AAA-1*, by [32])

[34] $\vdash all(a, b)\rightarrow(all(c, a)\rightarrow some(c, b))$ (i.e. *AAI-1*, by [33], (1) in Fact 2 and Rule 3)

[35] $\vdash all(c, a)\rightarrow(all(a, b)\rightarrow some(b, c))$ (i.e. *AAI-4*, by [34], [6] and Rule 2)

[36] $\vdash all(c, a)\rightarrow(\neg all(c, b)\rightarrow\neg all(a, b))$ (by [33], Rule 2 and Rule 5)

[37] $\vdash not\ all(c, b)\rightarrow(all(c, a)\rightarrow not\ all(a, b))$ (i.e. *OAO-3*, by [36], (2) in Table 2, Rule 2)

This ends the proof process of Theorem 1. In fact, the way to deduce the same valid syllogisms are not unique.

6. Conclusion

Logic not only studies thinking forms and their laws, but also studies the formal structures of thinking objects and the relations between structures. The article takes advantage of the trisection structure of categorical propositions such as *Q(a, b)*, the transformation relations between an Aristotelian quantifier and its inner and outer negation, the symmetry of the two Aristotelian quantifier (that is, *no* and *some*), and some inference rules in classical propositional logic, and derives the remaining 23 valid syllogisms from the syllogism *EIO-2*, so as to realize the reduction between different valid categorical syllogisms, and then establish a concise formal axiom system for categorical syllogistic logic.

The research method of this paper provides a concise mathematical paradigm for studying other kinds of syllogisms. Under the objective needs of the rapid development of artificial intelligence technology, the formal processing of natural language has gradually become indispensable. How to make full use of this research method to serve for natural language information processing remains to be discussed in depth.

Acknowledgements

This work was supported by the Scientific Research Projects of Colleges and Universities in Anhui Province, China under Grant No.SK2021A0023.

References

Cai, S. S. (1984). A formal system of Aristotle's syllogism different from that of Łukasiewicz. *Philosophical research*, (4), 33-41. (in Chinese)

Endrullis, J., & Moss, L. S. (2015). Syllogistic logic with 'most'. In V. de Paiva et al. (Eds.), *Logic, Language, Information, and Computation* (pp. 124-139). https://doi.org/10.1007/978-3-662-47709-0_10

Hao, Y. J. (2013). Language construction of mathematical existence: the research paradigm of structuralism. *Journal of Chongqing University of Technology (Social Science Edition)*, (3), 9-14. (in Chinese)

Hao, Y. J., & Kan, T. (2018). Modern logic from the perspective of structuralism. *Journal of Sichuan Normal University (Social Sciences Edition)*, (3), 78-83. (in Chinese)

Hellman, G. (2001). Three varietes of mathematical structuralism. *Philosophia Mathematica*, *9*(3), 184-211. https://doi.org/10.1093/philmat/9.2.184

Huang, M. Y., & Zhang, X. J. (2020). Assertion or rejection of Łukasiewicz's assertoric syllogism system ŁA. *Journal of Chongqing University of Science and Technology (Social Sciences Edition)*, (2), 10-18. (in Chinese)

Ivanov, N., & Vakarelov, D. (2012). A system of relational syllogistic incorporating full Boolean reasoning. *Journal of Logic, Language and Information*, (21), 433-459. https://doi.org/10.1007/s10849-012-9165-1

Johnson, F. (2004). Aristotle's modal syllogisms. *Handbook of the History of Logic*, *1*, 247-308. https://doi.org/10.1016/S1874-5857(04)80006-2

Kulicki, P. (2020). Aristotle's syllogistic as a deductive system. *Axioms*, *9*(2), 1-16.

https://doi.org/10.3390/axioms9020056

Łukasiewicz, J. (1957). *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*. Oxford: Clarendon Press.

Moss, L. S. (2008). Completeness theorems for syllogistic fragments. In F. Hamm, & S. Kepser (Eds.), *Logics for Linguistic Structures* (pp. 143-173). Berlin: Mouton de Gruyter.

Moss, L. S. (2010). Syllogistic logics with verbs. *Journal of Logic and Computation*, *20*(4), 947-967. https://doi.org/10.1093/logcom/exn086

Murinová, P., & Novák, V. (2012). A formal theory of generalized intermediate syllogisms. *Fuzzy Sets and Systems, 186*, 47-80. https://doi.org/10.1016/j.fss.2011.07.004

Patzig, G. (1969). *Aristotle's Theory of the Syllogism* (J. Barnes, Trans.). Dordrecht: D. Reidel. https://doi.org/10.1007/978-94-017-0787-9

Pereira-Fariña, M., Vidal, J. C., Díaz-Hermida, F., & Bugarín, A. (2014). A fuzzy syllogistic reasoning schema for generalized quantifiers. *Fuzzy Sets and Systems*, *234*(1), 79-96. https://doi.org/10.1016/j.fss.2013.02.007

Preston, S. (2020). Syllogistic reasoning as a ground for the content of judgment: A line of thought from Kant through Hegel to Peirce. *European Journal of Philosophy*, *29*(4), 864-886. https://doi.org/10.1111/ejop.12609

Tennant, N. (2014). Aristotle's syllogistic and core logic. *History and Philosophy of Logic*, *35*(2), 120-147. https://doi.org/10.1080/01445340.2013.867144

Westerståhl, D. (1989). Aristotelian syllogisms and generalized quantifiers, *Studia Logica*, *XLVII*(4), 577-585. https://doi.org/10.1007/BF00370209

Zhang, X. J. (2018). Axiomatization of Aristotelian syllogistic logic based on generalized quantifier theory. *Applied and Computational Mathematics*, *7*(3), 167-172. https://doi.org/10.11648/j.acm.20180703.23

Zhang, X. J. (2020a). Reducible relations between/among Aristotle's modal syllogisms. *SCIREA Journal of Computer*, *5*(1), 1-33.

Zhang, X. J. (2020b). Screening out all valid Aristotelian modal syllogisms. *Applied and Computational Mathematics*, *8*(6), 95-104. https://doi.org/10.11648/j.acm.20190806.12

Zhang, X. J., & Huang, C. Y. (2012). Relationship between the monotonicity of generalized quantifiers and that of their three negative quantifiers. *Journal of Anhui Normal University (Humanities and Social Sciences Edition)*, (6), 673-678. (in Chinese)

Zhang, X. J., & Li, S. (2016). Research on the formalization and axiomatization of traditional syllogisms. *Journal of Hubei University (Philosophy and Social Sciences)*, (6), 32-37. (in Chinese)

Zhang, X. J., & Wu, B. X. (2021). *Research on Chinese Textual Reasoning*. Beijing: People's Publishing House. (in Chinese)

Zhou, B. H., Wang, Q., & Zheng, Z. (2018). Aristotle's division lattice and Aristotelian logic. *Logic research*, (2), 2-20. (in Chinese)