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Original Paper

About Integral Equation for the Symmetrical Loop Antenna

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Abstract

The derivation and solution's problems of integra-differential equations for the currents in symmetrical loop antennas of rectangular, circular and other short-circuited forms with large length are considered. The results are compared with the properties of small antennas.

Keywords

antenna theory, integral equation, circular loop antenna

1. Introduction

There are problems that at first sight seem simple, but stubbornly oppose to rigorous and approximate decision. The symmetrical loop antenna (circular or rectangular) with a length, comparable with the wavelength, is an example of such a problem. As a rule, it is located vertically and in a horizontal plane has the important and often necessary peculiarity - the presence of a zero direction. At that the directional pattern of a circular loop antenna has the form of a figure-eight. This feature allows us to use such antennas for direction finding.

The properties of these antennas with dimensions, small in comparison with the wavelength, are well known: see, for example, Fradin (1977). Unfortunately, if the loop dimensions are comparable with the wavelength, the problem becomes much more complicated. It requires, as in the case of linear radiator, the solution of an integral equation.

Interesting results can be obtained, if analyze the structure of a circular loop antenna more precisely and compare it with the structure of a straight radiator with two arms (dipole). As seen from Fig. 1a, the dipole's structure is symmetrical relatively horizontal line, i.e. one consists of two identical elements, located towards.

From these observations one can make useful conclusions. The horizontal directional pattern of the straight radiator has the form of a circumference. The horizontal directional pattern of the loop antenna (circular and rectangular) has the form of figure-eight. Therefore, its radiated power is reduced approximately twice at the same length.

On Fig. 1b the other structure, located along a vertical line and consisting of two identical arms, is given, and each arm coincides with the right branch of the circular loop antenna. One can easily see that the loop antenna (Fig. 1c) is symmetrical relatively vertical line: it consists of the left and right branches, directed contrary along horizontal line.

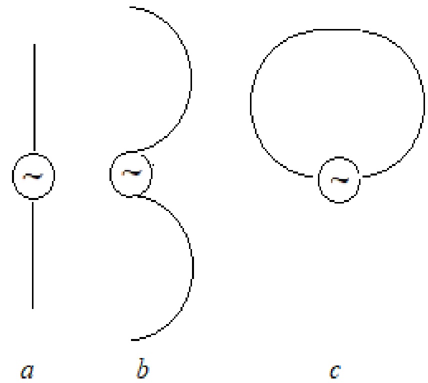

Fig. 1. Radiators with Two Straight (a) and Semi-circular (b) Arms, Circular Loop (c)

2. Straight and Loop Radiators

It is useful to compare the properties of the straight and loop radiators. As is known, the rigorous method of calculating the current's distribution and the input impedance of the straight radiator is given in Leontovich and Levin (1944) as the solution's result of the integral equation for the current along the antenna's conductor. The expression for current along the straight radiator takes the form of a series with terms located according to powers of a small parameter

$$\chi_1 = 1/(2 ln pa).$$

Here a is the radiator' radius. As is shown in Miller (1954), in the capacity of constant $1/p$ one should choose the distance to the nearest inhomogeneity, i.e., the smallest of three magnitudes: wavelength λ, antenna length $2L$ and radius R_0 of the curvature. In order to calculate the total sum of the series in case of a straight radiator, the length of which does not exceed the wavelength, one can consider that

$$1/p=2L, \text{ i.e. } \chi=-\chi_1=[2\ln(2L/a)]^{-1}.$$

The integral equation for the current in the straight radiator, considered in Leontovich and Levin (1944), is solved in the second approximation. Calculation of the series' terms is based on the assumption that the current is located on the radiator's axis. In order to calculate the total sum of a series, the integral equation was replaced by a key equation and solved by variational methods (Vainshtein, 1959-1961). The obtained results were presented in 1964 at a conference in Kharkov. They show that the total current weakly differs from the second approximation.

The simple and visual method of calculating the terms of a current series is offered in Levin (2025). It is based on the use of geometric progression. As is known, the geometric progression is sequence of numbers a_n, in which each sequential number a_{n+1} is equal to the preceding one, multiplied by the denominator q of the progression. If q<1, the progression is called by descending, and with grows of terms' quantity n its sum grows limitlessly and tends to the limit

$$\lim_{n\to\infty} s_n = s = \frac{a_1}{1-q}.$$

As follows from Leontovich and Levin (1944) and is shown in Levin (2025), the expression for current along the straight radiator takes the form of a series, in which each subsequent term is result of an additional emf, created by fields, radiated by previous term of this series. The subsequent term is equal to the previous one, multiplied by -χ, where χ is a small parameter. One can consider that the solution of this equation is accomplished by the method of sequential approximations. The given circumstances allow us to determine the total sum of the series as the sum of infinite geometric progression.

This peculiarity connected, as already mentioned, with the fact, that the current field, flowing along the antenna, creates in the point of the generator's location an additional emf, equal to -χe, which excites an additional current. This fact allows us to calculate the total current (as well as input impedance). The given method permits to simplify calculation and to refine results. As shown further, in the case of a loop antenna, the analogous effect depends on its structure.

3. Rectangular Loop Antenna

In order to determine the current's distribution along the antenna's conductor, one must write and solve the equation, in accordance with which the sum of extraneous emf and tangential components of the currents' fields is equal to zero. We begin from a rectangle loop antenna with a width b and a height l (Fig. 2). Here e is extraneous emf. The conductors form the two-conductor long line, short-circuited at the end. That is the generator's load.

As seen from the figure, the currents of the right and left branches have in symmetrically located points the same magnitudes. The currents along the vertical conductor and also between both horizontal conductors have opposite directions. The current of the right branch is equal to

$$J_1 = I_0 \begin{cases} \cos kx, 0 \le x \le \frac{b}{2}, z = l, \\ \cos\left[k\left(\frac{b}{2} + l - z\right)\right], 0 \le z \le l, x = \frac{b}{2}, \\ \cos[k(b + l - x)], 0 \le x \le \frac{b}{2}, z = 0. \end{cases}$$

In the first approximation one can consider that capacitances per unit length between the vertical conductors are const and equal to

$$C_1 = \frac{\pi\varepsilon_0}{ln(b/a)}.$$

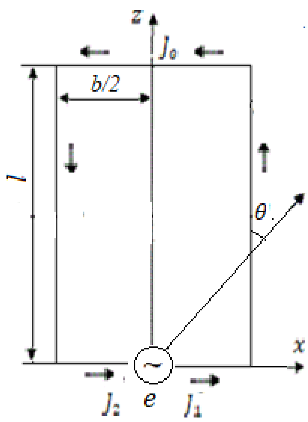

Fig. 2. Rectangular Loop Antenna

In this case the wave impedance of a line is

$$W = \omega C_1 = 120\ln(b/a),$$

the input impedance of an antenna is

$$Z_A = jW\tan kL + R_\Sigma,$$

where $L = l + b,\ R_\Sigma$ is the radiation resistance.

The input current's magnitude of each conductor's current is equal to

$$J_A = J_0\cos kL \quad -$$

in the total agreement with obtained results. These expressions are similar to expressions for the straight linear radiator. If to take into account the difference between sections of a long line, we will obtain expressions, similar to expression for the straight radiator, consisting of sections with different parameters.

The currents of antenna's branches radiate the fields, which created the additional emf and the additional currents. Each current is the field's source, but vertical conductors, located symmetrically, compensate one another in all points, including the points of a lower horizontal line. The currents of two upper horizontal wires create in the lower horizontal wire, located

$$E_1 = -j60\frac{J_0 e^{-jkl}}{kl}\int_0^{b/2}\cos kx dx.$$

The extraneous emf on the antenna's input is equal to

$$K_e = J_A Z_A = J_0(jW\sin kL + R_\Sigma\cos kL),$$

i.e.

$$\frac{E_1}{K_e} = -60\frac{exp(-jkl)\sin\left(\frac{kb}{2}\right)}{kL(W\sin kL - j\,R_\Sigma\cos kL)}.$$

As follows from this result, each emf creates additional currents and additional emfs, forming the infinite geometric progression, similar to progression for emf and current of a linear radiator. But at first it is need to determine the radiation's resistance. Since the power, radiated by a left branch of rectangular antenna to right in its plane, along axis x, is negligible in comparison with the power, radiator by the right branch, one can to consider, that the antenna's effective length is equal to the effective length of the right branch

$$h_e = \frac{1}{\cos(kb/2)} \int_0^l \cos kz dz,$$

and the radiation's resistance is equal to $20k^2 h_e^2$.

In the given case the geometric progression is infinitely descending, has the first term $a_1 = K_e$, the denominator is

$$q = E_1/K_e,$$

and the sum of currents' series tends to limit, equal to

$$\frac{a_1}{1-q} = \frac{K_e}{1-E_1/K_e}.$$

Obtained results practically correspond to derivation and solution of the integral equation for the rectangular loop antenna.

4. Circular Loop Antenna

The symmetrical circular loop antenna is given in Fig. 3. First of all, in this case one must to use other antenna's elements. This element is inductance per unit length of conductor

$$\Lambda = \frac{\mu_0 l}{2\pi}\left(\ln\frac{2L}{a} - 1\right).$$

Here a is conductor's radius. And the extraneous emf in this case is equal to

$$K_e = J_A(j\omega\Lambda \tan kL + R_\Sigma).$$

The currents of the antenna's conductors create the fields in opposite conductors. In particular, the current of an arc, located in the left upper quarter, creates the field in the right lower quarter. The identical current of an arc,

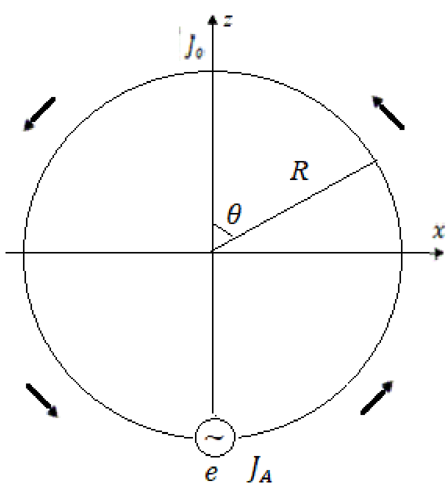

Fig. 3. Circular Loop Antenna

located in the right upper quarter, creates the same field in the left lower quarter (on the distance 2R). The total field is equal to

$$E_2 = \frac{60J_0}{j2kR} e^{-j2kR} \int_0^{\frac{\pi}{2}} \cos kR\theta d\theta =$$

$$\frac{30J_0 \sin\left(\frac{kR\pi}{2}\right)}{jk^2R^2} e^{-j2kR}.$$

The currents' fields create an additional emf, which reduces the extraneous emf. This result coincides with the results, obtained for rectangular loop antenna and brings to the infinitely descending geometric progression with denominator

$$q = \frac{E_2}{K_e} == -\frac{30 \sin\left(\frac{kR\pi}{2}\right) e^{-j2kR}}{k^2R^2 \,(\omega\Lambda \tan kL - jR_\Sigma)\cos kL},$$

i.e., the total sum of series is $\frac{a_1}{1-q} = \frac{K_e}{1-E_2/K_e}$.

As it is seen from obtained results, the current distribution in the large loop antenna of any form is substantially distinguished from the current distribution in the small loop, where the current amplitude is constant along the wire, and the created field is proportional to product of the generation current to the length of semi-perimeter.

5. Triangular Antenna

The asymmetrical loop antennas may have different forms. In this Section appointed variants of the antennas are considered as examples. The triangular antenna is presented in Fig. 4.

As already said, in order to determine the currents distribution along the antenna's conductor, one must to write and solve the equation, in accordance with which the sum of extraneous emf and tangential components of the currents' fields is equal to zero. The triangular loop antenna has a basis *b* and a side *l*. The extraneous emf is equal to *e*. As seen from the figure, the angles between the side conductors and a basis are the same

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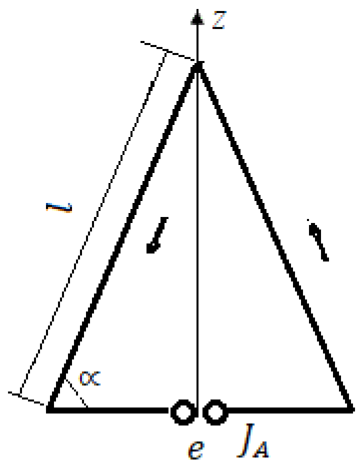

Fig. 4. The Symmetrical Triangular Antenna

and equal to $\propto$. The currents of a right and left branches are the same, and each current is equal to

$$J_1 = J_0 \begin{cases} \cos k(1 - z/\sin\alpha), 0 \le z \le l \sin \propto, \\ \cos k\left(l + \frac{b}{2} - x\right), z = 0. \end{cases}$$

The load of a generator is the short-circuited two-conductor long line. In the first approximation one can consider that

$$\pi\varepsilon_0 C_1 = \pi\varepsilon_0 / \ln[b/\cos \propto].$$

The input current of each conductor $J_A = J_0 \cos kL\ -$

in the total agreement with obtained results. In the first approximation one can consider, that capacitance per unit length between the conductors of this line is constant and equal to

$$C_1 = \pi\varepsilon_0 / \ln(b/a).$$

These expressions coincide with expressions for the straight line. In this case the wave impedance of a line changes along the antenna length in accordance with *a*

$$W = \omega C_1 = 120 \ln(b/a).$$

The input impedance of an antenna is

$$Z_A = jW \tan kL + R_\Sigma,$$

where $L = l + b/2.$ The input current of each conductor is

$$I_A = I_0 \cos kL.$$

The extraneous emf in this case is equal to the product of an input current on the input impedance

$$K_e = J_A(jW \tan kL + R_\Sigma).$$

The field, created by the currents, is

$$E_3 = -j\frac{60}{k}\int_0^{l \sin\alpha} \frac{e^{-jkz}}{z} \cos k(l - z) \sin\alpha dz.$$

The total sum of the series is $\frac{a_1}{1-q}=\frac{K_e}{1-E_3/K_e}$

6. Antenna with Compound Form

In accordance with the described method one can obtain the similar results in the case of antenna with compound form, shown in Fig. 5. In this case

$$J_1 = J_0 \begin{cases} \cos[k(d_1 - z)/\sin\alpha], d_2 \le z \le d_1, \\ \cos\left[k\left(l_1 + \frac{d_2 - z}{\sin\beta}\right)\right], 0 \le z \le d_2. \end{cases}$$

The input impedance of this antenna is

$$Z_A = jW \tan kL + R_\Sigma,$$

where L=$l_1 + l_2$, R_Σ is the radiation resistance,

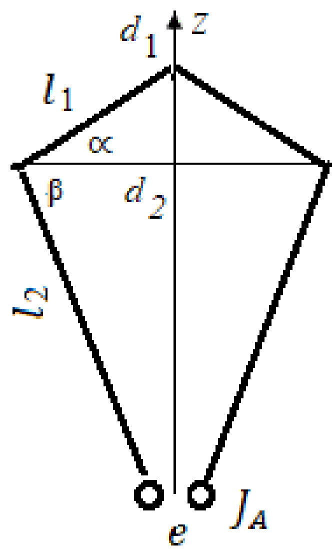

Fig. 5. Antenna of Compound Form

where L=$l_1 + l_2$, R_Σ is the radiation resistance, the input current is equal to $I_A = I_0 \cos kL$. The extraneous emf in this case is

$$K_e = J_A(jW \tan kL + R_\Sigma).$$

The currents' fields E_4 of upper conductors create additional emf on the lower conductors and reduces the extraneous emf. For the same vertical line on the upper section $x = l_1 \cos\alpha$,

on the lower section $x = l_2 \cos\beta$,

i.e., the distance between the points of upper and lower conductors with the same x is

$$z = x(\tan\alpha + \tan\beta),$$

and the field, created by the currents, is equal to

$$E_4 = -j\frac{60I_0}{k}\int_0^{l_1\cos\alpha}\frac{e^{-ikTx}}{Tx}\cos(kx/\cos\alpha)dx,$$

where $T = \tan\alpha + \tan\beta$.

The total sum of the current series is equal to

$$\frac{a_1}{1-q} = \frac{K_e}{1-E_4/K_e}.$$

7. About Integral Equations

The obtained results in fact consider the derivation and solution of the integral equations for the current of symmetrical loop antennas of different form (rectangular, circular, triangular, compound) with large length. The offered method of analysis is based on the results of Leontovich and Levin (1944) and Levin (2025). The analogical solution is proposed for different variants of short-circuited antennas.

In the case symmetrical loop antenna by contrast to rectilinear radiator the integral equation has the form

$$K_e = J_A(jW\tan kL + R_\Sigma).$$

The current is sought in the form of expansion into a series in powers of small parameters q. In the first approximation the current is equal to the relation of extraneous emf K_e to the antenna's input impedance, which is usually based on long line theory. For each variant the problem's solution is based on the calculation of the sum of input current's series. The series' members form infinite geometric progression. The calculation of currents in antenna wires allows to determine in the placement point of extraneous emf the additional emf created by wires' currents. Amplitudes of these emfs are inversely proportional to the distance from this point and depend on the angle between the wires. Additional emfs create weaker currents, etc. The use of geometric progression allows to determine the full current and full input impedance.

8. Conclusion

Obtained results allow us to generalize the proposed by M. A. Leontovich rigorous method of calculating rectilinear radiators to the radiators of intricate form. At that the used method is simplified and refined. These results allow us to compare the properties of great and small circular loop radiators.

Boris Levin was born in Saratov, Russia, in January of 1937. In 1960 he graduated from Leningrad Polytechnical institute, in 1969 he got the Ph.D. degree in radio physic from the Central Research Institute of Automatic Devices (Leningrad, Russia). In 1993 he defended the dissertation in order to get the Doctor of Sciences degree (professor) in physics and mathematics from Peter the Great

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From 1999 he lives in Lod (Israel). Here he worked in the Holon Technology University and in the company MARS. He is the author of 10 books. Three books are published in Russian, including "Monopole and Dipole Antennas for Marine-Vehicle Radio Communications (S.- Petersburg: Abris, 1998).

Seven books are published in English, including "The Theory of Thin Antennas and Its Use in Antenna Engineering" (Bentham Science Publishers, 2013), "Method of Complex Potential in Antenna Engineering" (LAP, 2014), "Inverse Problems of Antennas Theory" (LAP, 2014), "Antenna Engineering. Theory and Problems" (CRC Press, 2017), "Wide-range and Multi-frequency Antennas" (CRC Press, 2019), "Antennas: Rigorous Methods of Analysis and Synthesis" (CRC Press, 2021) and "Antennas: From the Theory of Long Lines to Integral Equations (CRC Press, 2024).

Also, he is the author of 95 papers in technical journals, of 102 papers on scientific conferences and of 43 patents.

His main research interests are in the fields of electromagnetic theory, the theory of antennas and antennas' optimization.

References

Fradin, A. Z. (1977). *Antenna-Feeder Devices*. Moscow: Communications. (in Russian).

Kalantarov, P. L., & Cheytlin, L. A. (1986). *Calculation of Inductances*. Leningrad: Energoatomizdat. (in Russian).

Leontovich, M. A., & Levin, M. L. (1944). On the theory of oscillation excitation in linear radiators. *Journal of Technical Physics*, *14*(9), 481-506. (in Russian).

Levin, B. M. (2025). *Antennas. From the Theory of Long Lines to Integral Equations*. London, New York: CRC Press.

Miller, M. A. (1954). Application of homogeneous boundary conditions to the theory of thin antennas. *Journal of Technical Physics*, *24*(8), 1483-1495. (in Russian).

Vainshtein, L. A. (1959-1961). Waves of a current in a thin cylindrical conductor." *Journal of Technical Physics*, *29*(6), 673-699, and *31*(1), 29-50. (in Russian).