Applied Science and Innovative Research
ISSN 2474-4972 (Print) ISSN 2474-4980 (Online)
Vol. 10, No. 1, 2026
www.scholink.org/ojs/index.php/asir

Original Paper

Input Impedances of Symmetrical Loop Antennas

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Abstract

The application of integral equations for currents in symmetrical loop antennas of arbitrary length and different form (circular, rectangular and another short-circuited forms) allows us to substantially raise the calculation's exactness of electrical characteristics for the appointed antennas' versions and to be convinced that the received result is substantially distinguished from the characteristics of small antennas.

Keywords

antenna theory, integral equation, symmetrical loop antenna

1. Introduction

Properties of small symmetrical loop antennas are widely known. Unfortunately, if their dimensions are comparable with the wavelength, the calculation of electrical characteristics becomes complicated. As in the case of rectilinear radiator, the problem's solution requires to employ the integral equations for currents. Such the equation for the circular loop antenna was written by professor A. Z. Fradin (1977), and their post-graduate students received numeral results (Bezkakotova, T. B., & Porivaev B. N., 1971). The applicated method didn't take into consideration the cross-section current of a loop's wire, and the author confirmed that this circumstance is the method's drawback. But the account of a cross-section current weakly changes result. The field of this current is essential in the case of direct antenna's wire and is small in comparison with the currents' fields in the structure of the several wires, located at an angle to each other.

The principal drawback of this method was another: calculation was very complicated and required to use the mainframe computer. The series for current diverges, and the current's magnitude is considered as the sum of a finite number of series' members.

The new method of an analysis was suggested in Levin, B. M. (2025a). It is based on the calculation of an additional emf, created in the excitation point of a loop antenna by the fields of currents, flowing along the antenna's wires, more exact along the left and right sections of an upper branch of a wire. The sum of additional emfs substantially reduces the main (extraneous) emf. The sum of created at that series for a current is calculated by means of an infinite geometric progression (Levin, B. M., 2025b).

This method was checked during terms' summation for the current in the solution of Leontovich's integral equation. It is need to mark that the current's series (the solution of this equation) is actually the total sum of two series (active and reactive components of current). These series don't connect with each other. Every term of each series is equal to preceding one, which is multiplied to χ. Therefore in the capacity of the first term of a common series it is expedient to take the sum of first terms of both series. That allows us to use the properties of an infinite geometric progression. And as it was demonstrated in Levin, B. M. (2025a), the new method may be applied to the different versions of loop radiators. In the first place we will consider the result of its application to the circular loop.

2. Circular Loop

The circuit of an antenna is given in Figure 1. An antenna consists of the left and right branches, connected with each other in the upper point. The current of each branch is distributed in accordance with cosine fashion:

$$J_A = J_0 \cos kR\theta.$$

Here $k = 2\pi/\lambda$, λ is the wave length. The length of each wire is equal to $L=\pi R$. The input impedance of an antenna is

$$Z_A = jW\, tankL + R_\Sigma,$$

where $W = j\omega\Lambda$ is the antenna's wave impedance, ω is the circular frequency, Λ is the inductance per unit of wire length, R_Σ is the radiation's resistance of an antenna. If, for example, $L= \lambda/4$, then $R = L/\pi = \lambda/(4\pi)$. Accordingly the excitement's emf is equal to

$e = I_A Z_A = J_A\ (jW\, tankL + R_\Sigma)$.

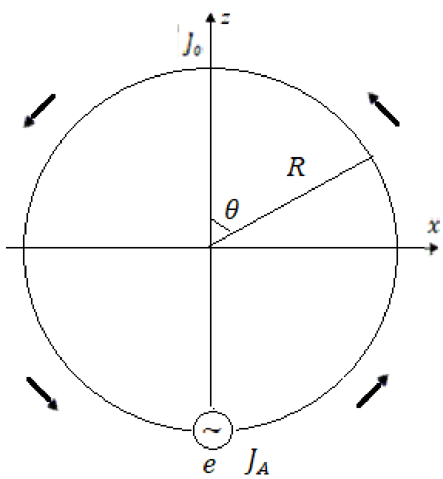

Figure 1. Circular Loop Antenna

In the book (Levin, B. M., 2025b) (chapter 4) it is shown that the effective length of the circular loop antenna is equal to

$$h_e = \frac{4kR^2}{1 - k^2R^2} \tan kL.$$

At that the antenna pattern in a horizontal plane has a shape of figure-eight. This means that the radiation's resistance of the loop antenna is half the radiation's resistance of a vertical linear antenna with the same effective height. If, for example, the wave impedance of the loop antenna is equal to W, then the reactive component of an antenna's input impedance, when $kL = \pi/4$ (at that $\lambda = 8L = 8\pi R, kR = 0.25$), is $X_A = jW$. In this case the resistance of a loop is

$R_A = 10k^2h_e^2 = 10 \cdot 16: 16^2 \cdot \frac{16^2}{15^2} = \frac{320}{225} \approx 0.71\ ohm.$

It is useful to compare this resistance with the magnitude, received by the strict method with application of the mainframe computer. Figure 2 demonstrates the curves, presented in the book (Fradin, A. Z., 1977). They show the current distribution along the loop antenna depending on the propagation constant k. The shape of curve depends from the wave impedance, the radiation resistance and the load. The total resistance changes approximately from W/10 до W, the propagation constant changes accordingly from 0.1 to 1. Unfortunately, Figure 2 don't allow to determine the magnitude of radiation resistance, since it is small in comparison with W=10, when the propagation constant is equal to 1.

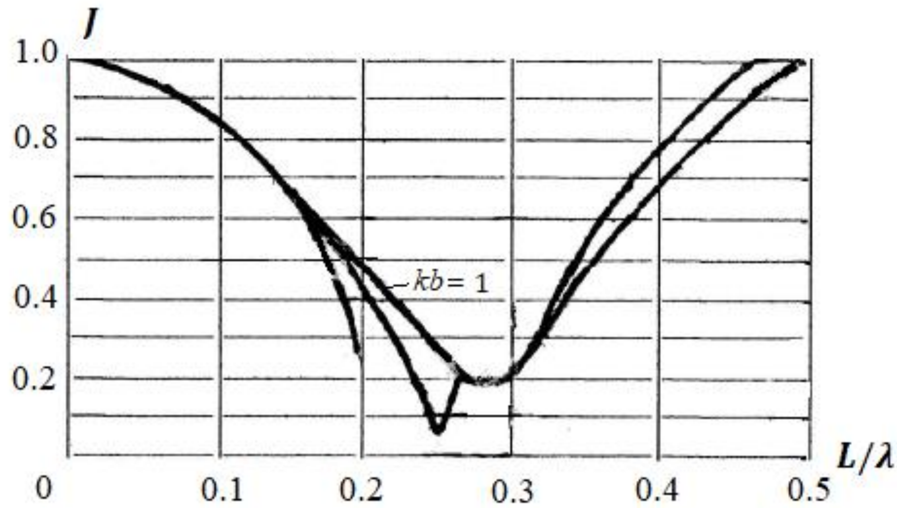

Figure 2. Current Distribution along the Loop Antenna

As it was shown in Levin, B. M. (2025a), the current on the upper conductor of a circular loop antenna creates in the point of excitement the additional emf of an opposite direction

E_1=- j30J_0 $\sin\left(\frac{kR\pi}{2}\right)\frac{exp(-j2kR)}{k^2R^2}$.

This result brings to the infinitely descending geometric progression with denominator

$q_1 = \frac{E_1}{e} = -\frac{30\sin\left(\frac{kR\pi}{2}\right)e^{-j2kR}}{k^2R^2\,(\omega\Lambda\tan kL - jR_\Sigma)\cos kL}$

and the total sum $e_{\Sigma 1} = \frac{e}{1-q_1}$.

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3. Other Forms of Loops

As it follows from the said, the additional emf in a circular loop arises as result of the same direction of currents along the left and right sections of an upper loop's wire. Symmetrical *T* -antenna (Figure 3a) and asymmetrical *Γ*-antenna (Figure 3b) don't have this effect, since the horizontal currents don't create the vertical fields.

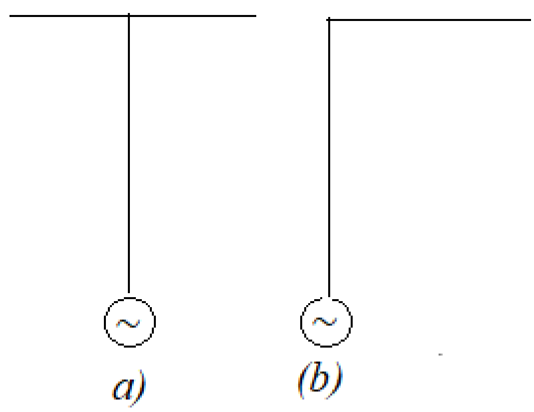

Figure 3. T-antenna (a) and Γ-antenna (b)

The currents of upper wires in rectangular wires, as is shown in Levin, B. M. (2025a), create the additional emf in the excitation point. The fields of inclined wires are proportional to cosine of the inclination angle. Therefore, during analysis of radiators' characteristics in triangular antennas (Figure 4a) and antennas of compound form (Figure 4b) one must to take these fields into account.

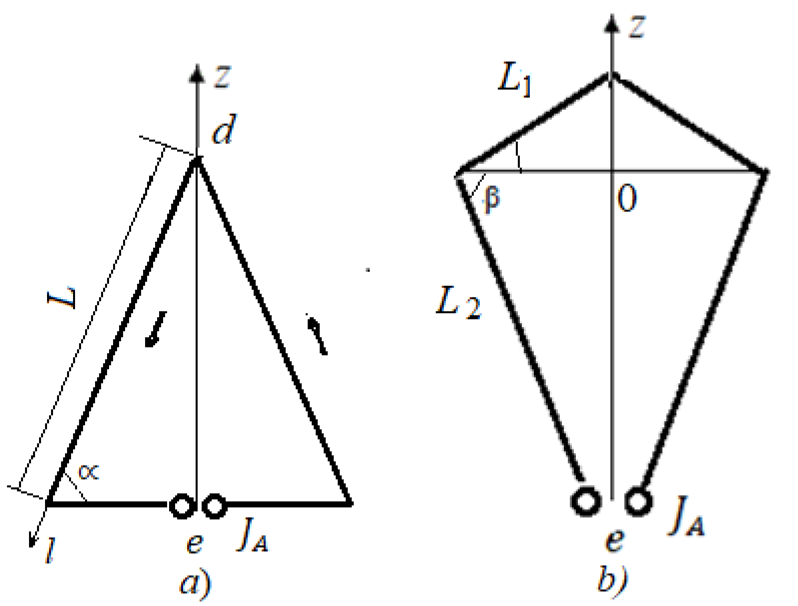

Figure 4. Triangular Antenna (a) and Antenna of Compound Form (b)

In the case of triangular antenna, the currents of the left and right branches create in opposite wires the fields, which compensate one another. Also, the current $J(l) = J_0 \cos kl$ of the left branch, located in a point at a height z= $d - l \sin \alpha$, creates in the antenna's excitation point the field

$\delta E = -j30J(l)\frac{e^{-jk(L-l)\sin\alpha}}{k(L-l)\sin\alpha}\cos\alpha\,\delta l.$

The total additional field, created by both branches, is equal to

$$E_2 = -j60\int_0^L J(l)\frac{e^{-jk(L-l)\sin\alpha}}{k(L-l)\sin\alpha}\cos\alpha\, dl.$$

The magnitudes e and E_2 are the first and second term of a geometric progression with the denominator $q_2 = E_2/e$ and the total sum (total emf $e_{\Sigma 2}$)

$$e_{\Sigma 2} = \frac{e}{1-q_2},$$

where $q_2 = \frac{E_2}{e}$.

Similarly for the antenna of compound form one can to receive in the first approximation the field in the wire, which creates the additional emf of an opposite direction. This field also decreases extraneous emf. As it is obvious from Figure 5, the distance between the upper wire and parallel wire of the same length in the point of contacts with the right wire is equal to $\frac{d_1-d_2}{\cos\alpha}$. By means of the successive indication of angles one can show, that the angle between the parallel wire and a horizontal is equal to α, and the angle between the parallel wire and the right wire is $\beta-\alpha$, i.e., the field, created in the lower wire is equal to

$E_3 = j60\int_0^L I_0 \cos kl \; \exp\left(-jk\frac{d_1-d_2}{\cos\alpha}\right)\cos(\beta-\alpha)dl.$

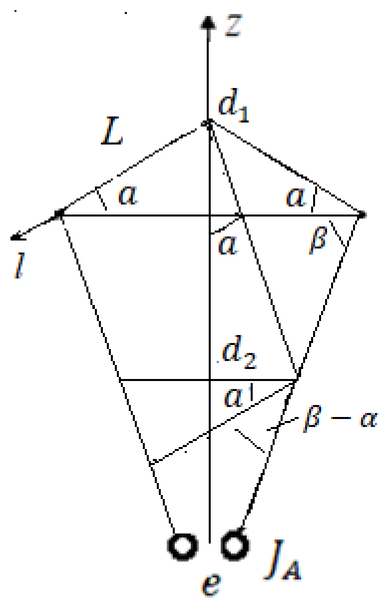

Figure 5. Angles between Wires of an Antenna with Compound Form

The magnitudes e $=J_A(jW\ tankL+R_\Sigma)$ and E_3 are the first and second terms of a geometric progression with the denominator $q_3=E_3/e$ and the total sum

$$e_{\Sigma 3}=\frac{e}{1-q_3},$$

where $q_3=\frac{E_3}{e}$.

In the case of a rectangular loop antenna (Figure 6), as it is shown in Levin, B. M. (2025a), the current of an upper horizontal wire creates the additional field

$$E_4=-j60\frac{J_0e^{-jkL}}{kL}\int_0^{b/2}\cos kxdx,$$

i.e., $e_{\Sigma 4}=\frac{e}{1-q_4}$, where $q_4=\frac{E_4}{e}$.

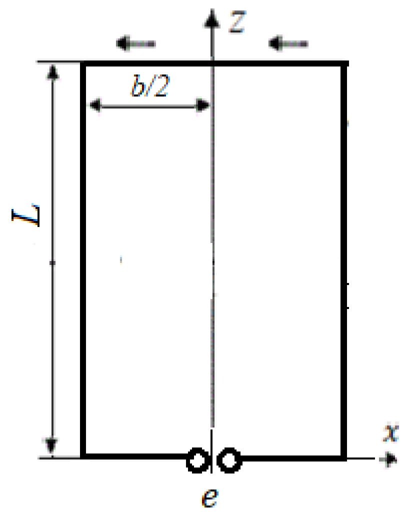

Figure 6. Rectangular Loop Antenna

4. Conclusion

As follows from the received results, the described method of a calculation may be applied to the close wire antennas of an arbitrary shape. The problem's solution allows us to obtain the series for the currents and emf with signs, varying from one member to another one, which form the infinite geometric progressions.

The calculation shows that the fields of loop antennas substantially depend from the angle φ and to a lesser extent from the radiator's radius. The change of wire's thickness has a weak influence on antenna's characteristics, since a wire's radius is small in comparison with the cross dimension of a loop. Stock-taking of these factors permits to simplify the analysis of electrical characteristics. Professor M. A. Leontovich widely used corresponding approach to considered problem, understanding that the majority of appeared mistakes is obvious less than the rough error, caused by use of $\delta-$ function in the capacity of an extraneous emf. In the process of the loop antenna's analysis, the all assumptions, used in the derivation of Leontovich's equation, are correct. One can consider, that the loop's current is axial wire's filament, which follows along wire's axis and depends only on a longitudinal coordinate, i.e. on the angle φ. Application of these assumption substantially simplifies the problem. In particular, in determination of the sum of a currents' series, one can to replace the variational calculus by a method, based on the properties of an infinite geometric progression. The result, received during the solution of a simple problem, confirm the famous opinion of Laplace: "Mathematical mill grinds very fine, but the quality of obtained flour depends from an initial product".

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