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Original Paper

How to Deduce the Other 91 Valid Aristotelian Modal Syllogisms from the Syllogism □I□A□I-3

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Abstract

This paper firstly formalizes Aristotelian modal syllogisms by taking advantage of the trisection structure of (modal) categorical propositions. And then making full use of the truth value definition of (modal) categorical propositions, the transformable relations between an Aristotelian quantifier and its three negative quantifiers, the reasoning rules of classical propositional logic, and the symmetry of the two Aristotelian quantifiers (i.e. some and no), this paper shows that at least 91 valid Aristotelian modal syllogisms can be deduced from □I□A□I-3 on the basis of possible world semantics and set theory. The reason why these valid Aristotelian modal syllogisms can be reduced is that any Aristotelian quantifier can be defined by the other three Aristotelian quantifiers, and the necessary modality (□) and possible modality (◇) can also be defined mutually. This research method is universal. This innovative study not only provides a unified mathematical research paradigm for the study of generalized modal syllogistic and other kinds of syllogistic, but also makes contributions to knowledge representation and knowledge reasoning in computer science.

Keywords

Aristotelian modal syllogisms, trisection structure, reduction, possible world, Aristotelian quantifiers

1. Introduction

Syllogistic reasoning plays an important role in the production and life of human society, so it has become one of the focuses in logic. There are many kinds of syllogisms, such as Aristotelian syllogisms (Patzig, 1969; Moss, 2008), generalized syllogisms (Murinová & Novák, 2012; Endrullis et al., 2015), relational syllogisms (Ivanov, 2012; Pratt-Hartmann, 2014), syllogisms with verbs (Moss, 2010), Aristotelian modal syllogisms (Johnson, 2004), and generalized modal syllogisms, and so on (Zhang, 2020a). Up to now, the most studied is Aristotelian syllogisms. So far, the author has not searched the literature of generalized modal syllogisms. Unless otherwise specified, the following modal syllogisms in this paper refer to Aristotelian modal syllogisms.

Aristotle was the first logician to systematically study modal syllogisms. In *Organon*, he discussed modal syllogisms more than non-modal syllogisms. After that, Łukasiewicz (1957), McCall (1963), Geach (1964), Johnson (1989) and others have discussed modal syllogisms and made some progress, but a formal axiom system of Aristotelian modal syllogistic has not yet been established. Thomason (1993, 1997), Thom(1996), Johnson (2004), Malink (2006, 2013) gave adequate semantic analysis or reconstruction of the syllogistic, but the prevailing view is that there are many faults and inconsistencies (Malink, 2006, p. 95). When studying this syllogistic, the difficulty lies in how to maintain its consistency. Therefore, this paper attempts to deduce the validity of other syllogisms by means of reduction on the basis of proving the validity of one Aristotelian modal syllogism (i.e. □I□A□I-3). This research method can well ensure the consistency of this logic. More specifically, this paper discusses the reduction between the Aristotelian modal syllogism □I□A□I-3 and other valid Aristotelian modal syllogisms.

In the syllogistic, the proposition "all *X*s are *Z*" is denoted by *all*(*X*, *Z*), and called by Propositions A; "no *X*s are *Z*" is denoted by *no*(*X*, *Z*), and called by Propositions E; "some *X*s are *Z*" is denoted by *some*(*X*, *Z*), and called by Propositions I; "not all *X*s are *Z*" is denoted by *not all*(*X*, *Z*), and called by Propositions O, in which *X* and *Z* represent (the set of) lexical variables. The symbols used in this paper are the basic symbols in modal logic (Chagrov, 1997) and set theory, for example, $\neg$, $\wedge$, $\rightarrow$, $\leftrightarrow$, □, and ◇ are symbols of negation, conjunction, conditionality, biconditionality, necessity, and possibility, respectively.

Since any premise and conclusion of Aristotelian modal syllogisms can be any of the 12 propositions A, E, I, O, □A, □E, □I, □O, ◇A, ◇E, ◇I and ◇O, and the middle term has four different positions (that is, four figures), there are (12×12×12×4−256=) 6656 Aristotelian modal syllogisms in natural language, of which 256 represents the number of Aristotelian syllogisms. Zhang (2020b) screened out 384 valid syllogisms from 6656 Aristotelian modal syllogisms. Through the way of reduction, this paper profoundly reveals the logical relations between/among 91 valid Aristotelian modal syllogisms.

2. Preliminaries

In the following propositions, *X*, *Y* and *Z* represent (the set of) lexical variables (subjects or predicates) involved in categorical propositions or modal categorical propositions. The definition of figures of Aristotelian modal syllogisms are similar to those of Aristotelian syllogisms.

Aristotelian modal syllogisms can be seen as extended syllogisms obtained by adding necessary modality □ or possible modality ◇ to Aristotelian syllogisms. Aristotelian syllogisms represent the semantic and reasoning properties of the four Aristotelian quantifiers (that is, *all*, *some*, *no* and *not all*). Similarly, Aristotelian modal syllogisms are syllogisms that just contain the four Aristotelian quantifiers, and at least contain one necessary modality (□) or possible modality (◇).

Example 1:

Major premise: Some apples are necessarily green apples.

Minor premise: All apples are necessarily fruits.

Conclusion: Some green apples are necessarily fruits.

The syllogism contains the Aristotelian quantifiers *some* and *all*, and contains three necessary modalities (□), and the middle term is the subject of the major and minor premise, so it is an Aristotelian modal syllogism, and the third figure. Let X is the set of green apples in a given domain, Y is the set of apples in the domain, and Z is the set of fruits in the domain. Example 1 of the third figure syllogism can be formalized by $\Box some(Y, Z) \wedge \Box all(Y, X) \Rightarrow \Box some(X, Z)$, abbreviated by □I□A□I-3. In all of the syllogisms in this article, X, Y and Z represent the minor, medium and major terms of syllogisms respectively.

The formalization of other modal syllogisms are similar to that of this syllogism. In fact, all modal syllogisms can be formalized into the forms similar to $Q_1(X, Y) \wedge Q_2(Y, Z) \Rightarrow Q_3(X, Z)$, where Q_1, Q_2 and Q_3 can be any of the following 12 generalized quantifiers: *all*, *some*, *no*, *not all*, □*all*, □*some*, □*no*, □*not all*, ◇*all*, ◇*some*, ◇*no* and ◇*not all*.

2.1 Relevant Definitions

According to the generalized quantifier theory (Peters & Westerståhl, 2006), set theory and possible world semantics (Chellas, 1980), one can give the truth value definition of the following propositions:

Definition 1 (truth value definition):

(1) $all(X, Z)$ is true if and only if $X \subseteq Z$ is true.

(2) $some(X, Z)$ is true if and only if $X \cap Z \neq \varnothing$ is true.

(3) $no(X, Z)$ is true if and only if $X \cap Z = \varnothing$ is true.

(4) $not\ all(X, Z)$ is true if and only if $X \not\subseteq Z$ is true.

(5) $\Box all(X, Z)$ is true if and only if $X \subseteq Z$ is true in any possible world.

(6) $\Diamond all(X, Z)$ is true if and only if $X \subseteq Z$ is true in at least one possible world.

(7) $\Box some(X, Z)$ is true if and only if $X \cap Z \neq \varnothing$ is true in any possible world.

(8) $\Diamond some(X, Z)$ is true if and only if $X \cap Z \neq \varnothing$ is true in at least one possible world.

(9) $\Box no(X, Z)$ is true if and only if $X \cap Z = \varnothing$ is true in any possible world.

(10) $\Diamond no(X, Z)$ is true if and only if $X \cap Z = \varnothing$ is true in at least one possible world.

(11) $\Box not\ all(X, Z)$ is true if and only if $X \not\subseteq Z$ is true in any possible world.

(12) $\Diamond not\ all(X, Z)$ is true if and only if $X \not\subseteq Z$ is true in at least one possible world.

In the following, D stands for the domain of lexical variables, Q for any of the four Aristotelian quantifiers (that is, *all*, *some*, *no* and *not all*), $\neg Q$ and $Q\neg$ for the outer and inner negative quantifier of Q, respectively. And the symbol $=_{def}$ indicates that the left can be defined by the right.

Definition 2 (inner negation): $Q\neg(X, Z) =_{def} Q(X, D-Z)$.

Definition 3 (outer negation): $\neg Q(X, Z) =_{def}$ It is not that $Q(X, Z)$.

2.2 Relevant Facts

Fact 1 (inner negation for Aristotelian quantifiers)

(1) $all(X, Z)=no\neg(X, Z)$; (2) $no(X, Z)=all\neg(X, Z)$;

(3) $some(X, Z)=not\ all\neg(X, Z)$; (4) $not\ all(X, Z)=some\neg(X, Z)$.

Fact 1 can be easily proved by Definition 2.

Fact 2 (outer negation for Aristotelian quantifiers):

(1) $\neg not\ all(X, Z)=all(X, Z)$; (2) $\neg all(X, Z)=not\ all(X, Z)$;

(3) $\neg no(X, Z)=some(X, Z)$; (4) $\neg some(X, Z)=no(X, Z)$.

Fact 2 can be easily proved by Definition 3.

The necessary modality □ and possible modality ◇ are mutually dual. Then let $Q(X, Z)$ is a categorical proposition, then $\Diamond Q(X, Z)=_{def}\neg\Box\neg Q(X, Z)$ and $\Box Q(X, Z)=_{def}\neg\Diamond\neg Q(X, Z)$, hence, one can obtain the following Fact 3.

Fact 3: (1) $\neg\Box Q(X, Z)=\Diamond\neg Q(X, Z)$; (2) $\neg\Diamond Q(X, Z)=\Box\neg Q(X, Z)$.

Fact 4 (a necessary proposition implies an assertoric proposition): $\vdash\Box Q(X, Z)\rightarrow Q(X, Z)$.

Fact 5 (a necessary proposition implies a possible proposition): $\vdash\Box Q(X, Z)\rightarrow\Diamond Q(X, Z)$.

Fact 6 (a universal proposition implies a particular proposition):

(1) $\vdash all(X, Z)\rightarrow some(X, Z)$; (2) $\vdash no(X, Z)\rightarrow not\ all(X, Z)$.

Fact 7 (symmetry of *some* and *no*): (1) $some(X, Z)\leftrightarrow some(Z, X)$; (2) $no(X, Z)\leftrightarrow no(Z, X)$.

All of the above facts are the general knowledge of generalized quantifier theory (Zhang, 2018) or modal logic (Zhang, 2020b), thus their proofs are omitted here.

2.3 Relevant Inference Rules

Aristotelian modal syllogistic is an extension of classical propositional logic (Hamilton, 1978), and the reasoning rules of the latter are also applicable to the former. The following rules are the basic rules of propositional logic, in which *p, q, r* and *s* are propositions.

(1) Rule 1 (subsequent weakening): If $\vdash(p\wedge q\rightarrow r)$ and $\vdash(r\rightarrow s)$, then $\vdash(p\wedge q\rightarrow s)$.

(2) Rule 2 (anti-syllogism): If $\vdash(p\wedge q\rightarrow r)$, then $\vdash(\neg r\wedge p\rightarrow\neg q)$ or $\vdash(\neg r\wedge q\rightarrow\neg p)$.

3. Reduction between the Modal Syllogism □I□A□I-3 and Other Valid Modal Syllogisms

There is reducibility between valid Aristotelian syllogisms of different figures and forms, similarly, there is also reducibility between valid Aristotelian modal syllogisms of different figures and forms. According to the following Theorem 1, the modal syllogism □I□A□I-3 is valid, therefore, all the following syllogisms derived from this syllogism are valid. In the following Theorem 2, □I□A□I-3⇒□I□A□I-4 means that the validity of syllogism □I□A□I-4 can be deduced from the validity of syllogism □I□A□I-3, hence one can say that there is reducibility between the two syllogisms. Other cases are similar.

Theorem 1 (□I□A□I-3): $\Box some(Y, Z)\wedge\Box all(Y, X)\Rightarrow\Box some(X, Z)$ is valid.

Proof: According to Example 1, □I□A□I-3 is the abbreviation of the third figure syllogism □*some*(*Y*, *Z*)∧□*all*(*Y*, *X*)→ □*some*(*X*, *Z*). Suppose that □*some*(*Y*, *Z*) and □*all*(*Y*, *X*) are true, then $Y \cap Z \neq \varnothing$ and $Y \subseteq X$ is true at any possible world in terms of the clause (7) and (5) in Definition 1. Now it follows that $X \cap Z \neq \varnothing$ is true at every possible world. Hence □*some*(*X*, *Z*) is true according to the clause (7) in Definition 1 again. This proves the claim that the third figure syllogism □*some*(*Y*, *Z*)∧□*all*(*Y*, *X*)→□*some*(*X*, *Z*) is valid, just as desired.

Theorem 2: The following valid modal syllogisms can be deduced from □I□A□I-3:

(2.1) □I□A□I-3⇒□I□AI-3

(2.2) □I□A□I-3⇒□I□A◇I-3

Proof: For (2.1). As pointed out earlier, □I□A□I-3 is valid, which is the abbreviation of the modal syllogism □*some*(*Y*, *Z*)∧□*all*(*Y*, *X*)→□*some*(*X*, *Z*). According to Fact 4: □*some*(*X*, *Z*)⇒*some*(*X*, *Z*). Then on the basis of Rule 1, it follows that □*some*(*Y*, *Z*)∧□*all*(*Y*, *X*)→*some*(*X*, *Z*) from □I□A□I-3. In other words, the modal syllogism □I□AI-3 is valid, as required. The proof of (2.2) is similar to that of (2.1), except that it needs to be based on Fact 5 instead of Fact 4.

Theorem 3: The following valid modal syllogisms can be deduced from □I□A□I-3:

(3.1) □I□A□I-3⇒□O□A□O-3⇒□O□AO-3⇒□O□A◇O-3

(3.2) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3

Proof: For (3.1). According to the clause (3) in Fact 1, *some*(*Y*, *Z*)=*not all*¬(*Y*, *Z*), and *some*(*X*, *Z*)=*not all*¬(*X*, *Z*), it follows that □*not all*¬(*Y*, *Z*)∧□*all*(*Y*, *X*)→□*not all*¬(*X*, *Z*) from □I□A□I-3. According to Definition 2, *not all*¬(*Y*, *Z*)=*not all*(*Y*, *D*–*Z*), *not all*¬(*X*, *Z*)=*not all*(*X*, *D*–*Z*). Therefore, one can derive that □*not all*(*Y*, *D*–*Z*)∧□*all*(*Y*, *X*)→□*not all*(*X*, *D*–*Z*). In other words, the syllogism □O□A□O-3 can be deduced from □I□A□I-3, just as desired. With the help of Fact 4 and Fact 5, it is easily seen that □O⇒O and □O⇒◇O, hence (3.1) can be proved according to Rule 1. The proof of (3.2) is similar to that of (3.1). In other words, 3.2 can be proved with the help of the above facts and rules.

Theorem 4: The following valid modal syllogisms can be derived from □I□A□I-3:

(4.1) □I□A□I-3⇒◇E□I◇O-2

(4.2) □I□A□I-3⇒□I□AI-3⇒E□I◇O-2

(4.3) □I□A□I-3⇒□I□A◇I-3⇒□E□I◇O-2

(4.4) □I□A□I-3⇒□O□A□O-3⇒◇A□O◇O-2

Proof: For (4.1). □I□A□I-3 is valid, and its expansion is that □*some*(*Y*, *Z*)∧□*all*(*Y*, *X*)→□*some*(*X*, *Z*). According to Rule 2, it can be seen that ¬□*some*(*X*, *Z*)∧□*some*(*Y*, *Z*)→¬□*all*(*Y*, *X*). In line with the clause (1) in Fact 3, it follows that ◇¬*some*(*X*, *Z*)∧□*some*(*Y*, *Z*)→◇¬*all*(*Y*, *X*). With the help of the clause (2) and (4) in Fact 2, i.e., ¬*some*(*X*, *Z*)=*no*(*X*, *Z*) and ¬all(*Y*, *X*)=not all(*Y*, *X*), one can deduce that ◇*no*(*X*, *Z*)∧□*some*(*Y*, *Z*)→◇*not all*(*Y*, *X*). Therefore, ◇E□I◇O-2 can be derived from □I□A□I-3, just as required. Others can be similarly proved on the basis of the above facts and rules.

Theorem 5: The following valid modal syllogisms can be followed from □I□A□I-3:

(5.1) □I□A□I-3⇒◇E□A◇E-1

(5.2) □I□A□I-3⇒□I□AI-3⇒E□A◇E-1

(5.3) □I□A□I-3⇒□I□A◇I-3⇒□E□A◇E-1

(5.4) □I□A□I-3⇒□O□A□O-3⇒◇A□A◇A-1

Proof: For (5.1). The expansion of □I□A□I-3 is that □*some*(*Y*, *Z*)∧□*all*(*Y*, *X*)→□*some*(*X*, *Z*). According to Rule 2, it follows that ¬□*some*(*X*, *Z*)∧□*all*(*Y*, *X*)→¬□*some*(*Y*, *Z*). In terms of the clause (1) in Fact 3, it can be seen that ◇¬*some*(*X*, *Z*)∧□*all*(*Y*, *X*)→◇¬*some*(*Y*, *Z*). According to the clause (4) in Fact 2, ¬*some*(*X*, *Z*)=*no*(*X*, *Z*) and ¬some(*Y*, *Z*)=no(*Y*, *Z*). Therefore, the syllogism ◇*no*(*X*, *Z*)∧□*all*(*Y*, *X*)→◇*no*(*Y*, *Z*) is valid. In other words, ◇E□A◇E-1 can be derived from □I□A□I-3. So far, the proof of (5.1) has been completed. Others can be similarly proved.

Theorem 6: The following valid modal syllogisms can be derived from □I□A□I-3:

(6.1) □I□A□I-3⇒□O□A□O-3⇒□O□AO-3⇒A□O◇O-2

(6.2) □I□A□I-3⇒□O□A□O-3⇒□O□A◇O-3⇒□A□O◇O-2

(6.3) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒◇A□E◇E-2

Proof: For (6.1). According to (3.1) □I□A□I-3⇒□O□A□O-3⇒□O□AO-3⇒□O□A◇O-3, it follows that □O□AO-3 is valid, and its expansion is that □*not all*(*Y*, *Z*)∧□*all*(*Y*, *X*)→*not all*(*X*, *Z*). According to Rule 2, it can be seen that ¬*not all*(*X*, *Z*)∧□*not all*(*Y*, *Z*)→¬□*all*(*Y*, *X*). In the light of the clause (1) in Fact 3, one can deduce that ¬*not all*(*X*, *Z*)∧□*not all*(*Y*, *Z*)→◇¬*all*(*Y*, *X*). In terms of the clause (1) in Fact 2, it follows that ¬*not all*(*X*, *Z*)=*all*(*X*, *Z*) and ¬all(*Y*, *X*)=not all(*Y*, *X*). Hence, the syllogism *all*(*X*, *Z*)∧□*not all*(*Y*, *Z*)→◇*not all*(*Y*, *X*) is valid. That is to say that A□O◇O-2 can be derived from □I□A□I-3, the proof of (6.1) has been completed. The proofs of (6.2) and (6.3) are similar to that of (6.1).

Theorem 7: The following valid modal syllogisms can be obtained from □I□A□I-3:

(7.1) □I□A□I-3⇒□O□A□O-3⇒□O□AO-3⇒A□A◇A-1

(7.2) □I□A□I-3⇒□O□A□O-3⇒□O□A◇O-3⇒□A□A◇A-1

(7.3) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒◇A□I◇I-1

Proof: For (7.1). According to (3.1), □O□AO-3 is valid, and its expansion is that □*not all*(*Y*, *Z*)∧□*all*(*Y*, *X*)→*not all*(*X*, *Z*). According to Rule 2, one can derive that ¬*not all*(*X*, *Z*)∧□*all*(*Y*, *X*)→¬□*not all*(*Y*, *Z*). In the light of the clause (1) in Fact 3, it follows that ¬*not all*(*X*, *Z*)∧□*all*(*Y*, *X*)→◇¬*not all*(*Y*, *Z*). In terms of Fact 2, it can be seen that ¬*not all*(*X*, *Z*)=*all*(*X*, *Z*) and ¬*not all*(*Y*, *Z*)=*all*(*Y*, *Z*). Therefore, the syllogism *all*(*X*, *Z*)∧□*all*(*Y*, *X*)→◇*all*(*Y*, *Z*) is valid. That is, A□A◇A-1 can be derived from □I□A□I-3, just as required. The two others can be similarly proved.

Theorem 8: The following valid modal syllogisms can be deduced from □I□A□I-3:

(8.1) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒□E□IO-3

(8.2) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒□E□I□O-4⇒□E□IO-4

(8.3) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒□E□I□O-1⇒□E□IO-1

(8.4) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒□E□I□O-2⇒□E□IO-2

Proof: For (8.1). According to (3.2) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3, it follows that

□E□I□O-3 is valid. According to Fact 4, it is easily seen that □O⇒O, so □E□IO-3 is valid. In other words, it can be derived from □I□A□I-3. Therefore, the proof of (8.1) has been completed. Others can be similarly proved.

Theorem 9: The following valid modal syllogisms can be inferred from □I□A□I-3:

(9.1) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒□E□IO-3⇒A□I◇I-1

(9.2) □I□A□I-3⇒◇E□A◇E-1⇒◇E□A◇O-1⇒□A□A□I-3

(9.3) □I□A□I-3⇒□O□A□O-3⇒◇A□A◇A-1⇒◇A□A◇I-1⇒□E□A□O-3

Proof: For (9.1). According to (8.1) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒□E□IO-3, it follows that □E□IO-3 is valid, and its expansion is that □*no*(*Y*, *Z*)∧□*some*(*Y*, *X*)→*not all*(*X*, *Z*). According to Rule 2, it can be derived that ¬*not all*(*X*, *Z*)∧□*some*(*Y*, *X*)→¬□*no*(*Y*, *Z*). in line with the clause (1) in Fact 3, one can deduce that ¬*not all*(*X*, *Z*)∧□*some*(*Y*, *X*)→◇¬*no*(*Y*, *Z*). In terms of the clause (1) and (3) in Fact 2, it follows that ¬*not all*(*X*, *Z*)=*all*(*X*, *Z*) and ¬*no*(*Y*, *Z*)=*some*(*Y*, *Z*). Thus, the syllogism *all*(*X*, *Z*)∧□*some*(*Y*, *X*)→◇*some*(*Y*, *Z*) is valid. That is, A□I◇I-1 can be derived from □I□A□I-3, as required. The two others can be similarly proved with the help of the above theorems, facts and rules.

Theorem 10: The following valid modal syllogisms can be deduced from □I□A□I-3:

(10.1) □I□A□I-3⇒□I□A□I-4

(10.2) □I□A□I-3⇒□A□I□I-3

(10.3) □I□A□I-3⇒□A□I□I-1

(10.4) □I□A□I-3⇒□I□A◇I-3⇒□I□A◇I-4

(10.5) □I□A□I-3⇒□I□A◇I-3⇒□A□I◇I-3

(10.6) □I□A□I-3⇒□I□A◇I-3⇒□A□I◇I-1

(10.7) □I□A□I-3⇒□I□AI-3⇒□I□AI-4

(10.8) □I□A□I-3⇒□I□AI-3⇒□A□II-3

(10.9) □I□A□I-3⇒□I□AI-3⇒□A□II-1

(10.10) □I□A□I-3⇒◇E□I◇O-2⇒◇E□I◇O-4

(10.11) □I□A□I-3⇒◇E□I◇O-2⇒◇E□I◇O-1

(10.12) □I□A□I-3⇒◇E□I◇O-2⇒◇E□I◇O-3

(10.13) □I□A□I-3⇒◇E□A◇E-1⇒◇E□A◇E-2

(10.14) □I□A□I-3⇒◇E□A◇E-1⇒□A◇E◇E-4

(10.15) □I□A□I-3⇒◇E□A◇E-1⇒□A◇E◇E-2

Proof: For (10.1). According to Theorem 1, □I□A□I-3 is valid, and its expansion is that □*some*(*Y*, *Z*)∧□*all*(*Y*, *X*)→□*some*(*X*, *Z*). According to the clause (1) in Fact 7, it can be seen that □*some*(*Y*, *Z*)↔□*some*(*Z*, *Y*). Hence, it follows that □*some*(*Z*, *Y*)∧□*all*(*Y*, *X*)→□*some*(*X*, *Z*). That is to say that □I□A□I-4 can be derived from □I□A□I-3, the proof of (10.1) has been completed. The remaining syllogisms can be similarly deduced from □I□A□I-3.

Theorem 11: The following valid modal syllogisms can be obtained from □I□A□I-3:

(11.1) □I□A□I-3⇒□O□A□O-3⇒□O□A◇O-3⇒□A□A◇A-1⇒□A□A◇I-1⇒

□A□A◇I-4

(11.2) □I□A□I-3⇒□O□A□O-3⇒□O□AO-3⇒A□A◇A-1⇒A□A◇I-1⇒ □AA◇I-4

(11.3) □I□A□I-3⇒□O□A□O-3⇒◇A□A◇A-1⇒◇A□A◇I-1⇒□E□A□O-3⇒ □E□A□O-4

Proof: For (11.1). According to (10.6) □I□A□I-3⇒□I□A◇I-3⇒□A□I◇I-1, □A□A◇I-1 is valid, and its expansion is that □*all*(*Y*, *Z*)∧□*all*(*X*, *Y*)→◇*some*(*X*, *Z*). In terms of the clause (1) in Fact 7, it can be seen that □*some*(*X*, *Z*)↔□*some*(*Z*, *X*). Therefore, one can easily derive that □*all*(*X*, *Y*)∧□*all*(*Y*, *X*)→□*some*(*Z*, *X*). In other words, □A□A◇I-4 can be derived from □I□A□I-3, just as required. Others can be similarly proved.

Theorem 12: The following valid modal syllogisms can be obtained from □I□A□I-3:

(12.1) □I□A□I-3⇒□I□AI-3⇒E□I◇O-2⇒E□I◇O-4

(12.2) □I□A□I-3⇒□I□AI-3⇒E□I◇O-2⇒E□I◇O-1

(12.3) □I□A□I-3⇒□I□AI-3⇒E□I◇O-2⇒E□I◇O-3

(12.4) □I□A□I-3⇒□I□AI-3⇒E□A◇E-1⇒E□A◇E-2

(12.5) □I□A□I-3⇒□I□AI-3⇒E□A◇E-1⇒□AE◇E-4

(12.6) □I□A□I-3⇒□I□AI-3⇒E□A◇E-1⇒□AE◇E-2

(12.7) □I□A□I-3⇒□I□A◇I-3⇒□E□I◇O-2⇒□E□I◇O-4

(12.8) □I□A□I-3⇒□I□A◇I-3⇒□E□I◇O-2⇒□E□I◇O-1

(12.9) □I□A□I-3⇒□I□A◇I-3⇒□E□I◇O-2⇒□E□I◇O-3

(12.10) □I□A□I-3⇒□I□A◇I-3⇒□E□A◇E-1⇒□E□A◇E-2

(12.11) □I□A□I-3⇒□I□A◇I-3⇒□E□A◇E-1⇒□A□E◇E-4

(12.12) □I□A□I-3⇒□I□A◇I-3⇒□E□A◇E-1⇒□A□E◇E-2

(12.13) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒□E□I□O-4

(12.14) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒□E□I□O-1

(12.15) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒□E□I□O-2

Proof: For (12.1). According to (4.2) □I□A□I-3⇒□I□AI-3⇒E□I◇O-2, E□I◇O-2 is valid, and its expansion is that *no*(*Z*, *Y*)∧□*some*(*X*, *Y*)→◇*not all*(*X*, *Z*). In the light of the clause (1) in Fact 7, it follows that □*some*(*X*, *Y*)↔□*some*(*Y*, *X*). Therefore, it is easily seen that *no*(*Z*, *Y*)∧□*some*(*Y*, *X*)→◇*not all*(*X*, *Z*). That is, E□I◇O-4 can be derived from □I□A□I-3, the proof of (12.1) has been completed. The others can be similarly followed from □I□A□I-3.

Theorem 13: The following valid modal syllogisms can be inferred from □I□A□I-3:

(13.1) □I□A□I-3⇒◇E□A◇E-1⇒◇E□A◇O-1

(13.2) □I□A□I-3⇒◇E□A◇E-1⇒◇E□A◇E-2⇒◇E□A◇O-2

(13.3) □I□A□I-3⇒◇E□A◇E-1⇒□A◇E◇E-4⇒□A◇E◇O-4

(13.4) □I□A□I-3⇒◇E□A◇E-1⇒□A◇E◇E-2⇒□A◇E◇O-2

(13.5) □I□A□I-3⇒□I□AI-3⇒E□A◇E-1⇒E□A◇O-1

(13.6) □I□A□I-3⇒□I□AI-3⇒E□A◇E-1⇒E□A◇E-2⇒E□A◇O-2

(13.7) □I□A□I-3⇒□I□AI-3⇒E□A◇E-1⇒□AE◇E-4⇒□AE◇O-4

(13.8) □I□A□I-3⇒□I□AI-3⇒E□A◇E-1⇒□AE◇E-2⇒□AE◇O-2

(13.9) □I□A□I-3⇒□I□A◇I-3⇒□E□A◇E-1⇒□E□A◇O-1

(13.10) □I□A□I-3⇒□I□A◇I-3⇒□E□A◇E-1⇒□E□A◇E-2⇒□E□A◇O-2

(13.11) □I□A□I-3⇒□I□A◇I-3⇒□E□A◇E-1⇒□A□E◇E-4⇒□A□E◇O-4

(13.12) □I□A□I-3⇒□I□A◇I-3⇒□E□A◇E-1⇒□A□E◇E-2⇒□A□E◇O-2

(13.13) □I□A□I-3⇒□O□A□O-3⇒◇A□A◇A-1⇒◇A□A◇I-1

(13.14) □I□A□I-3⇒□O□A□O-3⇒□O□AO-3⇒A□A◇A-1⇒A□A◇I-1

(13.15) □I□A□I-3⇒□O□A□O-3⇒□O□A◇O-3⇒□A□A◇A-1⇒□A□A◇I-1

(13.16) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒◇A□E◇E-2⇒◇A□E◇O-2

(13.17) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒◇A□E◇E-2⇒◇A□E◇E-4⇒ ◇A□E◇O-4

(13.18) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒◇A□E◇E-2⇒□E◇A◇E-2⇒ □E◇A◇O-2

(13.19) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒◇A□E◇E-2⇒□E◇A◇E-1⇒ □E◇A◇O-1

Proof: For (13.1). According to (10.14) □I□A□I-3⇒◇E□A◇E-1⇒□A◇E◇E-4, one can see that ◇E□A◇E-1 is valid. With the help of Fact 6, it follows that E⇒O, so ◇E□A◇O-1 is valid. In other words, ◇E□A◇O-1 can be derived from □I□A□I-3, as required. On the basis of the above theorems, facts and rules, all of the other syllogisms can be similarly deduced from □I□A□I-3.

Theorem 14: The following valid modal syllogisms can be derived from □I□A□I-3:

(14.1) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒◇A□E◇E-2⇒◇A□E◇E-4

(14.2) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒◇A□E◇E-2⇒□E◇A◇E-2

(14.3) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒◇A□E◇E-2⇒□E◇A◇E-1

(14.4) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒◇A□I◇I-1⇒◇A□I◇I-3

(14.5) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒◇A□I◇I-1⇒□I◇A◇I-4

(14.6) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒◇A□I◇I-1⇒□I◇A◇I-3

(14.7) □I□A□I-3⇒□O□A□O-3⇒◇A□A◇A-1⇒◇A□A◇I-1⇒□A◇A◇I-4

Proof: For (14.1). According to (13.19) □I□A□I-3⇒□A□I□I-3⇒□E□I□O-3⇒ ◇A□E◇E-2⇒□E◇A◇E-1⇒□E◇A◇O-1, it follows that ◇A□E◇E-2 is valid, and its expansion is that ◇*all*(Z, Y)∧□*no*(X, Y)→◇*no*(X, Z). in line with the clause (2) in Fact 7, it can be seen that □*no*(X, Y)↔□*no*(Y, X). Therefore, one can easily deduce that ◇*all*(Z, Y)∧□*no*(Y, X)→◇*no*(X, Z). That is, ◇A□E◇E-4 can be derived from □I□A□I-3. So far, the proof of (14.1) has been completed. The others can be similarly proved on the basis of the above theorems, facts and rules.

Theorem 15: The following valid modal syllogisms can be followed from □I□A□I-3:

(15.1) □I□A□I-3⇒□O□A□O-3⇒◇A□A◇A-1⇒◇A□A◇I-1⇒□E□A□O-3⇒ □E□AO-3

(15.2) □I□A□I-3⇒□O□A□O-3⇒◇A□A◇A-1⇒◇A□A◇I-1⇒□E□A□O-3⇒□E□A□O-4⇒□E□AO-4

(15.3) □I□A□I-3⇒◇E□A◇E-1⇒◇E□A◇O-1⇒□A□A□I-3⇒□A□AI-3

(15.4) □I□A□I-3⇒◇E□A◇E-1⇒◇E□A◇O-1⇒□A□A□I-3⇒□A□A◇I-3

(15.5) □I□A□I-3⇒□O□A□O-3⇒◇A□A◇A-1⇒◇A□A◇I-1⇒□E□A□O-3⇒□E□A◇O-3

(15.6) □I□A□I-3⇒□O□A□O-3⇒◇A□A◇A-1⇒◇A□A◇I-1⇒□E□A□O-3⇒□E□A□O-4⇒□E□A◇O-4

Proof: For (15.1). In terms of (11.3) □I□A□I-3⇒□O□A□O-3⇒◇A□A◇A-1⇒ ◇A□A◇I-1⇒□E□A□O-3⇒□E□A□O-4, it follows that □E□A□O-3 is valid. According to Fact 4, it is easily seen that □O⇒O, thus □E□AO-3 is valid. That is to say that □E□AO-3 can be derived from □I□A□I-3, just as required. Making the best of the above theorems, facts and rules, all of the other modal syllogisms can be inferred from □I□A□I-3.

4. Conclusion

Making full use of the truth value definition of (modal) categorical propositions, the transformable relations between an Aristotelian quantifier and their three negative quantifiers, the reasoning rules of classical propositional logic, and the symmetry of the two Aristotelian quantifiers (i.e. *some* and *no*), this paper shows that at least 91 valid Aristotelian modal syllogisms can be deduced from □I□A□I-3. The reason why the above 91 valid Aristotelian modal syllogisms can be reduced is that any Aristotelian quantifier can be defined by the other three Aristotelian quantifiers, and the necessary modality (□) and possible modality (◇) can also be defined mutually. This research method is universal. That is to say that other valid Aristotelian modal syllogisms can also be deduced similarly from some valid syllogisms other than □I□A□I-3, such as □I□A◇I-3, □I◇A◇I-3, and IA◇I-3.

All valid Aristotelian modal syllogisms obtained by adding modal operators to the valid Aristotelian syllogism IAI-3 are as follows: □I□A□I-3, □I□A◇I-3, □I◇A◇I-3, ◇I□A◇I-3, □I□AI-3, ◇IA◇I-3, I◇A◇I-3, □IA◇I-3, I□A◇I-3, □IAI-3, I□AI-3 and IA◇I-3. The validity of these syllogisms can be proved in the same way that Theorem 1 proves the validity of □I□A□I-3. If one can use the reduction method in this paper to derive all the other valid Aristotelian modal syllogisms from these 12 valid syllogisms, then it is possible to establish a formal axiom system for Aristotelian modal syllogistic. This issue needs further study.

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