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Original Paper

From Structure to Substance: A Systemwide K–8 Mathematics Review of Instructional Coherence, Content Knowledge, and Algebra Readiness in a Comprehensive TK–12 School District

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Abstract

This study presents findings from a systemwide K–8 mathematics review conducted in a K through 12 district of roughly 9000 students in the southern United States, October 2025. Using a qualitative district review design supported by descriptive quantitative summaries, data were collected through classroom observations, principal interviews, and mathematics coach surveys across 10 campuses. Findings indicate that the district has established many of the organizational conditions associated with improvement, including pacing guides, common assessments, instructional coaching, and data meetings. However, these structures have not yet translated into consistently strong mathematics instruction. The most significant barriers to improved student outcomes were uneven teacher and coach mathematical content knowledge, inconsistent differentiation, limited student discourse, and weak progression from foundational numeracy to algebraic thinking. Fractions and place value emerged as critical chokepoints. The study concludes that districts seeking stronger mathematics outcomes must move beyond structural coherence toward deeper content expertise, conceptually grounded intervention, and classrooms that position students as active mathematical thinkers.

Keywords

mathematics instruction, coherence, teacher knowledge, coaching, student discourse

1. Introduction

Across the United States, school districts continue to search for effective strategies to improve mathematics teaching and learning. In many systems, leaders have implemented pacing guides, common assessments, professional learning communities, intervention blocks, and instructional coaching to create coherence and improve student outcomes. Yet the existence of these structures does not automatically yield rigorous mathematics instruction or equitable access to deep conceptual understanding. Too often, instructional systems become organized without becoming intellectually powerful.

Research consistently demonstrates that strong mathematics teaching depends not only on general pedagogy, but also on teachers' knowledge of content, students, and curriculum (Ball et al., 2008; Hill et al., 2005). Likewise, students' ability to reason, justify, and engage in mathematical discourse is central to conceptual understanding and long-term achievement (Hiebert & Grouws, 2007; NCTM, 2014). When these elements are weak, instruction often defaults to procedural teaching, passive compliance, and surface-level engagement.

This article presents findings from a K–8 mathematics review conducted in a midsize school district in the southern United States in fall 2025. The study sought to understand the strengths and limitations of the district's mathematics system by examining classroom practice, leadership perspectives, and coaching structures across 10 campuses. The study was guided by the following questions:

1) What districtwide patterns characterize K–8 mathematics instruction in the District?
2) What organizational strengths and instructional barriers influence mathematics teaching and learning across the system?
3) What implications do these findings hold for district leaders seeking to strengthen mathematics outcomes and readiness for algebra by Grade 9?

The findings point to a district with strong foundational structures and a positive culture for improvement, yet with a pressing need to deepen mathematical content knowledge, strengthen Tier 2 instruction, elevate student discourse, and build a coherent progression toward algebra readiness.

1.1 Student Demographics and Assessment Data

The district serves over 8,500 students. Tables 1 and 2 provide an overview of enrollment by grade span and student performance for the 2024-2025 school year. The district also serves a high-need population, with approximately 70-80% of students identified as economically disadvantaged, 12-16% receiving special education services, and 3-6% classified as multilingual learners.

Table 1. District Enrollment by Grade Span

Grade Span	Grades	Estimated Enrollment
Elementary	PreK–5	4,500–5,500 students
Middle	6–8	2,000–2,300 students
High School	9–12	1,800–2,100 students
Total District	PreK–12	8,500–9,000 students

Table 2. District Students' Performance on State Assessment in Mathematics (Grades 3–8)

Proficiency Level Description		Grades 3-5 at Each Level	Grades 6-8 at Each Level
Level 1	Minimal Understanding	30–40%	35–45%
Level 2	Partial Understanding	30–35%	30–35%
Level 3	Proficient	20–25%	15–20%
Level 4	Advanced	5–10%	3–8%

Approximately 60 to 75% of elementary students and 70-80% of middle school students are below proficiency, reflecting compounding gaps from elementary grades, particularly in fractions, place value, and algebra readiness.

Across the District, these patterns are more pronounced among economically disadvantaged students, students with disabilities, and multilingual learners, highlighting persistent opportunity gaps.

1.2 Instructional Coherence and System Improvement

District leaders frequently rely on system structures such as pacing guides, assessments, and collaborative planning to improve instructional quality. These efforts can create greater coherence, especially in fragmented systems, by aligning expectations and routines across schools (Fullan, 2016; Bryk et al., 2015). Doan et al. (2025) and Covelli et al. (2026) demonstrate that coherence is strengthened when districts pair high-quality instructional materials with aligned professional learning that supports consistent classroom implementation. However, coherence alone is insufficient if educators do not have the content expertise required to use those systems well. A district may be organizationally aligned yet instructionally shallow.

1.3 Mathematical Knowledge for Teaching

A robust body of research has established that effective mathematics instruction requires specialized content knowledge for teaching. This includes not only knowing how to perform mathematical procedures, but also understanding underlying concepts, common misconceptions, developmental progressions, and multiple representations (Ball et al., 2008). Teachers with stronger mathematical knowledge for teaching are better positioned to support conceptual learning and respond to student thinking in real time (Hill et al., 2005). Hill et al. (2015) and Doan et al. (2025) further show that teachers' engagement with curriculum materials, when paired with strong content knowledge, significantly influences instructional quality and student outcomes.

1.4 Student Discourse and Conceptual Understanding

Mathematics learning is strengthened when students explain their reasoning, compare strategies, and engage in purposeful discourse. High-quality classrooms position students as sense-makers rather than passive recipients of procedures (NCTM, 2014; Smith & Stein, 2018). Smith and Stein (2018) emphasize that structured discussion routines are essential for maintaining cognitive demand, while the National Academies of Sciences, Engineering, and Medicine (2019) highlight discourse as a central mechanism for deep conceptual understanding and equitable participation. The use of meaningful discussion, error analysis, and justification supports deeper understanding, especially when paired with cognitively demanding tasks (Stein et al., 2008).

1.5 Differentiation and Intervention in Mathematics

Effective differentiation in mathematics extends beyond reducing group size or repeating instruction. It requires diagnosis of misconceptions and deliberate use of representations, tasks, and scaffolds matched to learner need (National Mathematics Advisory Panel, 2008). The National Academies of Sciences, Engineering, and Medicine (2019) underscore that effective intervention is most successful when it is conceptually grounded and integrated within core instruction, while Doan et al. (2025) highlight the importance of using high-quality materials to support differentiated learning within Tier 1 environments. Tiered intervention is most effective when it is standards-based, conceptually grounded, and informed by formative evidence.

1.6 Coaching and Professional Learning

Instructional coaching can be a powerful mechanism for improving teaching practice, especially when it includes modeling, co-teaching, feedback, and sustained cycles of support (Kraft et al., 2018). Darling-Hammond et al. (2017) identify content-focused, sustained professional learning as a key driver of instructional improvement, while Doan et al. (2024) demonstrate that coaching aligned to curriculum and classroom practice strengthens implementation. Yet the effectiveness of mathematics coaching depends on coaches themselves possessing strong mathematical understanding. Generalist coaching models may improve routines, but they cannot substitute for content-deep support in mathematics.

1.7 Conceptual Framework

This study is grounded in an integrated conceptual framework that brings together three complementary bodies of research: instructional coherence, mathematical knowledge for teaching, and cognitively demanding mathematics instruction.

1.7.1 Instructional Coherence

Instructional coherence refers to the alignment of curriculum, assessment, professional learning, and instructional practices within a system (Fullan, 2016). Districts often pursue coherence through structures such as pacing guides, common assessments, and professional learning communities. While coherence can reduce fragmentation and create consistency, it does not inherently guarantee instructional quality. As noted in the literature, systems may become procedurally aligned without ensuring that classroom instruction is conceptually rigorous or responsive to student thinking.

1.7.2 Mathematical Knowledge for Teaching

At the core of effective mathematics instruction is mathematical knowledge for teaching (MKT), a specialized form of content knowledge that enables teachers to represent ideas, anticipate misconceptions, and respond to student reasoning (Ball et al., 2008). MKT extends beyond procedural fluency to include deep conceptual understanding and knowledge of mathematical progressions. Teachers with strong MKT are better positioned to facilitate discourse, support multiple representations, and engage students in meaningful problem solving (Hill et al., 2005).

1.7.3 Cognitive Demand and Student Discourse

High-quality mathematics instruction is characterized by cognitively demanding tasks and opportunities for students to engage in reasoning, explanation, and justification (Stein et al., 2008; Smith & Stein, 2018). The National Council of Teachers of Mathematics (2014) emphasizes that student discourse is central to learning, as it allows students to make sense of mathematics and construct understanding collaboratively. When cognitive demand is reduced through excessive teacher talk, procedural teaching, or reliance on technology students are less likely to develop deep understanding.

1.7.4 Integrative Framework for This Study

This study conceptualizes effective mathematics systems as the intersection of these three elements:

- Coherent structures (e.g., pacing, assessment, coaching)
- Deep mathematical knowledge for teaching
- Instructional practices that sustain cognitive demand and student discourse

The framework posits that while coherence establishes the conditions for improvement, mathematical knowledge and instructional practice determine the quality of learning experiences and, ultimately, student outcomes. This perspective guided both the design of the study and the interpretation of findings, particularly the conclusion that coherence alone is insufficient without content expertise and cognitively rich instruction.

2. Method

2.1 Research Design

This study used a qualitative district review design supported by descriptive quantitative summaries. The aim was to synthesize multiple forms of evidence to identify districtwide patterns in mathematics instruction, leadership, and coaching.

2.2 Setting and Participants

The review took place in a TK-12 school district in October 2025. Ten campuses were included in the site review from TK to middle school participants that included one primary school and one intermediary school. Participants included eight principals, eleven mathematics instructional coaches, and teachers observed across approximately 50 classrooms, resulting in 69 recorded classroom observations.

2.3 Data Sources

Three tools were used:

- **Classroom observation tool.**
  - Observation data captured evidence related to lesson design, student engagement, differentiation, classroom culture, discourse, teacher talk, student artifacts, and instructional use of classroom walls.
- **Principal interview guide.**
  - Semi-structured interviews elicited principals' perceptions of accomplishments, challenges, mathematics-specific needs, and desired supports.
- **Coach survey.**
  - The survey collected information on coaches' educational backgrounds, instructional systems, coaching practices, perceived effectiveness, data use, supports, and ongoing needs.

2.4 Data Analysis

Data from all three sources were synthesized to identify recurring themes, strengths, and areas for growth. Open-ended responses and observation notes were reviewed for cross-cutting patterns. Descriptive averages were used where ratings were provided, including coach ratings of systems and strategies.

3. Results

3.1 Positive Culture and Core Structures Are Established

In this section we present our findings. The review of the findings found a broadly positive student culture across the district. Students were described as polite, mild-mannered, and generally eager to learn. Campuses were orderly, and many classrooms, particularly those led by newer teachers, were warm, engaging, and student friendly.

In addition, the District has established several important system structures. Principals consistently identified pacing guides, common assessments, data meetings, coaching cycles, and planning routines as major accomplishments. Coaches described similarly coherent systems, including weekly professional learning communities, progress monitoring, intervention structures, and lesson routines anchored in number sense, gradual release, and formative assessment.

These findings suggest that the District has already done the important work of building organizational infrastructure. The central question is no longer whether structures exist, but whether those structures are producing sufficiently deep mathematics instruction.

3.2 Mathematical Content Knowledge Is the Primary Instructional Limiter

Across classrooms, interviews, and surveys, the most significant barrier to stronger mathematics teaching was uneven mathematical content knowledge. Several observations indicated that instruction often remained at the procedural level, with limited conceptual depth. In a few cases, teachers demonstrated visible confusion about the content they were teaching. Principals repeatedly noted that many teachers were new, uncertified, or teaching outside of their formal training. Coaches similarly acknowledged that they were generally well prepared in pedagogy and leadership but not deeply trained in mathematics content.

This finding is consistent with prior research showing that teacher knowledge of mathematics shapes the quality of explanations, task design, responsiveness to misconceptions, and overall rigor of instruction (Ball et al., 2008; Hill et al., 2005). Without strong content knowledge, teachers may rely heavily on videos, curriculum scripts, or digital practice tools, reducing opportunities for student reasoning and mathematical sense-making.

3.3 Student Engagement Is Visible, but Discourse Remains Limited

Student engagement across classrooms was generally positive. Many teachers used manipulatives, number routines, partner talk, and guided practice to keep students active. However, the dominant pattern across grades, especially in upper elementary and middle grades, was one of teacher-directed instruction with limited student discourse.

Choral responses were common, but more substantive checks for understanding, such as student explanations, peer critique, board work, written justification, and consensus-building, were less common. As grade levels increased, teacher talk increased and student voice decreased. In many classrooms, a small group of students answered most questions, while quieter learners remained largely invisible.

This pattern matters because mathematical discourse is not peripheral to instruction; it is central to conceptual understanding (NCTM, 2014; Smith & Stein, 2018). Classrooms in which students primarily listen, repeat, or comply may appear orderly, but they do not necessarily cultivate deep reasoning.

3.4 Differentiation Is Inconsistent and Tier 2 Is too Often Procedural

Although workshop models, centers, and small groups were present in some classrooms, differentiation was inconsistent across the system. In many observations, instruction was primarily whole group, with little evidence of targeted scaffolding for English learners, students with disabilities, or students working below grade level. Extension opportunities for students ready for greater challenge were also limited.

A particularly important finding was that Tier 2 often functioned as procedural reteaching rather than conceptual intervention. Students were frequently asked to revisit the same content in smaller groups without a different representation, strategy, or diagnostic response to misconception. This suggests that the district's intervention structures are present, but their instructional quality needs strengthening.

3.5 Fractions, Place Value, and Algebra Readiness Are Districtwide Chokepoints

Principals and coaches consistently named fractions, place value, and basic fact fluency as major barriers to student success. Fractions emerged as the most prominent mathematical chokepoint, particularly because weak fraction understanding limited student access to later work in proportional reasoning and algebra.

Place value was similarly identified as a foundational area of weakness. Across grade spans, concerns about numeracy and conceptual understanding pointed to an incomplete developmental progression in mathematics. The findings also raised concern that readiness for Grade 9 algebra was being treated too narrowly. Although the state emphasis on numeracy is important, the review suggested the need for a clearer Pre-K–8 progression of algebraic ideas, including equivalence, patterning, generalization, and relational thinking.

These findings align with national research emphasizing the importance of strong foundations in whole number understanding, fractions, and proportional reasoning as precursors to algebraic success (Kilpatrick et al., 2001; NMAP, 2008).

3.6 Coaching Is a Major Asset, but Coaches Need Deeper Mathematics Preparation

The district's coaching corps emerged as a clear strength. Coaches described using modeling, co-teaching, micro-modeling, work sorts, non-evaluative scripting, video reflection, and data conversations. Both systems and coaching strategies were rated at an average of 4.0 out of 5.

At the same time, coaches reported that their greatest need was deeper mathematics content knowledge. Most held advanced degrees, but none held degrees in mathematics or mathematics education. They also requested more protected planning time, better administrator calibration during walkthroughs, and stronger support for tiered instruction and inclusion.

This suggests that the District has an unusually promising coaching platform for improvement, but one that now requires content-deep investment if it is to move from generally effective to highly impactful.

3.7 Classroom Environments Are Underused as Instructional Tools

The review found limited use of classroom walls and student artifacts as supports for mathematics learning. Many classrooms displayed posters, objectives, and commercially produced visuals, but relatively few featured accessible, co-created anchor charts, visible models of current concepts, or authentic student work that explained reasoning.

Similarly, student work displays were rare and more often associated with literacy than mathematics. The limited visibility of mathematical thinking in the environment suggested missed opportunities to reinforce vocabulary, celebrate student ideas, and make learning public.

4. Discussion

The findings from this review suggest that the District has achieved a meaningful degree of structural coherence in mathematics but has not yet converted that coherence into consistently high-quality mathematics instruction. This distinction is important. Structures such as pacing guides, common assessments, and coaching cycles are necessary, but they are not self-executing. Their effectiveness depends on the knowledge and skill of the educators using them.

The most striking implication is that mathematics improvement must be treated as content-specific work. General instructional reforms are helpful but insufficient. Teachers and coaches need a deep understanding of mathematical ideas, representations, and progressions if they are to teach for conceptual understanding rather than procedural completion.

The review also suggests that districts should resist conflating engagement with rigor. Students may appear busy, compliant, or even enthusiastic without engaging in meaningful mathematical sense-making. The next phase of district improvement should therefore focus on discourse, reasoning, and the public visibility of student thinking.

Finally, the findings highlight the importance of a coherent trajectory toward algebra readiness. Preparing students for Grade 9 algebra is not simply a middle school task. It requires a carefully articulated developmental pathway beginning in the early grades and extending through upper elementary and middle school mathematics.

4.1 Limitations

While this study provides valuable insights into systemwide mathematics instruction, several limitations should be considered when interpreting the findings.

First, the study was conducted within a single midsize school district in the southern United States. Although the patterns identified are consistent with national trends in mathematics education, the findings may not be fully generalizable to districts with different demographic compositions, governance structures, or resource levels. The purpose of this study was not to generalize broadly, but to provide a detailed, context-rich analysis that may inform similar districts facing comparable challenges.

Second, the data collection relied primarily on qualitative methods, including classroom observations, principal interviews, and coach surveys. While these methods allowed for a comprehensive and nuanced understanding of instructional practices and system conditions, they are inherently subject to observer interpretation and potential bias. To mitigate this, multiple data sources were triangulated; however, the absence of extensive quantitative analysis limits the ability to establish causal relationships between observed practices and student outcomes.

Third, classroom observations were conducted during a limited time window and represent snapshots of instruction rather than longitudinal patterns. Instructional practices may vary across the school year due to pacing, assessment cycles, or professional learning initiatives. As such, the observations should be interpreted as representative trends rather than exhaustive accounts of daily practice.

Fourth, the study did not include direct measures of teacher mathematical content knowledge or student cognitive demand through formal assessment instruments. Instead, inferences about content knowledge and instructional rigor were drawn from observed practices, leader perspectives, and coaching data. Future research could strengthen these findings by incorporating validated measures of mathematical knowledge for teaching and student-level performance analyses.

Finally, while student performance data from the State Comprehensive Assessment Program were used to contextualize findings, this study did not conduct a detailed statistical analysis linking specific instructional practices to student achievement outcomes. As a result, the conclusions emphasize patterns and relationships rather than definitive causal claims.

Despite these limitations, the study offers a coherent and triangulated view of systemwide mathematics instruction, highlighting critical leverage points for improving teaching and learning. The findings contribute to a growing body of evidence suggesting that structural coherence, while necessary, must be paired with deep mathematical knowledge and intentional instructional practice to produce meaningful gains in student outcomes.

4.2 Implications for Practice

Several implications emerge from this study. They are detailed below.

Districts should invest in sustained mathematics-specific professional learning for teachers, coaches, and principals particularly around place value, fractions, proportional reasoning, and algebraic thinking.

Teacher led instruction should replace video teaching with a clear progression of learning from discovery, to conceptual, to procedural to real world application. Districts should map and communicate a Pre-K–8 progression of learning tied to algebra readiness so that teachers understand how current content connects to future mathematics.

Leaders should strengthen Tier 2 mathematics support by ensuring that intervention is tied to diagnosed misconceptions and conceptually different instructional approaches, rather than repetition of whole-group lessons. Moreover, teachers must have deep math knowledge to differentiate instruction.

Walkthroughs, coaching cycles, and school improvement plans should explicitly prioritize student discourse, including explanation, justification, comparison of strategies, and mathematical argumentation.

Finally, schools should transform classroom walls, journals, and artifacts into living instructional tools that document and reinforce student thinking.

4.3 Future Research

The findings of this study suggest several important directions for future research in mathematics education and district-level improvement.

First, future studies should examine the relationship between teacher mathematical content knowledge and student outcomes using validated instruments for measuring mathematical knowledge for teaching. While this study identified content knowledge as a primary constraint through observational and perceptual data, more precise measurement could strengthen causal claims and inform targeted professional learning.

Second, there is a need for longitudinal research that tracks the impact of content-focused professional development and coaching on instructional practice and student achievement over time. Such studies would provide insight into how sustained investments in teacher and coach learning influence systemwide improvement.

Third, future research should explore the design and implementation of high-quality Tier 2 mathematics interventions, particularly those that move beyond procedural reteaching to address conceptual misunderstandings. Investigating how diagnostic assessment, flexible grouping, and representation-based instruction impact student learning would be especially valuable.

Fourth, additional research is needed to better understand how districts can effectively build coherent Pre-K–8 learning progressions aligned to algebra readiness. This includes examining how early numeracy, place value, and fraction understanding develop over time and how instruction can intentionally support that progression.

Fifth, given the increasing presence of digital tools and artificial intelligence in classrooms, future studies should investigate how these tools influence student cognition, attention, and mathematical reasoning. Specifically, research should examine the balance between technology use and teacher-led instruction in fostering deep learning.

Finally, comparative studies across districts with varying levels of performance could provide further insight into the conditions under which instructional coherence translates into improved student outcomes. Such research would help clarify how context, leadership, and capacity interact to influence mathematics achievement.

4.4 Conclusion

The District has many of the ingredients necessary for significant mathematics improvement: positive student culture, committed educators, established coaching structures, and districtwide systems for planning and data use. These strengths are real and important. Yet the findings of this review indicate that stronger student outcomes will depend on moving from structure to substance, from routines to deep mathematics, from compliance to reasoning, and from general support to content-specific instructional capacity.

The district's next phase of improvement should focus on strengthening mathematical knowledge for teaching, sharpening the quality of differentiation and intervention, elevating student discourse, and clarifying the progression to algebra readiness. In doing so, the District has the opportunity not only to improve its own outcomes, but also to offer a useful model for similarly situated districts seeking to build mathematics systems that are both coherent and intellectually rigorous.

The central takeaway is clear: systemwide mathematics success requires the integration of coherent structures with deep mathematical knowledge for teaching. Beyond induction programs and licensures, teachers need on-going professional development and coaching from math experts. Coaches need training from math experts and principals need training in how to support both coaches and teachers in math instruction.

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