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Original Paper

Arithmetic Fact Fluency Supported by Artificial Intelligence

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Abstract

The main purpose of this study was to investigate to what extent students aged eight years developed their arithmetic fact fluency by using artificial intelligence to practice number combinations. The study compares the effects of three different ways of practicing number combinations: Artificial Intelligence (AI), Memorization (Mem), and Guided Learning (GL). The design was a split-plot factorial design with group as a between-subject factor and time (i.e., before and after a six-week intervention) as a within-subject factor. Pre- and post-test were performed to assess students' fluency with respect to basic number combinations, i.e., addition, such as 3 + 4, 2 + 1, 6 + 3, etc. The results show that students developed their fluency significantly with respect to basic number combinations when practicing this skill with support of artificial intelligence. It seems that the technique is effective at analyzing and recommending content based on students' learning patterns and what has worked best for similar students. The results therefore strengthen the findings of previous studies that artificial intelligence presents great opportunities to offer individual support to maximize learning. The results also show that practicing number combinations with artificial intelligence is more effective compared to practicing with a focus on memorization and guided learning.

Keywords

artificial intelligence, mathematics, arithmetic fact, fluency, learning

1. Introduction

Several studies have investigated how teachers and students can best use computers to support students' mathematics learning. Technological development is constantly creating new conditions that can affect students' learning. Not least, researchers argue that artificial intelligence presents significant opportunities such as assessing students' knowledge and offering individual support to maximize learning (Hwang & Tu, 2021).

New technology has always exerted a captivating force on the human intellect (Samuelsson, 2006). This psychological side of technology raises many questions about the conditions of learning. "Can technology help more people to learn more in less time?" is one such question. Today, artificial intelligence is starting to become a reality in schools in Sweden. Artificial intelligence is predicted to revolutionize our society, and education is no exception. A research overview of AI in mathematics teaching states that the technology is being used more and more in this area (Hwang & Tu, 2021), giving mathematics teachers an opportunity to adapt teaching in a whole new way. The technique, AI, can be adapted to the student by analyzing the student's abilities and then recommending content, based on the student's abilities, and learning patterns and on what has worked best for similar students (Chen et al., 2007; Hwang et al., 2014; Hwang et al., 2020). Exercises can be tailored to the student's needs, both in terms of degree of difficulty and number of exercises. There have been various studies on how AI can be used as an intelligent learning support (Hwang et al., 2021). These studies have primarily focused on cognition, in terms of learning (e.g., Bartelet et al., 2016), but also the extent to which AI can support affective aspects of learning (e.g., Nye et al., 2018) such as attitudes, motivation and confidence in one's own abilities. Hwang et al. (2021) conclude their research review by stating:

> *It is important to investigate the effectiveness of using AI in mathematics learning activities from different perspectives by taking rarely considered research foci into account, such as cognitive load, collaboration and communication competencies and learning anxiety (p. 12).*

This study examines how AI can be used to support the development of students' fluency with respect to basic number combinations. The main purpose of the study was to investigate to what extent students aged eight years developed their fluency by using artificial intelligence to practice number combinations.

1.1 Arithmetic Fact Fluency

Learning arithmetic involves different forms of knowledge: a) declarative, b) procedural and c) conceptual (Hudson & Miller, 2006). Declarative knowledge means knowledge that is retrieved directly from memory without hesitation; this is when you know something with fluency (Hudson & Miller, 2006). Students who master basic number combinations and arithmetic facts, and understand the position system and the various arithmetic operations, have a good basis for handling the four arithmetic methods with multi-digit numbers (McIntosh, 2008). Having difficulty reproducing arithmetic facts is a common problem both for students with specific arithmetic difficulties and for students with milder math difficulties (Andersson, 2010; Geary, et al., 2012). Procedural knowledge is about being able to manipulate numbers in, for example, an addition such as 123 + 23 = __. Conceptual knowledge means that the student has knowledge of number relations, e.g., which number is greater, 4 or 7?

The importance of fluency with respect to arithmetic facts is stressed in several studies. For example, fluency with basic number combinations has been shown to explain differences in Chinese and American students' performance when solving multi-digit additions (Vasilyeva, Laski, & Shen, 2015). Another study shows that fluency in calculations predicts results in future, more advanced mathematics studies (Rathmell & Gabriele, 2011). Fluency also presents a significant challenge for students with specific arithmetic difficulties (Geary, 2013; Skagerlund & Träff, 2016). Thus, fluency in basic arithmetic seems to be important with respect to mathematical development.

There are broadly two positions in mathematics didactics that advocate two different positions in terms of how best to teach students arithmetic facts (Baroody, Bajwa, & Eiland, 2009). The first method is called passive learning, which involves memorizing number combinations by heart. Memorizing by heart is based on the idea that recall improves when given the chance to repeat what is to be remembered. In other words, the more times a student actively responds to stimuli such as 3 + 4 = __, the better the student will remember the sum 7. We acknowledge that most of the existing research would recommend the use of timed exercises (e.g., Baroody et al., 2014). The second method is conceptualized as active construction or number sense-based learning (number sense). Baroody et al. (2009) argue that the student should actively explore the relationship between numbers, with for instance image support, and thereby improve number perception which in turn improves the reproduction of arithmetic facts. There are also proponents of this position who claim that time-limited tests should not be used as a basis for assessing students' fluency in number combinations (Kling & Bay-Wiliams, 2014), arguing that timed tests can lead to math anxiety. However, it has been shown that exploratory learning without supporting elements such as scaffolding and feedback can have a negative effect on learning (Mayer, 2004). On the other hand, Alfieri, Brooks, Aldrich and Tenenbaum (2010) show that exploratory learning together with support is an effective way to learn. It is important to note that their meta-analysis shows that this type of learning is more effective for adults than for children and that the researchers are not specific either to students in need of support or to students without that need.

Thus, few studies have evaluated memorization-focused practice vs conceptual focused practice. Tournaki (2003) tested strategy instruction vs drill and practice procedure in a sample of second graders with respect to fluency development. Strategy instruction meant teaching the minimum addend strategy (starting with the larger number and adding the smaller number by counting). Drill and practice used two forms of tasks that should be solved during a lesson. The researchers examined the effect of eight lessons (15 minutes) and compared development on children with a mathematical learning disability and on children without a learning disability. The result showed that both conditions were better than the control group regardless of whether the child had a learning disability (Cohens d = 1.53 for strategy and Cohens d = 1.35 for drill and practice). Results also showed that children with a learning disability achieved better results if they received strategy instruction than if they worked with drill and practice (Cohens d = 1.52). The results seem to favor strategy instruction over drill and practice, although both strategy instruction and drill and practice conditions did emphasize speed for the children during practice.

Fuchs et al. (2009) have compared two programs with respect to fluency on number combinations. One program focused on drill and practice, while the other focused on conceptual aspects and involved some fluency training. The results showed that drill and practice developed equally as conceptual tutoring, and both developed significant more than control group (Drill and practice Cohens d = 0.55, conceptual Cohens d = 0.62). Baroody, Purpura, Eiland, and Reid (2014) have investigated whether unguided practice, guided ten strategy practice, or guided subtraction practice affect first graders' fluency with respect to basic number combinations. In the unguided group the student practiced number combinations without any guidance. The students in the ten-strategy group were guided in the ten-strategy, while the subtraction group were guided to use addition to solve subtraction combinations. The result show that subtraction practice increased the fluency significant of untrained number combinations (Hedges' g = 0.50 compared to use-a-ten strategy, 0.45 compared to unguided). However, fluency on trained number combinations developed more in the guided subtraction group and unguided practice group compared to ten-strategy group (Hedges' g = 1.46 and 0.95). Using addition to solve a subtraction combination could help student doing memorization.

Greene et al. (2018) investigated the effect of fluency training through peer tutoring with flash cards. The training was carried out three times a week, 30 minutes each session, for eight weeks and the children who participated in the intervention attended grades three and four. The intervention had a high degree of memorization focus while the control group was given standard lessons. The result showed that the intervention group developed significant more than the control group. (Cohens d = 1.77).

For students with math learning difficulties, Baroody et al. (2009) and Gersten et al. (2009) show that, among other things, explicit instructions, self-controlled teaching, computer-based teaching, and interventions carried out by researchers together with teachers, seemed to be beneficial. Codding, Burns and Lukito (2011) conducted another meta-analysis that focused on students with Math Learning Disabilities (MLD) in terms of fluently reproducing basic number combinations. They also did a component analysis of the studies to determine which type of teaching was most effective. The teaching methods that featured in the studies were as follows: modeling, hiding the answer, recording the answer with the task, showing the results, comparing answers to tasks, flash cards, worksheets with increasing difficulty, independent work, self-assessment, reinforcement, sound recording of tasks, and combinations of the above. They divided the methods into four categories: practice with models, practice without models, tests (drill), and self-directed activity. The results showed that the best effect was produced by practicing with models and conducting tests.

Fuchs et al. (2012) tested a tutoring program for at-risk learners in grade one. The tutoring took place outside the ordinary classroom in the form of 30-minute one-to-one sessions, three times a week. The study randomized 300 low-risk student and 206 at-risk students to a control condition group (121), speeded practice (195), and to non-speeded practice (190). The results showed that the condition with speeded (memorization-focused) training was superior to the non-speeded training condition, with an effect size of 0.51 (Cohens d = 0.43). Both intervention groups were significantly better than the control group.

Fluency in basic arithmetic seems to be important to develop. It predicts results in future, more advanced mathematics studies (Rathmell & Gabriele, 2011) and is significant challenge for students with specific arithmetic difficulties (Geary, 2013; Skagerlund & Träff, 2016). Few studies have investigated how students should practice in order to gain fluency and as far as we know, no study have compared fluency development practicing with support of artificial intelligence (cf. Hwang, 2021), practicing in order to memorize and practicing with focus on conceptual aspects of number combinations (cf. Baroody, Bajwa, & Eiland, 2009; Greene et al., 2018).

Accordingly, the following two research questions were formulated:

A) To what extent do eight-year-old students develop their fluency with respect to number combinations when practicing with a) Artificial Intelligence (AI), b) mathematical symbols to memorize (Mem), and c) mathematical symbols and images (Guided Learning, GL)?

B) To what extent do eight-year-old students with MLD develop their fluency with respect to number combinations when practicing with a) Artificial Intelligence (AI), b) mathematical symbols to memorize (Mem), and c) mathematical symbols and images (Guided Learning, GL)?

2. Method

The design was a split-plot factorial design with group (i.e., practice with AI, Mem, GL, and control group) as a between-subject factor, and time (i.e., before and after the six-week intervention) as a within-subject factor. Pre- and post-test were used to assess students' fluency with respect to basic number combinations, i.e., addition, such as 3 + 4, 2 + 1, 6 + 3, etc.

A total of 1,006 students in Year 2 participated in the study. Data was collected over two years. In the first year, 877 students participated. The aim was then to investigate to what extent the students developed their fluency in number combinations when practicing with mathematical symbols in order (memorizing) and with image support (guided learning). The students were randomly assigned, at class level, to one of three interventions: a) exercises with mathematical symbols (N = 320), b) exercises with image support (N = 386), and c) control group (171). When the opportunity to practice with AI also became possible, schools in a medium-sized municipality were asked if there were any classes that were interested in practicing number combinations with the support of AI (N = 129). All students performed fluency tests before and after the six-week intervention period. The students were asked to answer as many number combination tasks as possible for 90 seconds. Test reliability α = .98.

2.1 Practice of Fluency

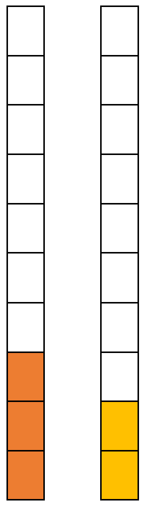
This study compares how students' performance regarding fluency in number combinations developed in four different groups: a) AI, practice with AI engine, b) Mem, practice with mathematical symbols, c) GL, practice with image support, and d) control group. The project was carried out over a six-week period. In the intervention groups, each mathematics lesson ended with students practicing number combinations for 10 minutes. All teachers scored to what extent they followed the practice plan after each lesson, on a ten-point Likert scale. Fidelity was calculated to .88. The following four groups' fluency development over six weeks was investigated.

- AI: Students practice at a computer with an AI engine. The AI engine collects data on how the individual student handles the tasks, identifies patterns in this and gradually learns what the student's knowledge needs look like. It picks up tasks that respond directly to these needs, without the student noticing that AI is involved (cf. Chen et al., 2007; Hwang et al., 2014; Hwang et al., 2020).

- Mem: Students only practice basic number combinations with mathematical symbols of the type (3 + 2, 3 + 6, 6 + 3) in short, timed exercises (cf. Baroody et al., 2014).

- GL: In addition to numbers, students are also presented with an image representing the number combinations (cf. Baroody et al., 2009). For instance:

3 + 2 =



- Control group: Mathematics teaching is conducted as usual.

Means and effect size Cohen's *d* were calculated for each student regarding fluency on pre- and post-test.

Finally, repeated measure analysis of variance (MANOVA) was employed to investigate development differences between groups. Analyses were made of the entire material to answer our first research question and of the lowest performing 25 percent of students to answer our second research question. We used the cut-off of 25 percent to define our group with special needs in mathematics (MLD, Mathematics Learning Disabilities) (cf. Cowan & Powell, 2013; Geary et al., 2012; de Smedt & Gilmore, 2011).

3. Results

The results will be presented in two sections. The first section provides descriptive statistics of mean before and after the intervention. We also show the intervention effect with respect to mean change and effect size. The second section contains the results of two mixed ANOVAs.

3.1 Descriptive Statistics

The results, presented in Table 1, show that there were significant improvements during the intervention for all groups. Students who practiced with AI support had the greatest improvement with respect to effect size measure Cohen's *d* (.80). The second best development was shown by students focused on memorization (.69), with guided learning (.21) coming third and the control group (.11) last.

Table 1. Mean, Standard Deviation before and after Intervention and Effect Size with Respect to Fluency in Each Group

			Before			After		Intervention effect	
Test	Group	*n*	*M*	*SD*	*n*	*M*	*SD*	*M*	*Cohen's d*
Fluency	Control	171	14.85	7,.05	171	15.54	5.64	.69	.11
Fluency	GL	386	13.74	5.79	386	16a.37	6.48	2.63	.21
Fluency	Mem	320	13.63	5.94	320	17.72	5.99	4.09	.69
Fluency	AI	129	11.67	6.94	129	18.21	9.25	6.54	.80

The analyses, presented in Table 2, of the group we defined as having special needs show that students who focused on memorization developed their fluency the most regarding effect size measure Cohen's *d* (1.94). The second best development was shown by students who practiced their fluency with support of AI (.69), with guided learning (1.16) coming third and the control group (1.10) last.

Table 2. Mean, Standard Deviation before and after Intervention and Effect Size with Respect to Fluency in Each Group (MLD Students)

			Before			After		Intervention effect	
Test	Group	*n*	*M*	*SD*	*n*	*M*	*SD*	*M*	*Cohen's d*
Fluency	Control	32	6.81	1.82	32	10.44	4.21	3.63	1.10
Fluency	GL	89	6.85	2.17	89	11.11	4.73	4.26	1.16
Fluency	Mem	77	7.09	2.14	77	14.20	4.72	7.11	1.94
Fluency	AI	48	6.69	1.95	48	10.88	3.05	4.19	1.63

3.2 Mixed ANOVA

The next step in our analysis was to perform a mixed ANOVA, with group (i.e., AI, memorization, guided learning and control) as a between-subject factor and time (i.e., before and after the six-week intervention) as a within-subject factor. Thus, the primary data come from changes in fluency ability.

The main issue of interest was the extent to which the four groups had made differential progress on this performance measure. Analyses to address this question need to consider the fact that the groups are not perfectly matched for their fluency ability. In a study like this, it was not possible to match the groups exactly.

To assess the effects of three different methods for practicing fluency, two analyses of variance (ANOVA) with group as a between-subject factor and time as a within-subject factor were performed. The first was an analysis of the development of fluency for all students, and the second was an analysis of students that fell into the bottom 25 percent on pre-test.

An ANOVA with total fluency scores as dependent measures revealed a significant main effect for time, $F(1, 1002) = 340.58$, $p < 0.001$, suggesting that fluency with respect to number combinations was improved across teaching groups. There were no main effects of group, $F(3, 1002) = .849$, $p = 0.467$, but there was an interaction effect between group and time, $F(3, 1002) = 32.59$, $p < 0.001$. These findings suggest that there was a general effect of methods, and that total improvement in fluency was different across groups.

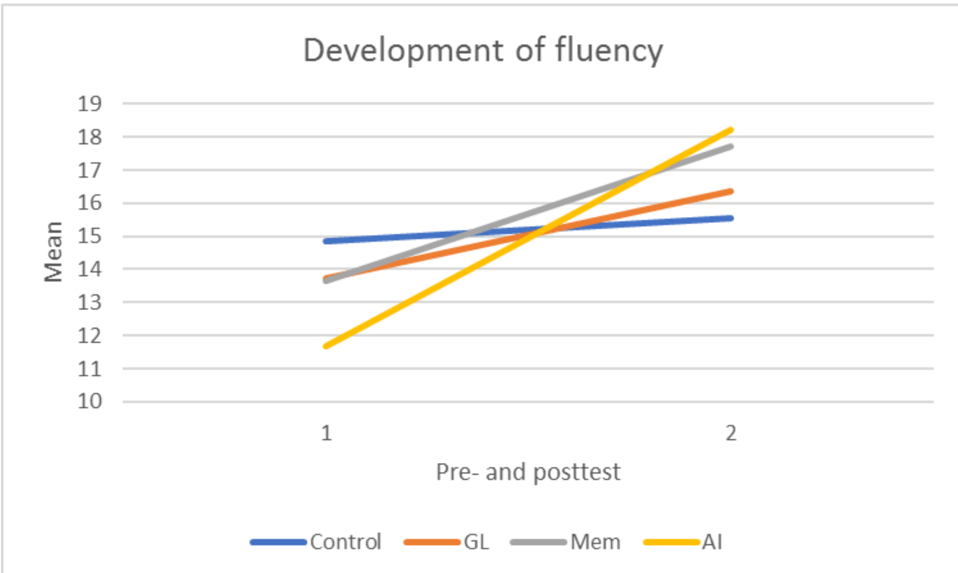


Figure 1. Development of Fluency (All Students)

A second ANOVA, with total fluency scores for students who performed lowest (cut-off 25 percent) on pre-test as dependent measures, revealed a significant main effect for time, $F(1, 241) = 300.81$, $p < 0.001$, suggesting that fluency with respect to number combinations was improved across teaching groups. There was a main effect of group, $F(3, 241) = 7.33$, $p < .001$. The analysis revealed that there was an interaction effect between group and time, $F(3, 241) = 9.89$, $p < 0.001$. These findings suggest that there was a general effect of methods with respect to the lowest performing students, and that total improvement in fluency was different across groups.

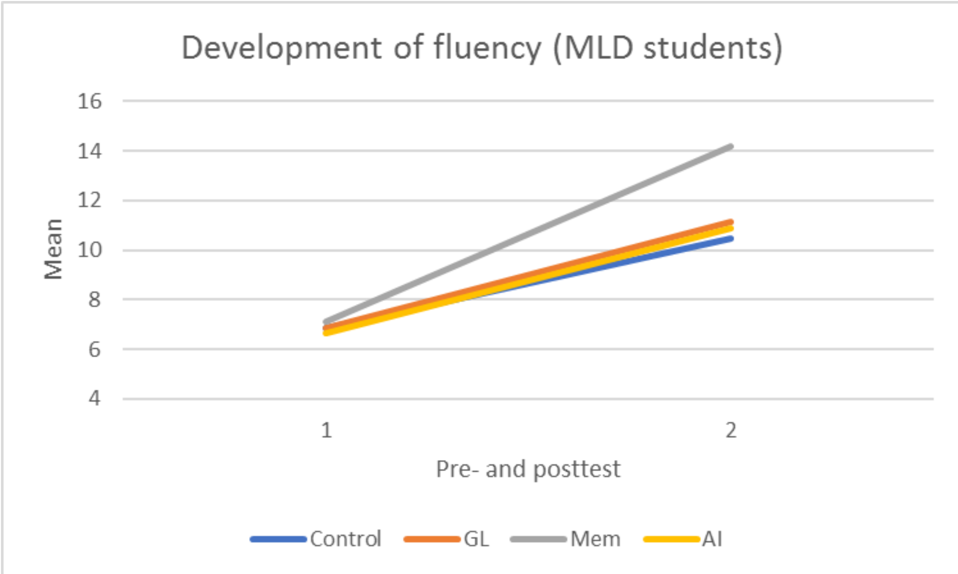


Figure 2. Development of Fluency (MLD Students)

To sum up, the results of this study show that students benefited from practicing number combinations with AI support in terms of fluency development compared to students working with other methods. Analysis of fluency development among students with MLD also showed a positive effect of working with AI. Thus, MLD students who focused on memorization saw the greatest development in fluency.

4. Discussion

The results of this study show that students developed their fluency significantly with respect to basic number combinations when practicing this skill with AI support. It seems that the technique is effective at analyzing student's abilities and recommending content based on students' learning patterns and what has worked best for similar students (Chen et al., 2007; Hwang et al., 2014; Hwang et al., 2020). The result therefore strengthens the position that AI presents great opportunities for offering individual support to maximize learning (Hwang & Tu, 2021) with respect to fluency. It gives mathematics teachers an opportunity to modify teaching in an entirely new way that will effectively support students' learning of basic number combinations.

The results also show that practicing with AI is more effective compared to practicing with a focus on memorization or guided learning. Earlier debates about whether to use learning where the student memorizes by heart (Baroody, Bajwa, & Eiland, 2009) or conceptualize as active construction or number sense-based learning (Codding, Burns, & Lukito, 2011) are challenged by a third option: practicing with AI. The results of the present study therefore strengthen existing research arguing that

AI can be used as an intelligent learning support (Hwang et al., 2021) to design exercises tailored to student's needs, both in terms of degree of difficulty and the number of exercises. Our result also shows that practicing memorizing was significantly better than using images to conceptualize the number combinations. This result therefore strengthens earlier studies result promoting memorization practice (Fuchs et al., 2012; Greene et al., 2018).

This study also shows that MLD students significantly developed their fluency as a result of practicing with AI support. This is an important finding, since fluency also presents a significant challenge for students with specific arithmetic difficulties (Geary, 2013; Skagerlund & Träff, 2016). The MLD group displayed a different pattern with respect to how best to practice fluency in basic number combinations. Students in the MLD group developed most when they practiced by memorizing (cf. Tournaki, 2003; Fuchs et al., 2012). This may be because this group is academically weak in general, which may affect their handling of the computer. Writing by hand might therefore allow them to complete more tasks, which is important according to Baroody, Bajwa and Eiland (2009), and therefore students will achieve better fluency on basic number combinations.

5. Conclusion

To sum up, artificial intelligence has become a reality in schools in Sweden. In this study, it is proven that practicing with AI has an impact on students' performance with respect to fluency in basic number combinations. This is important, since we know that fluency in basic number combinations predicts results in future, more advanced mathematics studies (Rathmell & Gabriele, 2011; Vasilyeva, Laski, & Shen, 2015). AI presents teachers with a powerful new tool for helping students develop important prerequisites for more advanced mathematics.

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