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Original Paper

Innovation Concentration and Patent Market Structure

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Abstract

Two empirical facts concerning the increase in innovation concentration levels of U.S. industries over the last two decades have been established: innovating firms with larger market shares invest more in R&D and enter into more industries. This paper presents a model of an imperfectly competitive patent market with heterogeneous firms to show that R&D itself increases industry concentration. In the context of an imperfectly competitive patent market structure, firms generate endogenous variable markups. In particular, the price of firms' newly invented knowledge, the profit and survival rate of the innovating firms depend on the market share of the knowledge stock. Firms pay a random fixed cost in each period which together with market share determine the decision on whether to innovate in different sectors. Firms enter different sectors sequentially: they start developing new varieties in a sector with minimal competition, building up private knowledge, and then venturing into other sectors using R&D investment and accumulated knowledge. Finally, we prove that the stationary firm innovation size distribution exhibits a Pareto tail in the steady state.

Keywords

innovation concentration, patent market structure, variable markup, indirectly additive preference, pareto distribution

1. Introduction

The nature of competition has potentially been altered by structural changes in product markets in recent decades (e.g., Grullon, 2019; Hsieh, 2019; Hopenhayn, 2018). This paper shows that the phenomenon that the product market has become more concentrated in U.S. industries since the beginning of the 21st century also occurs in R&D and patent markets. The majority of theoretical and empirical papers focus on the impact of industry concentration or industry competition on R&D (e.g., Cohen, 1992; Cohen; 1996; Gu, 2016). In contrast, this paper focuses on the impact of R&D on industry concentration. We demonstrate that industry concentration can be increased by R&D itself.

Empirical work on product markets suggests that the deregulation of antitrust policy enforcement could have a direct impact on the competition in the product market Grullon (2019). Different from their work, which mainly focuses on the impact of deregulation on product market competition through M&A, we show that the increase of market power in R&D and patent markets could happen through the channel of R&D process itself. And further promote the increase in product market concentration.

The imperfectly competitive patent market is crucial in explaining this self-fulfilling mechanism. In the context of an imperfectly competitive patent market, the price of a firm's newly invented knowledge, the profit, and the survival rate of the innovating firm depend on the market share of that firm's knowledge stock. Product and R&D markets are not completely separated, on the contrary, they are complementary. We demonstrate that this is true in both empirical and theoretical ways.

Two empirical facts have been established regarding the increase in innovation concentration levels of U.S. industries over the last two decades: innovating firms with larger market shares invest more in R&D, and enter into more industries. The main objective of this paper is to develop a general equilibrium model of multi-sector firm innovation to explain these two empirical facts and to draw aggregate implications.

In contrast to much theoretical work on endogenous growth, which treats technologies as equally influential, this paper pays attention to various patent adoptions and different knowledge pricing strategies of heterogeneous firms in an imperfectly competitive patent market. The allocation of resources and conduct of research is determined by the market power and structure of these firms. Imperfect competition seems to be a better fit for many markets than perfect competition, given the variety of market structures exist. What is the significance of imperfect competition, particularly oligopolistic competition, in understanding firms' different pricing strategies and decision-making about whether to innovate or in which sectors? This paper analyzes this question as its main focus.

Intuitively, firms that own a large number of blueprints in a sector have more market power to set prices, which results in greater profit from innovation and production. Their survival time will be longer than that of smaller firms. Despite being large enough to have market power, firms behave as if prices were given in the typical model, which restricts the nature of competition within a sector. That is, firms do not internalize their market power when making decisions. This paper explores the oligopolistic market structure of sectors in which firms do consider the impact of their decisions on the price and quantity of patents.

Most of the literature on innovation and growth are based on the assumption of monopolistic competition market environment and CES preferences as described in Stiglitz (1977), which implies that individual firms charge constant markups over their marginal costs when license patent application rights to other firms. While extremely convenient from an analytical point of view, constant markups are incompatible with the empirical evidence. Moreover, constant markup models neglect the significantly different pricing power of heterogeneous innovating firms in the same industry. Different firms possess varying amounts of knowledge stock. Firms with a larger share of the knowledge stock are dominant in the sector and will charge higher prices than smaller firms.

In this paper, we extend traditional growth models in two directions. First, we relax the assumption of an efficient and competitive knowledge licensing market. Specifically, we consider an oligopolistically competitive patent market to introduce heterogeneity that firms would bid a firm-specific knowledge price depending on the amount of knowledge stock share and the set of sectors in which the firm innovates. This modification adds more realism and depth to the model, capturing the complexity and dynamic nature of the patent market and its relationship with firm-specific knowledge pricing. Second, we introduce a class of indirectly additive preferences into the product market to generate endogenous markups which reconcile with a strictly positive growth rate of the number of varieties.

The model is rooted in variety expansion rather than quality ladder setup: firms invent new blueprints by adapting prior knowledge across various sectors through research and development (R&D). Applicable technologies not only enhance the innovative productivity of R&D, but also contribute to a diverse range of innovations in many sectors. In the process of adapting prior knowledge, firms can leverage their own proprietary knowledge, public knowledge, and acquired licenses to use the private knowledge of other firms in different sectors. This occurs within an imperfectly competitive licensing market. Specifically, if a firm decides not to innovate in sector i during a given period, it finds it optimal, in equilibrium, to license the application rights of its prior knowledge in sector j to other innovating firms in sector i during that period, assuming perfect protection of intellectual property rights. Within each sector, a finite number of heterogeneous firms face oligopolistic competition and set variable markups, following the approach of Atkeson (2008). The markup charged increases with the patent share of the firm: larger firms impose higher markups.

A firm is obligated to pay an idiosyncratic fixed cost to conduct research in any sector at any given time. Failure to pay this cost results in the cessation of developing new blueprints in that sector. This continuation cost can be interpreted as a license fee, representing the financial outlay for maintaining a research lab or as an entry cost. The equilibrium licensing fees, which clear the market, thus mirror the "application value" of source knowledge j for innovation in sector i. The presence of a knowledge market allows for the utilization of all knowledge in equilibrium, either by its original inventor or by other firms that have acquired its application rights. Consequently, the equilibrium value of knowledge capital in sector j is not only determined by the profit it generates in its own sector, as in conventional models. It also hinges on its application value across all sectors. A higher application value, coupled with a greater market share, entices firms to invest in R&D in that sector. A firm engages in research in a particular sector only if its knowledge is sufficiently applicable and its market share is substantial enough to generate an expected value exceeding its fixed costs. Given that the fixed costs faced by each firm are stochastic and idiosyncratic, the research decision is determined only by the expected value of innovation. This expected value is also an increasing function of the firms' share of the knowledge stock.

In any given industry, incumbent firms innovate and expand in size as they introduce new varieties.

Nevertheless, innovation may halt or cease when these firms encounter a sequence of adverse R&D shocks or face high fixed costs. This dynamically results in the organic sorting of firms. Larger firms are expected to have a higher probability of innovating, making it easier for them to expand. Conversely, smaller firms may struggle to survive as the cost of entry rises. However, once they achieve success, smaller firms are more inclined to defend and maintain their established status.

In addition, prospective innovators are inclined to enter the market if they face low fixed costs and have amassed sufficient knowledge capital, either through internal knowledge creation or external knowledge acquisition, especially in related sectors. Incumbent firms demonstrate a higher likelihood of innovation within an environment characterized by oligopolistic competition. In contrast, within monopolistic or perfectly competition environments, firms tend to be too small to wield significant market power. Consequently, larger firms with a more substantial share of blueprints in a sector possess greater pricing influence and derive more research-related profits. This circumstance leads to a polarization in R&D: firms with a larger market share tend to develop more blueprints, while smaller firms may engage in the opposite behavior or even exit the market altogether.

Furthermore, as firms expand and amass more private knowledge, they extend their reach into all accessible and promising sectors. Interestingly, innovative firms tend to enter more new sectors when entry costs increase. With the escalation of entry costs, the resources held by firms become relatively scarce. However, firms have the potential to allocate their own profits to support other research projects encountering challenges or to acquire new market shares as a strategy to overcome difficulties. It will be demonstrated that monopoly profits have the capacity to offset entry costs.

Our model extends the existing literature on dynamic firm innovation and growth with endogenous markups to a multi-sector discrete-time framework Boucekkine (2017), aligning with the tradition of variety-expanding models, as seen in Romer (1990) and others. While most models of monopolistic competition with endogenous markups Etro (2013) suggest that the markup is a variable function of the number of varieties, these models often fail to generate long-run growth. This limitation does not apply to the class of indirectly additive preferences. By employing the general class of indirectly additive preferences in an otherwise perfectly standard expanding product-variety model, we establish a framework that incorporates both endogenous markups and a strictly positive growth rate along the Balanced Growth Path (BGP).

Another rationale for the aforementioned extension is our aim to explore the intricate relationship between the product market and the R&D market. In our investigation, innovating firms not only devise blueprints for new varieties but also engage in the manufacturing of the products they invent. This integration serves to bridge the gap between the product market and the R&D market. As elucidated in the model, on the one hand, large firms in the R&D market are still large firms in the product market, and large firms in the product market would charge a higher price for their product so that they can charge a higher profit, which is a part of the value of their knowledge on the other hand. In essence, it explains why both markets exhibit similar trends in industry concentration.

2. Literature Review

The findings of this paper hold significance and contribute to the existing body of research on the evolution of market competition. Grullon (2019) documents a notable trend over the last 15 years, wherein the level of product market concentration in the U.S. has increased across most industries. This shift is attributed to the consolidation of public firms into mega firms. Zheng (2022) observes a similar phenomenon in the R&D and patent market. In a related vein, Autor (2017) present empirical patterns suggesting that globalization and technological changes favoring the most productive firms lead to an increase in product market concentration. This trend results in industries being increasingly dominated by superstar firms characterized by high profits and a low share of labor in firm value-added and sales. Similarly, Hsieh (2019) establish that the surge in national industry concentration in the U.S. from 1977 to 2013 is limited to three broad sectors, primarily driven by a new industrial revolution. Their model describes the availability of a new set of fixed-cost technologies that enable adopters to produce at lower marginal costs across all markets.

Aghion (2023) develope a framework to analyze the impact of regulation on innovation. They demonstrate that the prospect of regulatory costs discourages firms just below the threshold from engaging in innovative activities. Gutierrez (2019) investigate the entry and exit patterns of firms across U.S. industries over the past 40 years, concluding that lobbying and regulations have impeded free entry. Crouzet (2019) illustrate that the surge in intangibles is primarily driven by industry leaders, coinciding with an increase in their market share and, consequently, a rise in industry concentration. Hopenhayn (2018) reveal that the interplay of population and firm demographics can account for the decline in startup rates, firm exit rates, and the increase in average firm size in the US. This paper emphasizes the role of imperfectly competition in patent market as a potentially significant new channel for explaining industry concentration in innovation.

In recent years, a plethora of theoretical and empirical literature has emerged, focusing on models of monopolistic competition featuring variable markups. Behrens (2007) and Zhelobodko (2012) center their attention on additively separable preferences, while Melitz (2008) and Demidova (2017) embed a non-separable quadratic utility into a quasi-linear framework. Notably, these models mentioned above have demonstrated tractability in a static framework, as highlighted by Boucekkine (2017). However, extending them to a dynamic growth model poses challenges.

In the context of an expanding variety model necessitating a continual increase in blueprints for long-run growth, the aforementioned models fall short in generating sustained growth. This limitation does not apply to the class of indirectly additive preferences. Models with indirectly additive preferences can produce an endogenous markup that reconcile with the growth rate of varieties. While Boucekkine (2017) present a one-sector continuous-time model, our approach involves a multi-sector discrete model. We extend their model due to the foundation of our model lying in the knowledge network of inter-sectoral linkages.

A second strand of literature concerning endogenous variable markups centers on market structure.

Atkeson (2008) employ a nested constant elasticity substitution demand system that generates variable markups under imperfectly competition, with a focus on international pricing dynamincs. Grassi (2018) develops a tractable multi-sector heterogeneous-firm general equilibrium model featuring oligopolistic competition and input-output network. His primary emphasis lies in demonstrating how firm-level productivity shocks can cascade into sector- and macroeconomic-level outcomes. Moreover, the structural importance of a firm is determined by the interaction of the sector-level competition intensity, the firm's sector position in the I-O network and the firm size. In contrast to these papers, our work introduces an imperfectly competition structure into knowledge market, wherein firms innovate by applying existing knowledge from related sectors. Zheng (2022) accentuates the position of industry in the knowledge network. Their exploration delves into the oligopolistic market structure of sectors in which firms do take into account the effect of their decisions on patent price and quantity through the knowledge network.

Finally, this paper aligns with a body of literature centered on multi-product firms, conceptualized as entities engaging in the innovation of multiple patents or venturing into diverse industries and sectors. Bernard (2010) examines a novel aspect of firm adjustment-the “extensive margin”-by investigating the reassignment of resources within surviving firms as they both add and drop (i.e., “switch”) products. However, empirical examination of firms' product mixes reveals a tendency for firms to co-manufacture products within the same industry, or within “linked” industries. Building on this theme, Neary (2010) presents a new model of multi-product firms and flexible manufacturing, in which they predict that selection effects operate at the product level, with firms encouraged to focus on the “core competence” and drop marginal high-cost varieties. The work of Klette (2004) establishes connections between growth theories and findings from firm-level and sectoral-level studies of innovation. Akcigit (2018) study a model of innovation within multi-sector firms, with a focus on internal versus external innovation and its correlation with the distribution of firm sizes.

3. Model

3.1 Consumption

There is a unit measure of identical infinitely-lived households. The preference of a representative household is :

$$U = \sum_{t=0}^{\infty} \beta^t \log W_t$$

Where is the discount factor and is a lifetime stream of indirect utility of the sector composite final good.

The indirect utility of the sector composite final good is generated by combining all types of sectoral intermediate indirect utility according to a C-D function:

$$\log W_t = \sum_{i \in \mathcal{I}} s^i \log(V_t^i)$$

Where e presents the share of sector i that contributes to the composite utility. $\mathcal{I}$ is the set of all the sectors and K is the total number of sector.

We can rewrite the preference of a representative household as:

$$U = \sum_{i \in \mathcal{I}} s^i \sum_{t=0}^{\infty} \beta^t \log(V_t^i)$$

A typical household inelastically supplies a fixed unit of labor L, which is allocated to produce goods, to conduct research or to maintain research labs (fixed costs of research). Households have access to a one-period risk-free bond with interest rate and in zero aggregate supply.

There are K sectors in the economy. Each sector contains a set of varieties that are invented by different firms at different times. We denote the set of varieties by $[0, n_t^i]$, is the measure of the varieties in sector i at time t. Different firms possess different proportions of the varieties.

A representative consumer derives from her consumption an indirect utility $V_t^i = V(\mathbb{P}_t^i, E_t^i)$, where is the price vector of the available varieties and is the overall individual expenditure in sector i at time t, with and denoting the price and consumption of variety k in sector i at time t, respectively. We assume that the indirect utility is additively separable (and using the degree-zero homogeneity property met by any indirect utility function), we are able to further characterize in the following way:

$$V_t^i = \int_0^{n_t^i} v\left(p_{k,t}^i, E_t^i\right) dk = \int_0^{n_t^i} v\left(\frac{p_{k,t}^i}{E_t^i}\right) dk$$

Further assuming that is thrice differentiable, decreasing and convex.

The representative consumer maximizes her utility under the intertemporal budget constraint:

$$B_t = w_t L^i + (1 + r_{t-1}) B_{t-1} - E_t^i$$

Where is the individual stock of risk-free bond and is the wage share in sector i. The Euler condition takes the following form:

$$\frac{E_{t+1}^i}{E_t^i} \frac{\mu\left(\frac{E_{t+1}^i}{p_{t+1}^i}\right)}{\mu\left(\frac{E_t^i}{p_t^i}\right)} = (1 + r_t)\beta$$

With $\mu(z) \equiv -\frac{v\left(\frac{1}{z}\right)}{v'\left(\frac{1}{z}\right)} z > 0$.

This Euler condition is different from the one obtained in the case of the standard expanding-variety model featuring a CES-type utility function. More precisely, it differs by one term: We refer to as a measure of consumer's preference for variety (PFV). It represents the change in expenditure following a variation in the number of varieties when the consumer wishes to keep her utility constant. We apply Roy's identity to determine the demand for each variety:

$$x_{k,t}^{i} = \frac{v'\left(\frac{p_{k,t}^{i}}{E_t^{i}}\right)}{\int_0^{n_t^i} v'\left(\frac{p_{k,t}^{i}}{E_t^{i}}\right)\frac{p_{k,t}^{i}}{E_t^{i}}dk}$$

The total market demand for one variety is then of the form:

3.2 Production

Firms undertake two distinct activities. They can not only create blueprints for new varieties of different products but also manufacture the products that have been invented. The firm inventing a new variety is the unique supplier of that variety. We assume that each differentiated good is manufactured according to a common technology: to produce one unit of any variety requires one unit of labor.

The firm producing variety k in sector i faces a residual demand curve. Wage is normalized to one: $w_t = 1$. Producers maximize the following profits:

$$\pi\left(p_{k,t}^{i}, E_t^{i}\right) = \frac{\left(p_{k,t}^{i} - w_t\right)v'\left(\frac{p_{k,t}^{i}}{E_t^{i}}\right)}{\int_0^{n_t^i} v'\left(\frac{p_{k,t}^{i}}{E_t^{i}}\right)\frac{p_{k,t}^{i}}{E_t^{i}}dk}$$

Profit maximization with respect to price, $p_{k,t}^{i}$, yields the following FOC:

$$\frac{\left(p_{k,t}^{i} - w_t\right)v''\left(\frac{p_{k,t}^{i}}{E_t^{i}}\right)}{E_t^{i}} + v'\left(\frac{p_{k,t}^{i}}{E_t^{i}}\right) = 0$$

Without heterogeneity in production and consumption, all varieties in the same sector are completely symmetric: they charge the same price and are sold in the same quantity. So We write and instead of and respectively.

This FOC yields the following formula for the relative markup M_t:

$$\frac{\left(p_t^{i} - w_t\right)}{p_t^{i}} = -\frac{v'\left(\frac{p_t^{i}}{E_t^{i}}\right)}{v''\left(\frac{p_t^{i}}{E_t^{i}}\right)\frac{p_t^{i}}{E_t^{i}}} = M_t$$

If the second-order condition is satisfied. We can define the individual overall consumption level as follows:

$$X_t^i = n_t^i x_t^i = \frac{E_t^i}{p_t^i}$$

Then we can obtain the following expression for the price charged by each symmetric firm at time t:

$$p_t^i = \frac{w_t}{1 - M_t} = \frac{1}{1 - M_t}$$

Contrary to a CES model, the price-elasticity of demand hence varies with overall individual consumption. The associated profits are:

$$\pi(p_t^i, E_t^i) = (p_t^i - w_t)q_t^i = \frac{M_t}{1 - M_t}\frac{X_t^i}{n_t^i}$$

4. Innovation

There are K sectors in our economy. Each sector i contains firms and the total number of firms in the economy is $M = \sum_{i=1}^{K} M_i$. Each firm develops new varieties and produces goods in a set of sectors. The firm k in sector i combines R&D and other sector's knowledge to create new blueprints. The R&D investment of firm k in sector i is denoted by $R_{k,t}^i$. The R&D is measured by the amount of researchers. The firm l in sector j's knowledge that is used by firm k to innovate in sector i is used for the invention of sector j's knowledge in a CES form:

$$Z_t(i,k,j) = \left\{\sum_{l=1}^{M_j} Z_t(i,k,j,l)^{\frac{\varepsilon_j - 1}{\varepsilon_j}}\right\}^{\frac{\varepsilon_j}{\varepsilon_j - 1}}$$

Where is the elasticity of substitution across different knowledge within industry j. Note that the elasticities of substitution are the same for all users of an industry's knowledge.

Firm k's knowledge capital in sector i in one time period accumulates as follows:

$$Z_{k,t+1}^i = Z_{k,t}^i + \Delta Z_{k,t}^i$$

Where the newly invented knowledge, $\Delta Z_{k,t}^i$, is the composite of prior related knowledge in all sectors and R&D investment. Apparently, knowledge linkages across sectors in the network are heterogeneous. In the sector level, "central sectors" act as knowledge suppliers and the others are demanders. In the firm level, each innovating firm acquires knowledge from different firms in other sectors. The knowledge network is thus formed based on the knowledge flows across sectors.

Formally, new knowledge in sector i is created based on the knowledge creation function:

$$\Delta Z_{k,t}^i = \alpha_i A_i \left(R_{k,t}^i\right)^{r_i} \prod_{j=1}^{K} Z_t(i,k,j)^{\beta_{ij}}$$

The parameter governs the share of R&D in the knowledge creation. Here, is the share of sector j's knowledge in the knowledge creation in sector i. Thanks to constant return-to-scale,

$\beta_i = \sum_{j=1}^{K} \beta_{ij} = 1 - r_i$. denotes the licensing fee of using sector j's knowledge to innovate in sector i, then represents the aggregate licensing fee of using other sectors' knowledge to innovate in sector i. The matrix of ω_{ij}, denoted by $\mathbb{W}$, determines the network-structure of this economy. The parameter captures the share of prior knowledge in the knowledge creation. The normalization constant makes the mathematics simpler and is equal to . This knowledge creation function is similar with Grassi (2018) of the constant return-to-scale Cobb-Douglas type of firm's production function. There are two main differences between our models. Production is standard in his model: part of the products produced by each firm is consumed by the final consumer, and the remaining part is sold to other firms. However, in our model, firms can not only create blueprints for new varieties of different products but also manufacture the products that have been invented. The products that manufactured are then finally consumed by a representative consumer. More importantly, R&D are cumulative in a sense that knowledge can be used in every lasting period and never fade away. Therefore, the knowledge stock of innovating firms is constantly increasing.

4.1 Fixed Costs

In order to develop blueprints in any sector $i \in \mathcal{I}$, the firm k must pay a per period sector-specific fixed cost of $\Omega_{k,t}^{i} > 0$, measured in units of labor. For simplicity, we assume the sector-specific fixed cost is the same across firms in this sector, $\Omega_{k,t}^{i} = \Omega_{t}^{i}$. This sector-specific cost of innovation can be interpreted as legal barrier to entry or the cost of maintaining a research lab.

4.2 Timing

A firm k begins period t with a knowledge portfolio $Z_{k,t} = \left(Z_{k,t}^{1}, Z_{k,t}^{2}, \cdots, Z_{k,t}^{K}\right)$. If $Z_{k,t}^{i} > 0$, firm k retains knowledge in sector i at time t without depreciation. indicates that the firm has never entered into this sector and consequently doesn't own any knowledge stock. At the beginning of time t, the firm faces a set of the fixed costs. It then decides on whether to conduct R&D in each sector $i \in \mathcal{I}$. If it innovates in i, the firm would optimally acquire more related knowledge and decide its optimal R&D. new blueprints are then created which will generate profit in the next period and the firm updates its knowledge capital in sector i to $Z_{k,t+1}^{i}$. If the firm pauses its R&D activity in sector i at time t, it continues producing and making profit using its existing knowledge, and at the same time licenses the application rights of all its related knowledge to other firms. A similar process takes place in every sector, and the firm enters period t+1 with a knowledge portfolio $Z_{k,t+1}$. Each firm compares the value of these two options to make decisions in each sector based on the realized fixed costs and its existing knowledge $Z_{k,t}$.

4.3 Patent Market Structure

We assume that the individual patents developing firms are engaged in imperfectly competition. In most of the results that follow, We take as a baseline case a model of imperfectly competition based on the following two assumptions.

Assumption 1: patents are imperfectly substitutes: $\varepsilon_j < \infty, \forall j$.

Assumption 2: Firms play a static game of quantity competition. Specifically, each firm k chooses its patent quantity sold in industry i taking as given the quantities chosen by the other firms in the economy, as well as the aggregate patent price and quantity Z_t. Note that under this assumption, each firm does recognize that sectoral theoretical price index and quantity index vary when that firm changes its quantity $Z_t(i,k)$.

We solve the firm's pricing problem under the assumptions above as follows. Putting the assumptions and profit maximization conditions together allows us to give the following characterization of innovating firm's pricing strategy in term of endogenous variable markup.

Proposition 1 (Firm's Knowledge Pricing)

Under assumptions 1 and 2, firm k in sector i sets a price $P_t(i,k)$, a markup and has a patent share that satisfies the following system of equations:

$$P_t(i,k) = \psi_t(i,k)\mu_t(i,k)$$

$$\mu_t(i,k) = \frac{\varepsilon_i}{\varepsilon_i - 1 - (\varepsilon_i - 1)s_t(i,k)}$$

$$s_t(i,k) = \frac{P_t(i,k)Z_t(i,k)}{P_t^i Z_t^i}$$

The first thing to note in the above proposition is that firms charge a markup over their marginal cost $\psi_t(i,k)$. The markup charged is increasing in the patent share of the firm: larger firms charge a higher markup. When $s_t(i,k) \to 0$, $\mu_t(i,k) \to \frac{\varepsilon_i}{\varepsilon_i - 1}$. That is to say, as a firm becomes atomistic, its markup approaches the one under monopolistic competition.

4.4 Knowledge Value

Let denote the market value of per variety of firm k in sector i at time t. Since all existing knowledge is used to create new knowledge by its original inventor or its licensee in the economy at any period, the market value of a given unit of knowledge capital is not only given by the present discounted value of its future profits in its own sector through manufacturing products but also by its application values in other sectors. Therefore,

$$V_t(i,k) = \sum_{\tau=1}^{\infty} \frac{1}{(1+r)^{\tau-1}} \{\pi_{t+\tau}^{i} + E_t[P_{t+\tau}(i,k)]\}$$

Where is the price of per patent of firm k in sector i at time and is again the profit of manufacturing products based on this patent/variety. Given the nature of patents, the application right of time t created variety cannot be sold to other firms until next period. Hence, the market value of time t created variety is calculated from time t+1. We take expectation operator before patent price because firm can only determine own newly invented knowledge, thus the firm don't know exactly how much knowledge other firms have invented or its knowledge share in the next period.

4.5 Firm's R&D, Sectoral Entry and Exit Decisions

A firm k, given its existing knowledge and value of per variety, makes three decisions--optimal R&D investment $R_{k,t}^{i}$, optimal sectoral knowledge acquisition and innovation decisions in every sector i. The decisions are made by weighing the tradeoff between the expected gain from conducting research in that sector and the sector specific fixed cost of Ω_t^i. Therefore the firm solves the following maximization problem in i:

$$\mathrm{Max}_{R_{k,t}^{i},\{Z_t(i,k,j)\}_{j\in\mathcal{J}},I_{k,t}^{i}} \left\{ \frac{1}{1+r} E_t\left[V_t(i,k)\Delta Z_{k,t}^{i}\right] - R_{k,t}^{i} - \sum_{j\in\mathcal{J}} P_t(i,k,j)\, Z_t(i,k,j), \Omega_t^{i} \right\}$$

Subject to the knowledge creation function and value function per variety. Here, refers to the theoretical price index of sectoral knowledge and $P_t(i,k,j) = \left\{\sum_{l=1}^{M_j} P_t(j,l)^{1-\varepsilon_j}\right\}^{\frac{1}{1-\varepsilon_j}} \equiv P_t^{j}$.

Conducting research in sector i entails an researcher cost $R_{k,t}^{i}$, knowledge acquisition cost $\sum_{j\in\mathcal{J}} P_t(i,k,j)\, Z_t(i,k,j)$. But the effort creates additional blueprints of $\Delta Z_{k,t}^{i}$, which generates a present value of in expectation. On the other hand, if firm k does not innovate in sector i, it saves fixed costs while has no innovation profits. The firm would innovate in this sector if and only if the latter is larger than the former.

We solve the optimization problem by two steps. We calculate the maximum value of innovating in sector i in the first step. In the second step, we compare the value of innovate or not to make the innovation decision.

Firstly, suppose firm k does innovate in sector i, then the firm solves the optimization problem:

$$\lambda_{k,t}^{i} = \mathrm{Max}_{R_{k,t}^{i},\{Z_t(i,k,j)\}_{j\in\mathcal{J}}} \left\{ \frac{1}{1+r} E_t\left[V_t(i,k)\Delta Z_{k,t}^{i}\right] - R_{k,t}^{i} - \sum_{j\in\mathcal{J}} P_t(i,k,j)\, Z_t(i,k,j) \right\}$$

Note that we assume the knowledge creation function is constant returns to scale. Thanks to the good properties of such function, the optimization problem can be simplified as follows:

$$\lambda_{k,t}^{i} = \mathrm{Max}_{\Delta Z_{k,t}^{i}} \left\{ \frac{1}{1+r} E_t\left[V_t(i,k)\Delta Z_{k,t}^{i}\right] - \psi_t(i,k)\Delta Z_{k,t}^{i} \right\}$$

Where is marginal cost of knowledge creation defined above.

To simplify the analysis, We made two following assumptions:

Assumption 3 $E_t[P_{t+\tau}(i,k)] = E_t[P_{t+1}(i,k)]\ \forall \tau \geq 1$

Assumption 4 $E_t[s_{t+1}(i,k)] = s_t(i,k)$

These two assumptions express the same thing: since the firms don't know whether the cost of entry in the next period will fall or rise. On average, they expect a constant entry cost, then their market share keeps unchanged.

Proposition 2 (Innovation Rate)

Under assumptions 3 and 4, larger firms are likely to invest relatively more in R&D and are more active in the knowledge market than small firms:

$$\frac{\Delta Z_{k,t}^{i}}{Z_t(i,k)} \propto s_t(i,k)$$

We can obtain innovation profit $\lambda_{k,t}^{i} = \lambda_{k,t}^{i}\big(s_t(i,k)\big) \propto s_t(i,k)$.

Secondly, in deciding whether to innovate in sector i, the firm compares the innovation profit against its fixed costs Ω_t^{i}. The firm would innovate in i if and only if $\lambda_{k,t}^{i} > \Omega_t^{i}$. It means that there is a market share cutoff $s_t^{i}(\Omega_t^{i})$, below which firms would not innovate in this sector. The dynamics of firm's market share is presented in the following proposition.

Proposition 3 (Dynamics of Market Share)

$$\begin{cases} s_{t+1}(i,k) > s_t(i,k)\,, \text{if } s_t(i,k) > s_t^{i} \\ s_{t+1}(i,k) < s_t(i,k)\,, \text{if } s_t(i,k) < s_t^{i} \end{cases}$$

$$s_{t+1}(i,k) > s_{t+1}(i,j)\,, \text{if } s_t(i,k) > s_t(i,j)$$

Proposition 3 shows the dynamics of firm's market share which illustrates that when the market share of firm k in sector i is bigger than the cutoff at time t, then the market share of firm k grows over time, vice versa. More importantly, the order of market share of any two firms does not change over time. Small firms are getting smaller, and large firms are getting bigger. As a result, industrial concentration is increasing. Since HHI is used as index for industrial concentration, we construct the positive correlation between sector-level markup and sector-level HHI as follows.

An important variable is the sector-level markup. This markup is defined as the sector-level price divided by the sector-level marginal cost. For a given sector i, the marginal cost is defined as $\psi_t^i = \frac{dTC_t^i}{dZ_t^i}$, where is the total cost in sector i: $TC_t^i = \sum_{k=1}^{M_i} \psi_t(i,k) Z_t(i,k)$. Note that in the context of constant return-to-scale, the marginal cost is also equal to the average cost: $\psi_t^i = \frac{TC_t^i}{Z_t^i} = \sum_{k=1}^{M_i} \psi_t(i,k) \frac{Z_t(i,k)}{Z_t^i}$. After using the fact that firm-level price is a markup over the marginal cost, it is easy to see the sector-level markup is

$$\mu_t^i = \frac{P_t^i}{\psi_t^i} = \left[\sum_{k=1}^{M_i} \mu_t(i,k)^{-1} s_t(i,k)\right]^{-1}$$

The sector's markup is a patent-share-weighted harmonic average of firm-level markups. This expression is valid as long as firms charge a markup over the marginal cost. Lemma 1 below shows the sector-level markup is a function of the sector-level concentration index. In particular, the directly observable HHI, the sum of the patent share squared, plays an important role.

Lemma 1 (Sector-Level Markup)

The sector i's markup is equal to $\mu_t^i = \frac{\varepsilon_i}{\varepsilon_i - 1}(1 - HHI_t^i)^{-1}$.

Where is the sector i's Herfindahl-Hirchman-Index.

The above lemma shows that under imperfectly competition, the HHI entirely determines the sector's markup. The intuition is as follows: when the sector's concentration is high, that is when the HHI is high, large firms have a higher market share and thus they can use this higher market power to charge higher markups, which in turn aggregate to a higher sector's markup. An important implication of Lemma 1 is that it links empirically observable variables, such as the HHI, to the sector-level markup.

Using the result in the above proposition, it is easy to derive some comparative statics of the markup with respect to the HHI while keeping everything else constant.

$$\frac{\partial \mu_t^i}{\partial HHI_t^i} = \frac{\varepsilon_i - 1}{\varepsilon_i}(\mu_t^i)^2 > 0$$

A higher sector's HHI always implies a higher sector's markup. This relationship is stronger for low-competitive, high-markup sectors. In this framework, given the demand system and the assumed market structure, sector concentration is a measure of sector competition.

5. Equilibrium

5.1 Knowledge Market Clearing

The knowledge market clears for each source-application sector pair. We can express the knowledge of firm l in sector j that is utilized by other firms as $\sum_i \sum_k Z_t(i,k,j,l)$. Therefore, knowledge market clearing implies:

$$\sum_l \sum_i \sum_k Z_t(i,k,j,l) = n_t^j, \forall j \in \mathcal{J}$$

5.2 Labor Market Clearing

The population inelastically supplies L units of labor in each period which is allocated in three areas: to produce goods, to conduct research or to maintain research labs (fixed costs of research). Formally, the labor market clearing condition is:

$$\sum_i \frac{E_i}{p_i} + \sum_i \sum_k R_k^i + \sum_i \sum_k \Omega_k^i = L$$

5.3 General Equilibrium

Definition: A stationary balanced growth path (BGP) is an equilibrium path in which output, consumption and innovation grow at constant rates in every sector. It is given by: time paths of aggregate quantities and prices $\{X_t^i, p_t^i, w_t, r_t\}_{t=0}^{\infty}$; time paths of sectoral numbers of varieties, numbers of firms, quantity and price of rental application rights and knowledge value $(n_t^i, M_t^i, P_t^i, Z_t^i, V_t(i,k))_{i\in\mathcal{J},k\in M_i,t=0}^{\infty}$; time paths of firms' R&D investment $(R_{k,t}^i)_{i\in\mathcal{J},k\in M_i,t=0}^{\infty}$, number of blueprints $(Z_{k,t}^i)_{i\in\mathcal{J},k\in M_i,t=0}^{\infty}$; and time paths of firm's sectoral entry and exit decisions $(I_{k,t}^i)_{i\in\mathcal{J},k\in M_i,t=0}^{\infty}$, such that:

1. Given w_t, and P_t^i, the representative household maximizes life-time utility subject to an inter-temporal budget constraint.

2. Given w_t, r_t, the individual firm decides on the quantity and prices of goods produced and production labor needed.

3. Given w_t, r_t, and Z_t^i, firms decide on optimal R&D investment, optimal acquisition of external knowledge, and sectoral entry and exit decision.

4. Knowledge markets clear.

5. Labor markets clear.

5.4 Comparative Statics

Given that a considerable body of research already examines firm creation and destruction, we focus on the features of variety switching by surviving firms. In particular, we neither treat exiting firms as those that drop all their patents, nor entering firms as those that add their knowledge without private blueprints.

Non-R&D firms with private blueprints do not exit the R&D market. Since knowledge doesn't depreciate, non-R&D firms still possess patents and sell their application rights to other firms. Once opportunity occurs, they will develop knowledge with private and other firms' blueprints. As documented in the model above, market share and entry cost determine the amount of R&D companies. Therefore, changes in market share and entry costs will lead to changes in the number of firms in the current sectors.

Proposition 4 (Market Share Cutoff Transition)

$$\frac{\partial s_t^i(\Omega_t^i)}{\partial \Omega_t^i} > 0$$

This proposition illustrates that higher entry cost induces higher barrier to entry, only the firms with sufficient market power can stay for R&D. Resulting in fewer firms to conduct research in current period.

5.5 Multi-Product Firms in Knowledge Network

This paper relates a strand of literature about multi-product firms. Multi-product firms are firms that produce multiple goods, and therefore have to deal with allocating inputs more properly in order to attain higher production levels. In this paper, we treat multi-product firms as firms that innovate multiple patents or innovate in different industries and sectors. Bernard (2010) examines a new, “extensive margin” of firm adjustment, the reassignment of resources that takes place within surviving firms as they add and drop (i.e., “switch”) products. Empirical examination of firms' product mix, however, reveals that firms are more likely to co-manufacture products within the same industry, or within “linked” industries. ECKEL (2009) presents a new model of multi-product firms and flexible manufacturing, in which they predict that selection effects operate at the product level, with firms encouraged to focus on their “core competence” and drop marginal high-cost varieties. Bernard (2011) find that more productive firms export more products to more destinations, and that trade liberalization is accompanied by firms focusing on their core competencies.

When the entry cost is relatively low, firms extend their tentacles to all accessible and promising sectors. However, as entry costs increasing, various resources occupied by enterprises become relatively scarce. Even if a company succeeds in its field, there is no way to make itself an expert company in the field by increasing research and development or deepening market share. Because the company must contribute its own profits to help other research projects that are facing difficulties or are opening up new market areas to overcome difficulties.

5.6 Sequential Sectoral Entry

Firms enter different sectors sequentially: they start developing new varieties in a sector that pay the least entry cost and minimum competition, building up private knowledge and then venturing into other sectors using R&D investment and accumulated knowledge. Suppose sectors are ranked by their sector-level markup, and $\mu_1 < \mu_2 < \cdots < \mu_K$. If firms all draw the same entry cost Ω, every new firm enters sector 1 first. Entry stops when the net value of entry is zero. Next, the firm accumulates more private knowledge and innovation profit to fill up the gap between the entry cost and sector competition. Since firms are facing idiosyncratic shocks to fixed costs, not all firms follow the exact same path expanding across the technology space. Yet, their entries are all path-dependent.

Proposition 5 (Relationship Between Product Market and Patent Market)

$$\widetilde{\mathrm{sale}}_k^i = \tilde{z}_k^i$$

Where is sales share of firm k in sector i and is firm k's share of patents in sector i. The first thing to note in the above proposition is that: for a given firm in a specific sector, the market share of sales and patents are equal. As an inference, the industry concentration of product and patent markets are the same.

5.7 Firm Size Distribution

The stationary firm innovation size distribution exhibits a Pareto tail in the steady state:

$$\Pr\left(\tilde{z}_k^i > z\right) \sim \left(\frac{z}{k_i}\right)^{-\mu_i}$$

Where is the scale parameter and is the shape parameter, and Pr stands for the steady state probability. The economic implication of this result is intuitive. When knowledge creation is stationary, it accumulates additively to knowledge stock. However, since existing knowledge stock is also used in the knowledge creation process, the multiplicative process of knowledge accumulation then leads to the Pareto tail distribution of firm size. Firms which have realized an extended series of high positive innovation shocks and low fixed costs populate the tail in the firm size distribution.

To gain further insights on the shape of the Pareto distribution, note that the lower bound of the distribution is associated with imitated new varieties. The existence of public knowledge plays an important role in attenuating the size dispersion generated by idiosyncratic innovation shocks such that the minimum firm size would not become too small. Since the R&D and production of the firms are carried out simultaneously. We can expect that the stationary firm sales distribution showing a similar steady state with firm innovation size distribution.

Proposition 6 (Pareto Distribution)

The stationary firm sales distribution exhibits a Pareto tail in the steady state:

$$\Pr\left(\mathrm{sale}_k^i > s\right) \sim \left(\frac{s}{E_i k_i}\right)^{-\mu_i}$$

Where is sales of firm k in sector i and is expenditure of sector i in the steady state. As before, is the scale parameter and is the shape parameter respectively. A lower means a higher degree of inequality in the distribution: it implies a greater probability of finding very large firms and higher HHI.

6. Conclusion

This paper documents that over the last 20 years the level of R&D market concentration in the US has increased across most industries. We show that increasing barriers into patent markets have implications to firm performance, as they affect profitability, innovation, and returns to investors. First, the increase in entry costs is associated with small firms to exit. Since small scale firms have less profit to offset the entry costs. Second, the remaining large firms generating higher profits survival rates through higher market power. Consistent with the idea that market power considerations are becoming a key source of value during the increase in industry concentration levels. Finally, the increase in concentration levels consolidates the market power of the incumbent firms furthermore to enter into more sectors. Firms enter into new sectors by tradeoff market competition and entry cost. After entering the new sectors, these firms gradually consolidate their position in order to earn more profit and enter into more sectors. In general, our findings suggest that despite popular beliefs, competition could have been fading over time.

We bridge the gap between the product market and the R&D market. As interpreted in the model, on the one hand, large firms in the R&D market are still large firms in the product market, and large firms in the product market would charge higher price on their product so that they can charge higher profit which is a part of the value of their knowledge on the other hand.

More broadly, the findings that firms in industries that have become more concentrated generate higher profit margins, and enjoy better investment opportunities should be of interest to policy makers. While at least parts of these gains appear to be transferred to the firms' shareholders, it is not clear whether the higher market concentration benefits consumers or other stakeholders. The increase in profit margins without a corresponding economically significant increase in efficiency may suggest the opposite. Although it is possible that a more concentrated nature of R&D and product markets improve the quality or variety of patents and products offered, it is unclear whether those changes are sufficient to compensate customers for the higher profit margins that firms enjoy. Our findings may motivate policy makers to examine the impact of the increased concentration further. This paper is a starting point for studying the quantitative implications of increasing concentration levels. What are the appropriate government policies to mitigate the potential inefficiencies? We leave these questions for future research.

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