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*Original Paper*

# Predicting Exchange Rates of Morocco Using an Econometric and a Stochastic Model

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***Abstract***
*To predict the exchange rate EUR / MAD & USD / MAD in Morocco we used two most answered methods in the theory: the Box-Jenkins econometric model and the stochastic model of Vasicek then the comparison of the forecasted data for the month of March 2018 of the two methods with the exchange rates actually observed allowed us to retain the econometric the autoregressive integrated moving average model ARIMA (2,1,2) for EUR / MAD and (3,1,2) for USD / MAD rather than the Vasicek model.*
***Keywords***
*exchange rate, EUR/MAD USD/MAD Time Series, Box-Jenkins, ARMA model, Morocco, Vasicek model*

## 1. Introduction

The exchange rate is a tool of the economic policy of any country, open to the outside world, it is considered both a means of monetary regulation and an ideal instrument of external competitiveness, it is in this sense that Morocco has opted for a more flexible exchange rate regime which presents a risk to be managed by the banks and the insurance companies following an indexation of their results on exchange rates or elements of the assets or liabilities which are denominated in currency. may manifest itself in the form of capital losses as a result of the interconnection of international markets, exacerbating the volatility of foreign exchange markets.

To help policymakers and ALM committee choose the best model for predicting USD / MAD and EUR MAD exchange rate developments we have performed an empirical study of the two best-regarded models in the field of exchange rate prediction namely the Box-Jenkins model and the Vasicek model.

## 2. Literature Review

*2.1 Box-Jenkins (Econometric) Model*

The use of the econometric model to predict the exchange rate has been a subject of considerable academic scrutiny over the past few decades. A study by Alam (2012) that in case of in-sample the ARMA (1,1) model, whereas both the ARMA (1,1) and AR(1) models are capable to add value significantly to the forecasting and trading BDT/USD exchange rate in the context of statistical performance measures. (Ghalayini, 2013) has construct an econometric models capable to generate consistent and rational forecasts for the dollar/euro exchange rate; (Liuwei, 2006) use different methods, such as AR, MA, and ARIMA to forecast the exchange rate of US Dollar / Euro in the month of February 2005. And a lot of other works like (Al-Hamidy, 2010; Alam, 2012; Cheung & Lai, 2008; Etuk, 2012; Ghalayini, 2013; Liuwei, 2006; Olatunji & Bello, 2015; Reddy SK, 2015; Weisang & Awazu, 2008).

*2.2 Stochastic Model (Vasicek)*

Amini (2012) uses the vasicek model to calibrate stochastic interest rate model Ayranci and Özgürel (2014) modeled time series of TRLIBOR interest rates with Vasicek Model and calibrated through OLS method, Hamilton and James (2001), and many other works discuss the Vasicek models.

## 3. Data and Estimation Techniques

*3.1 Data*

In order to compare the two models ( Box-Jenkins and Vasicek) for predicting the exchange rate we use two time series EUR/MAD and USD/MAD can be taken directly from Casablanca Stock Exchange url http://www.casablanca-bourse.com/bourseweb/index.aspx the period covered is from 03/01/2000 to 09/03/2018( 4742 observations)

*3.2 Model Specification*

**For the Box-Jenkins model**

A time series has the property that neighboring values are correlated. This tendency is called autocorrelation. It is said to be stationary if it has a constant mean, constant variance and autocorrelation that is a function of the lag separating the correlated values. The autocorrelation expressed as a function of the lag is called the autocorrelation function (ACF).

A stationary time series {Xt} is said to follow an autoregressive moving average model of orders p and q (denoted by ARMA(p,q) ) if it satisfies the following difference equation (Note 1)

$$X_t+\alpha_1 X_{t-1}+\alpha_2 X_{t-2}+...+\alpha_p X_{t-p}=\varepsilon_t+\beta_1\varepsilon_{t-1}++\beta_2\varepsilon_{t-2}+...+\beta_q\varepsilon_{t-q} \tag{1}$$

Or

$$A(L)X_t = B(L)\varepsilon_t \tag{2}$$

where $\{\varepsilon_t\}$ is a system of uncorrelated random variables with zero mean and constant variance, called a white noise process, and the $\alpha_i$'s and $\beta_j$'s constants;

$$A(L) = 1 + \alpha_1 L + \alpha_2 L^2 + ... + \alpha_p L^p$$

and

$$B(L) = 1 + \beta_1 L + \beta_2 L^2 + ... + \beta_q L^q$$

and L is the backward shift operator defined by $L^k X_t = X_{t-k}$.

If p = 0, model (1) becomes a moving average model of order q (denoted by MA(q)). If, however, q=0 it becomes an autoregressive process of order p (AR(p)). An AR(p) model of order p may be defined as a model for which a current value of the time series Xt depends on the immediate past p values: $X_{t-1}, X_{t-1}, ....X_{t-p}$. On the other hand, an MA(q) model of order q is whereby the current value Xt is a linear combination of the immediate past q values of the white noise process: $\varepsilon_{t-1}, \varepsilon_{t-1}, ....\varepsilon_{t-q}$.

An AR(p) can be modeled by:

$$X_t + \alpha_{p1} X_{t-1} + \alpha_{p2} X_{t-2} + ... + \alpha_{pp} X_{t-p} = \varepsilon_t$$

Then the sequence of the last coefficients { $\alpha_{ii}$ } is called the partial autocorrelation function of (PACF) (Note 2) of {Xt}. The ACF of an MA(q) model cuts off after lag q whereas that of an AR(p) model is a mixture of sinusoidals tailing off slowly. On the other hand, the PACF of an MA(q) model tails off slowly whereas that of an AR(p) model tails off after lag p.

AR and MA models are known to have some duality characteristics. These include:

1) A finite order of the one type is equivalent to an infinite order of the other type.

2) The ACF of the one type exhibits the same behavior as the PACF of the other type.

3) An AR model is always invertible but is stationary if (L) = 0 has zeros outside the unit circle.

4) An MA model is always stationary but is invertible if (L) = 0 has zeros outside the unit circle.

Parametric parsimony consideration in model building entails preference for the mixed ARMA fit to either the pure AR or the pure MA fit. Stationarity and invertibility conditions for model (1) or (2) are that the equations A(L) = 0 and B(L) = 0 should have roots outside the unit circle respectively.

If a time series is non-stationary, Box and Jenkins (1976) proposed that differencing of an appropriate order could render a non-stationary series {Xt} stationary. Suppose the degree of differencing necessary for stationarity is equal to d. Such a series {Xt} may be modelled as

$$(1+\sum_{i=1}^{p} a_i B^i)\nabla^d X_t = B(L)_{\varepsilon t} \tag{3}$$

where $\nabla = 1 - L$ and in which case $A(L) = (1+\sum_{i=1}^{p} aiB^i)\nabla^d = 0$ shall have unit roots d times. Then differencing to degree d renders the series stationary. The model (3) is said to be an autoregressive integrated moving average model of orders p, d and q and denoted by ARIMA(p, d, q).

**For the vasicek model**

$$dx_t = \alpha(\mu - xt)dt + \delta dW_t \tag{4}$$

Where:

$\alpha$ : the Mean reversion speed

$\mu$ : Long term mean/mean reversion parameter

$\delta$ : Standard deviation that determines the volatility of the rate of exchange

$W_t$ : Wiener process that models the risk factor of random market

Solving the Ornstein-Uhlenbeck Stochastic Differential Equation includes taking the derivative of which yields so:

The conditional mean and variance of $x_t$ given $x_0$ (see Appendix 1 for demonstration)

$$E_0\left[x_t\right] = \mu + (x_0 - \mu_t)e^{-\alpha t} \quad \text{and} \quad Var_0\left[x_t\right] = \frac{\sigma^2}{2\alpha}(1-e^{-2\alpha t}); \alpha > 0$$

The conditional mean and variance of $x_t$ given $x_s$

$$E_0\left[x_t\right] = \mu + (x_0 - \mu_t)e^{-\alpha(t-s)} \quad \text{and} \quad Var_0\left[x_t\right] = \frac{\sigma^2}{2\alpha}(1-e^{-2\alpha(t-s)}); \alpha > 0$$

If time increases the mean tends to the long-term value and the variance remains bounded, implying mean reversion. The long-term distribution of the Ornstein-Uhlenbeck process is stationary and is Gaussian with mean $\mu$ and variance $\sigma^2 / 2\alpha$

*3.3 Vasicek Model calibration: Ordinary Least Squares Estimation*

The linear relationship between two consecutive observations $x_{ti+1}$ and $x_{ti}$ is linear with independent identical random values $\varepsilon$ such that:

$$x_{ti+1} = ax_{ti} + b + \varepsilon$$

Where: $a = e^{-\alpha \Delta t}$ ; $b = \mu(1 - e^{-\alpha \Delta t})$ ; $\varepsilon_{sd} = \sigma \sqrt{\frac{1 - e^{-2\alpha \Delta t}}{2\alpha}}$

Express these equations in terms of the parameters $\mu$ , $\alpha$ and $\sigma$ which yield:

$$\mu = -\frac{\ln(a)}{\Delta t} \quad , \quad \mu = \frac{b}{1-a} \quad \text{et} \quad \sigma = \varepsilon_{sd} \sqrt{\frac{-2\ln(a)}{\Delta t(1-a^2)}}$$

The following formulas are used to simplify further calculations:

$$S_x = \sum_{i=1}^{n} x_{t_{i-1}} \; , \; S_y = \sum_{i=1}^{n} x_{t_i}$$

$$S_{xx} = \sum_{i=1}^{n} x_{i-1}^2 \; , \; S_{yy} = \sum_{i=1}^{n} x_{t_i}^2 \quad \text{et} \quad S_{xy} = \sum_{i=1}^{n} x_{t_{i-1}} x_{t_i}$$

The ordinary least square (OLS) estimates $\hat{\mu}$ , $\hat{\alpha}$ and $\hat{\sigma}$ are

$$\hat{\alpha} = \frac{\ln\left(\frac{nS_{xy} - S_x S_y}{nS_{xx} - S_x^2}\right)}{\Delta t} \quad , \quad \hat{\mu} = \frac{S_y - \left(\frac{nS_{xx} - S_x S_y}{nS_{xx} - S_x^2}\right) S_x}{n\left[1 - \left(\frac{nS_{xy} - S_x S_y}{nS_{xx} - S_x^2}\right)\right]} \quad ,$$

$$\hat{\sigma} = \hat{\varepsilon}_{sd} \sqrt{\frac{-2\ln\left(\frac{nS_{xy} - S_x S_y}{nS_{xx} - S_x^2}\right)}{\Delta t\left(1 - \left(\frac{nS_{xy} - S_x S_y}{nS_{xx} - S_x^2}\right)^2\right)}}$$

## 4. Empirical Results

### *4.1 Model Estimation*

The involvement of the white noise terms in an ARIMA model necessitates a nonlinear iterative process in the model estimation. An optimization criterion like the least squares, maximum likelihood or maximum entropy is used. An initial estimate is usually used and each iteration is expected to be an improvement of the previous one until the estimate converges to an optimal one. However, for pure AR and pure MA models linear optimization techniques exist (See for example Box and Jenkins (1976), Oyetunji (1985)). There are attempts to propose linear methods to estimate ARMA models (See for example, Etuk (1987, 1998)). We shall use Eviews software which employs the least squares approach to analyze the data.

*4.2 Diagnostic Checking*

The model that is fitted to the data should be tested for goodness-of-fit. The automatic order determination criteria AIC and SIC are themselves diagnostic checking tools. Further checking can be done by the analysis of the residuals of the model. If the model is correct, the residuals would be uncorrelated and would follow a normal distribution with mean zero and constant variance.

*4.3 Results and Discussion*

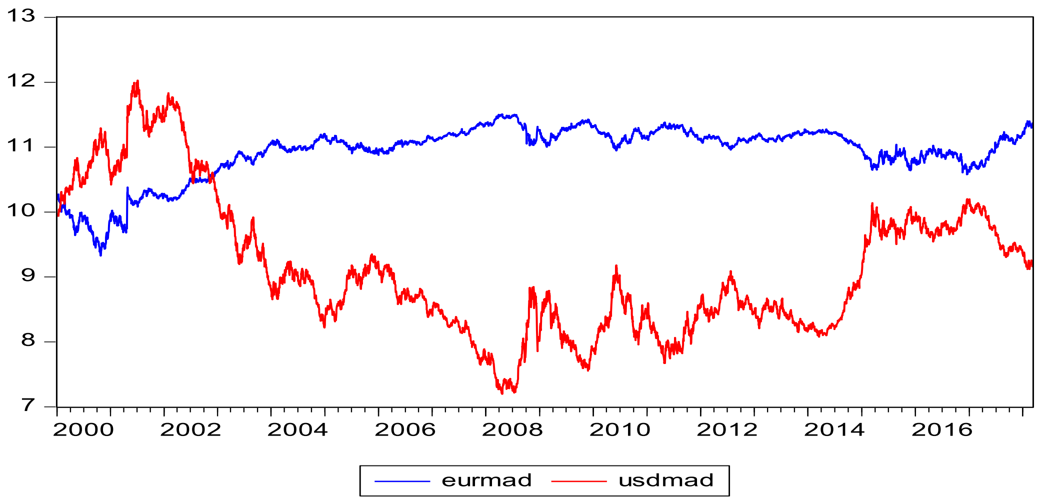



**Figure 1. Graph EUR/MAD & USD/MAD from 2000 to 2018**

Source: Established by as data from Casablanca Stock Exchange from 03/01/2000 to 09/03/2018

Figure 1 shows that the two series EUR / MAD and USD / MAD apparently are not stationary to the correlogram and the unit root test confirm the non-stationarity of the two series and none of them contain a trend as it confirmed in the ADF test (see Appendix 2). So, we move to differentiation as proposed by Box and Jenkins (1976) and covariance analysis shows the result of a negative correlation) that's explain the movement in the two curves in the opposite directions and TABLE1 shows that the covariance of the two series EUR / MAD and USD / MAD are < 0.

**Table 1. Covariance analysis EURMAD and USD MAD**

| Covariance | | |
|---|---|---|
| | USDMAD | EURMAD |
| USDMAD | 1.125525 | -0.398731 |
| EURMAD | -0.398731 | 0.176766 |

Null Hypothesis: D(USDMAD) has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=31)

| | | t-Statistic | Prob.* |
|---|---|---|---|
| Augmented Dickey-Fuller test statistic | | -72.46790 | 0.0001 |
| Test critical values: | 1% level | -2.565450 | |
| | 5% level | -1.940891 | |
| | 10% level | -1.616655 | |

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(USDMAD,2)
Method: Least Squares
Date: 03/23/18 Time: 15:07
Sample (adjusted): 1/05/2000 3/09/2018
Included observations: 4740 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| D(USDMAD(-1)) | -1.051308 | 0.014507 | -72.46790 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | 0.525654 | Mean dependent var | -2.32E-07 |
| Adjusted R-squared | 0.525654 | S.D. dependent var | 0.065786 |
| S.E. of regression | 0.045308 | Akaike info criterion | -3.350442 |
| Sum squared resid | 9.728423 | Schwarz criterion | -3.349078 |
| Log likelihood | 7941.546 | Hannan-Quinn criter. | -3.349962 |
| Durbin-Watson stat | 1.998675 | | |

**Figure 2. Unit Root Test of D(USD/MAD)**

Source: Established by as data from Casablanca Stock Exchange from 03/01/2000 to 09/03/2018

The series D(USD/MAD) is stationary we have $\left|T - statistic\right| = 72.46 > \left|2.56\right|$ and the probability p = 0.0001, so we accept the hypothesis of the stationarity of D(USD/MAD) series.

Null Hypothesis: DEURMAD has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=31)

| | | t-Statistic | Prob.* |
|---|---|---|---|
| Augmented Dickey-Fuller test statistic | | -80.29489 | 0.0001 |
| Test critical values: | 1% level | -2.565450 | |
| | 5% level | -1.940891 | |
| | 10% level | -1.616655 | |

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(DEURMAD)
Method: Least Squares
Date: 03/23/18 Time: 15:09
Sample (adjusted): 1/05/2000 3/09/2018
Included observations: 4740 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| DEURMAD(-1) | -1.152035 | 0.014348 | -80.29489 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | 0.576356 | Mean dependent var | -1.38E-05 |
| Adjusted R-squared | 0.576356 | S.D. dependent var | 0.035625 |
| S.E. of regression | 0.023188 | Akaike info criterion | -4.690171 |
| Sum squared resid | 2.548035 | Schwarz criterion | -4.688807 |
| Log likelihood | 11116.71 | Hannan-Quinn criter. | -4.689692 |
| Durbin-Watson stat | 2.010191 | | |

**Figure 3. Unit Root Test of D(EUR/MAD)**

Source: established by as data from Casablanca Stock Exchange from 03/01/2000 to 09/03/2018

We have a $|T - statistic| = 80.29 > |2.56|$ and the probability p = 0.0001, so we accept the hypothesis of the stationarity of D(EUR/MAD series.

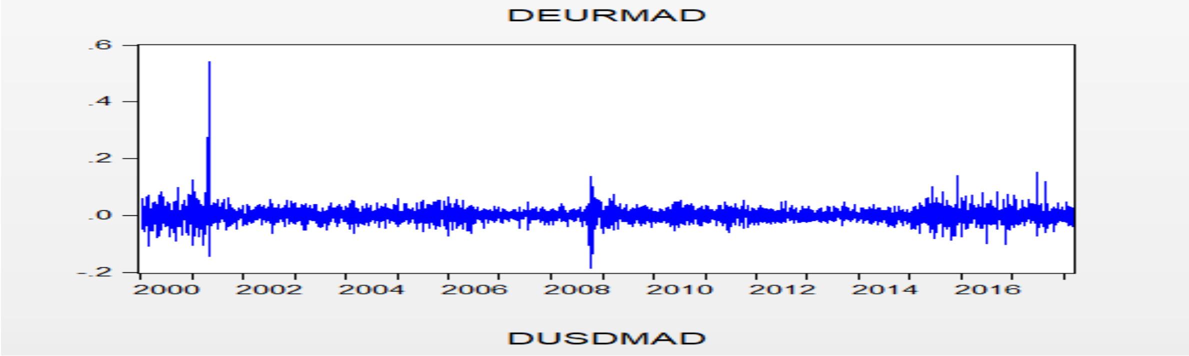



**Figure 4. Graph of D(EUR/MAD)**

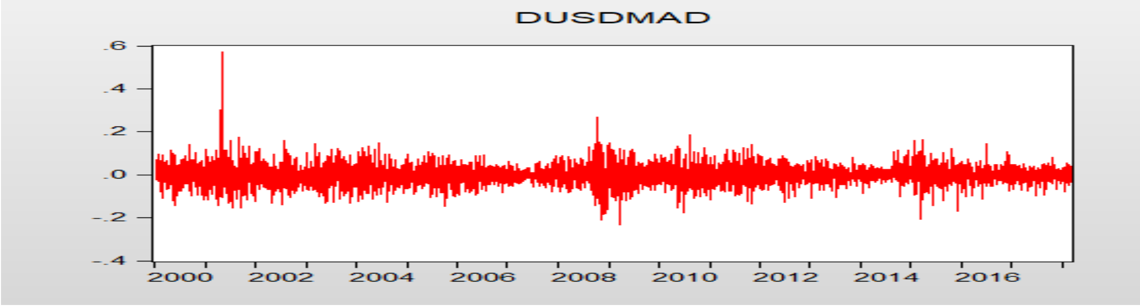



**Figure 5. Graph of D(EUR/MAD)**

**The two series are DS**

From 4.3 the two series (EUR/MAD & USD/MAD) are a nonstationary stochastic trend (random walk) and hence, they should be modeled as a first difference stationary (DS) process.

The Augmented Dickey-Fuller tests, approve the stationarity of each series D(USD/MAD) et D(EUR/MAD)

Date: 03/23/18 Time: 15:22
Sample: 1/03/2000 3/09/2018
Included observations: 4741

| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | -0.152 | -0.152 | 109.82 | 0.000 |
| 2 | -0.008 | -0.032 | 110.11 | 0.000 |
| 3 | -0.033 | -0.040 | 115.31 | 0.000 |
| 4 | 0.037 | 0.026 | 121.66 | 0.000 |
| 5 | -0.004 | 0.004 | 121.74 | 0.000 |
| 6 | 0.004 | 0.004 | 121.81 | 0.000 |
| 7 | -0.029 | -0.026 | 125.80 | 0.000 |
| 8 | 0.011 | 0.002 | 126.41 | 0.000 |
| 9 | -0.032 | -0.032 | 131.33 | 0.000 |
| 10 | 0.004 | -0.007 | 131.42 | 0.000 |
| 11 | -0.018 | -0.018 | 132.87 | 0.000 |
| 12 | -0.003 | -0.011 | 132.91 | 0.000 |
| 13 | -0.002 | -0.003 | 132.93 | 0.000 |
| 14 | -0.007 | -0.010 | 133.18 | 0.000 |
| 15 | 0.021 | 0.019 | 135.22 | 0.000 |
| 16 | 0.005 | 0.009 | 135.32 | 0.000 |
| 17 | 0.002 | 0.005 | 135.35 | 0.000 |
| 18 | -0.012 | -0.011 | 136.07 | 0.000 |
| 19 | 0.010 | 0.005 | 136.54 | 0.000 |
| 20 | -0.026 | -0.026 | 139.67 | 0.000 |
| 21 | 0.015 | 0.005 | 140.71 | 0.000 |
| 22 | 0.004 | 0.008 | 140.78 | 0.000 |
| 23 | 0.028 | 0.029 | 144.50 | 0.000 |
| 24 | -0.014 | -0.002 | 145.44 | 0.000 |
| 25 | 0.001 | -0.001 | 145.44 | 0.000 |
| 26 | -0.000 | 0.002 | 145.44 | 0.000 |
| 27 | 0.016 | 0.013 | 146.64 | 0.000 |
| 28 | -0.009 | -0.003 | 147.02 | 0.000 |
| 29 | -0.013 | -0.016 | 147.85 | 0.000 |
| 30 | -0.006 | -0.009 | 148.05 | 0.000 |
| 31 | -0.020 | -0.026 | 149.88 | 0.000 |
| 32 | 0.012 | 0.006 | 150.60 | 0.000 |
| 33 | -0.029 | -0.027 | 154.66 | 0.000 |
| 34 | -0.002 | -0.010 | 154.69 | 0.000 |
| 35 | 0.009 | 0.009 | 155.06 | 0.000 |
| 36 | 0.002 | 0.002 | 155.08 | 0.000 |

**Figure 6. Correlogram of DEURMAD**

From the Correlogram the **ARIMA model (2,1,2)** may be the appropriate model of DEURMAD that we will validate by adopted estimates tests

Date: 03/23/18 Time: 15:32
Sample: 1/03/2000 3/09/2018
Included observations: 4741

| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | -0.051 | -0.051 | 12.495 | 0.000 |
| 2 | 0.015 | 0.013 | 13.634 | 0.001 |
| 3 | -0.021 | -0.020 | 15.798 | 0.001 |
| 4 | 0.002 | -0.000 | 15.814 | 0.003 |
| 5 | 0.015 | 0.016 | 16.948 | 0.005 |
| 6 | -0.003 | -0.002 | 16.988 | 0.009 |
| 7 | 0.022 | 0.022 | 19.357 | 0.007 |
| 8 | 0.011 | 0.014 | 19.923 | 0.011 |
| 9 | -0.022 | -0.021 | 22.191 | 0.008 |
| 10 | -0.016 | -0.018 | 23.476 | 0.009 |
| 11 | 0.011 | 0.011 | 24.095 | 0.012 |
| 12 | 0.021 | 0.021 | 26.244 | 0.010 |
| 13 | 0.015 | 0.016 | 27.261 | 0.011 |
| 14 | -0.009 | -0.007 | 27.637 | 0.016 |
| 15 | -0.003 | -0.004 | 27.687 | 0.024 |
| 16 | -0.001 | 0.000 | 27.688 | 0.034 |
| 17 | 0.006 | 0.007 | 27.867 | 0.047 |
| 18 | 0.013 | 0.013 | 28.715 | 0.052 |
| 19 | 0.027 | 0.026 | 32.072 | 0.031 |
| 20 | 0.004 | 0.006 | 32.162 | 0.042 |
| 21 | -0.013 | -0.011 | 32.913 | 0.047 |
| 22 | 0.007 | 0.008 | 33.118 | 0.060 |
| 23 | -0.000 | 0.000 | 33.119 | 0.079 |
| 24 | 0.023 | 0.020 | 35.693 | 0.059 |
| 25 | -0.008 | -0.007 | 36.015 | 0.071 |
| 26 | -0.007 | -0.009 | 36.262 | 0.087 |
| 27 | 0.025 | 0.025 | 39.125 | 0.062 |
| 28 | -0.015 | -0.011 | 40.162 | 0.064 |
| 29 | -0.001 | -0.004 | 40.171 | 0.081 |
| 30 | -0.006 | -0.006 | 40.344 | 0.098 |
| 31 | 0.042 | 0.039 | 48.768 | 0.022 |
| 32 | 0.009 | 0.012 | 49.137 | 0.027 |
| 33 | -0.017 | -0.014 | 50.439 | 0.027 |
| 34 | -0.022 | -0.022 | 52.693 | 0.021 |
| 35 | 0.009 | 0.006 | 53.090 | 0.026 |
| 36 | -0.024 | -0.025 | 55.809 | 0.019 |

**Figure 7. Correlogram of DUSDMAD**

From the Correlogram the **ARIMA model (3,1,3)** may be the appropriate model of DUSDMAD series that we will validate by adopted estimates tests.

Dependent Variable: DEURMAD
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 03/23/18 Time: 16:05
Sample: 1/04/2000 3/09/2018
Included observations: 4741
Convergence achieved after 48 iterations
Coefficient covariance computed using outer product of gradients

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| AR(1) | -0.582782 | 0.144679 | -4.028093 | 0.0001 |
| AR(2) | 0.205237 | 0.046975 | 4.369025 | 0.0000 |
| MA(1) | 0.424684 | 0.144173 | 2.945650 | 0.0032 |
| MA(2) | -0.303315 | 0.047539 | -6.380370 | 0.0000 |
| SIGMASQ | 0.000537 | 1.89E-06 | 283.3286 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | 0.025998 | Mean dependent var | 0.000239 |
| Adjusted R-squared | 0.025175 | S.D. dependent var | 0.023473 |
| S.E. of regression | 0.023176 | Akaike info criterion | -4.690349 |
| Sum squared resid | 2.543805 | Schwarz criterion | -4.683532 |
| Log likelihood | 11123.47 | Hannan-Quinn criter. | -4.687954 |
| Durbin-Watson stat | 1.995583 | | |

| | | |
|---|---|---|
| Inverted AR Roots | .25 | -.83 |
| Inverted MA Roots | .38 | -.80 |

**Figure 8. D(EUR/MAD) ARMA Estimation**

The estimation of the ARIMA model as shown in Figure 8 of the series D(EUR/MAD) gives us AR(p=2) and MA(q=2) so the model to adopt is the ARIMA(2.1, 2) model

Dependent Variable: DUSDMAD
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 03/23/18 Time: 16:17
Sample: 1/04/2000 3/09/2018
Included observations: 4741
Convergence achieved after 66 iterations
Coefficient covariance computed using outer product of gradients

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| AR(1) | -0.701592 | 0.287946 | -2.436542 | 0.0149 |
| AR(2) | -0.674835 | 0.257369 | -2.622057 | 0.0088 |
| AR(3) | -0.045353 | 0.015852 | -2.861041 | 0.0042 |
| MA(1) | 0.651483 | 0.287580 | 2.265395 | 0.0235 |
| MA(2) | 0.655361 | 0.245890 | 2.665266 | 0.0077 |
| SIGMASQ | 0.002050 | 2.10E-05 | 97.48692 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | 0.003573 | Mean dependent var | -0.000157 |
| Adjusted R-squared | 0.002521 | S.D. dependent var | 0.045363 |
| S.E. of regression | 0.045306 | Akaike info criterion | -3.349498 |
| Sum squared resid | 9.719131 | Schwarz criterion | -3.341317 |
| Log likelihood | 7945.984 | Hannan-Quinn criter. | -3.346623 |
| Durbin-Watson stat | 2.000159 | | |

| | | | |
|---|---|---|---|
| Inverted AR Roots | -.07 | -.31-.73i | -.31+.73i |
| Inverted MA Roots | -.33-.74i | -.33+.74i | |

**Figure 9. D(USD/MAD) ARMA Estimation**

The estimation of the ARIMA model as shown in Figure 8 of the series D(USD/MAD) gives us AR(p=3) and MA(q=2) so the model to adopt is the ARIMA(3.1, 2) model

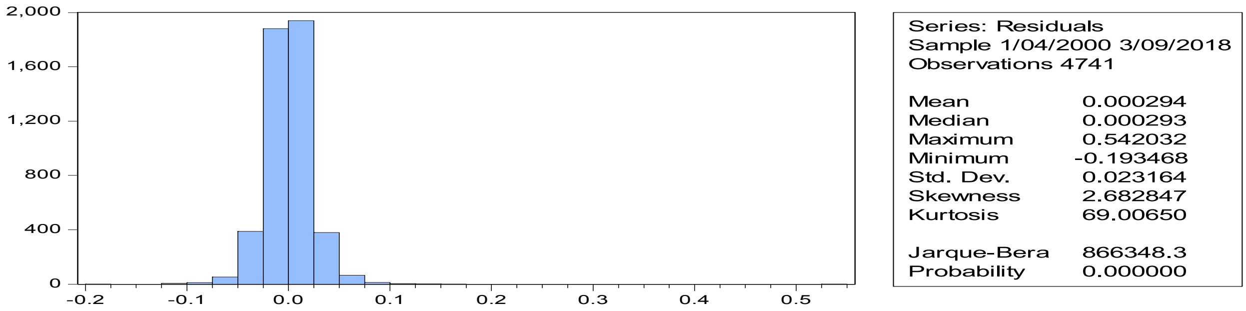



**Figure 10. Historgam of Residuals D(USD/MAD)**

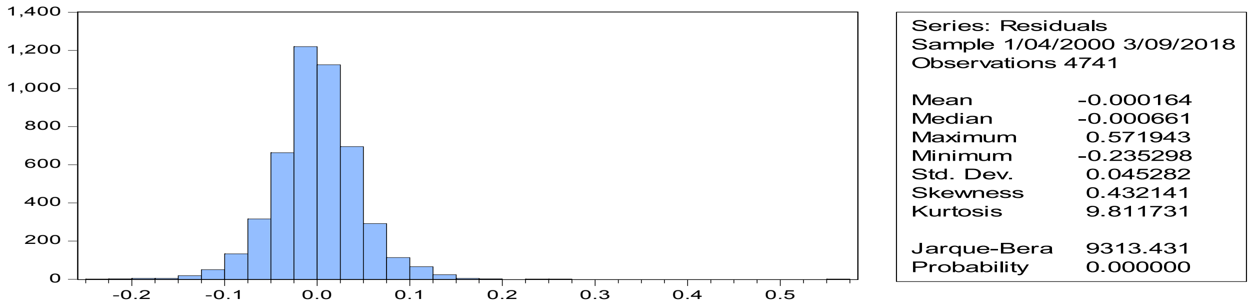



**Figure 11. Historgam of Residuals D(USD/MAD)**

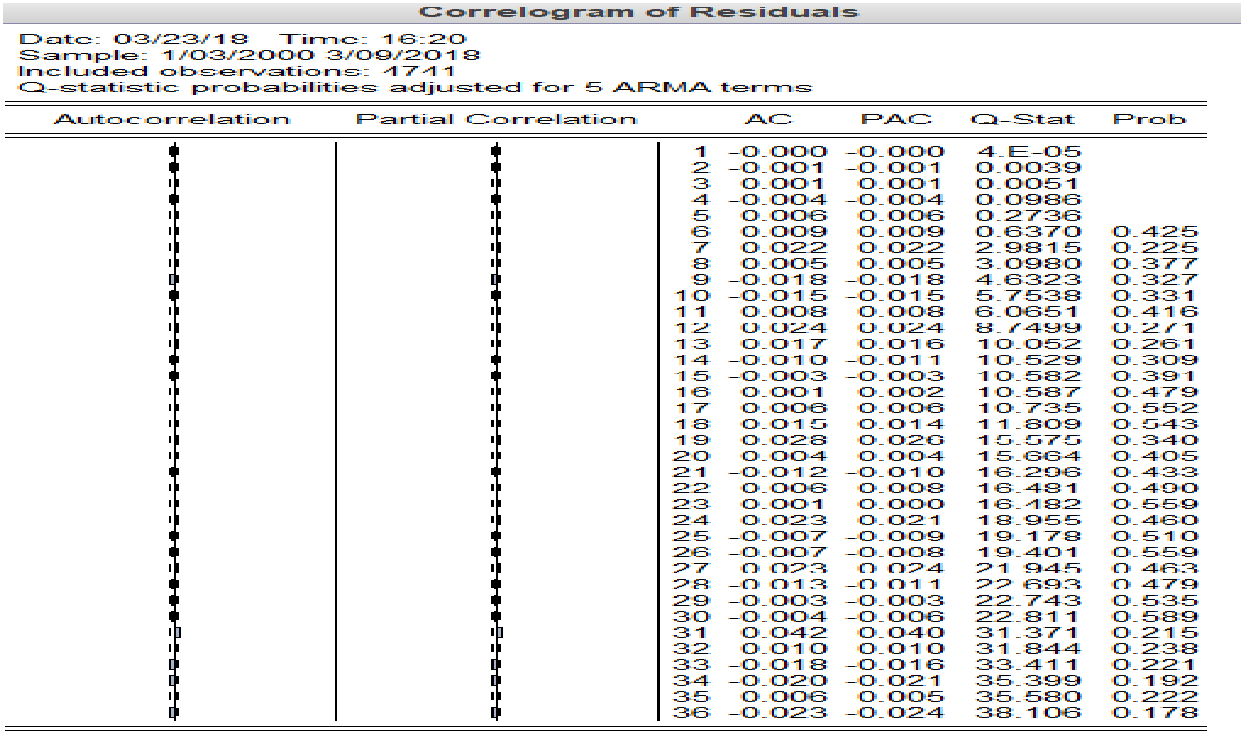
Correlogram of Residuals

Date: 03/23/18 Time: 16:08
Sample: 1/03/2000 3/09/2018
Included observations: 4741
Q-statistic probabilities adjusted for 4 ARMA terms

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| | | 1 | 0.001 | 0.001 | 0.0086 | |
| | | 2 | -0.007 | -0.007 | 0.2428 | |
| | | 3 | 0.000 | 0.000 | 0.2434 | |
| | | 4 | 0.020 | 0.020 | 2.1180 | |
| | | 5 | 0.014 | 0.014 | 3.0336 | 0.082 |
| | | 6 | -0.011 | -0.011 | 3.6432 | 0.162 |
| | | 7 | -0.020 | -0.020 | 5.5767 | 0.134 |
| | | 8 | -0.005 | -0.006 | 5.6998 | 0.223 |
| | | 9 | -0.028 | -0.029 | 9.3727 | 0.095 |
| | | 10 | -0.009 | -0.008 | 9.7246 | 0.137 |
| | | 11 | -0.016 | -0.015 | 10.941 | 0.141 |
| | | 12 | -0.010 | -0.009 | 11.383 | 0.181 |
| | | 13 | -0.001 | -0.000 | 11.387 | 0.250 |
| | | 14 | -0.007 | -0.007 | 11.620 | 0.311 |
| | | 15 | 0.022 | 0.022 | 13.989 | 0.234 |
| | | 16 | 0.007 | 0.006 | 14.221 | 0.287 |
| | | 17 | 0.002 | 0.002 | 14.241 | 0.357 |
| | | 18 | -0.011 | -0.013 | 14.831 | 0.390 |
| | | 19 | 0.006 | 0.004 | 14.976 | 0.453 |
| | | 20 | -0.023 | -0.025 | 17.389 | 0.361 |
| | | 21 | 0.013 | 0.012 | 18.181 | 0.377 |
| | | 22 | 0.010 | 0.010 | 18.642 | 0.414 |
| | | 23 | 0.027 | 0.028 | 22.240 | 0.272 |
| | | 24 | -0.009 | -0.007 | 22.626 | 0.308 |
| | | 25 | -0.000 | 0.000 | 22.627 | 0.364 |
| | | 26 | 0.002 | 0.002 | 22.649 | 0.422 |
| | | 27 | 0.014 | 0.012 | 23.564 | 0.428 |
| | | 28 | -0.009 | -0.009 | 23.991 | 0.462 |
| | | 29 | -0.017 | -0.017 | 25.363 | 0.442 |
| | | 30 | -0.012 | -0.011 | 26.031 | 0.461 |
| | | 31 | -0.023 | -0.024 | 28.462 | 0.387 |
| | | 32 | 0.005 | 0.007 | 28.599 | 0.433 |
| | | 33 | -0.031 | -0.029 | 33.088 | 0.274 |
| | | 34 | -0.004 | -0.002 | 33.179 | 0.315 |
| | | 35 | 0.008 | 0.010 | 33.481 | 0.348 |
| | | 36 | 0.006 | 0.006 | 33.633 | 0.388 |

**Figure 12. Correlogram of Residuals D(EUR/MAD)**

The correlogram shows the adequacy of the model. All the residual autocorrelations are not significantly different from zero.



Correlogram of Residuals

Date: 03/23/18 Time: 16:20
Sample: 1/03/2000 3/09/2018
Included observations: 4741
Q-statistic probabilities adjusted for 5 ARMA terms

| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | -0.000 | -0.000 | 4.E-05 | |
| 2 | -0.001 | -0.001 | 0.0039 | |
| 3 | 0.001 | 0.001 | 0.0051 | |
| 4 | -0.004 | -0.004 | 0.0986 | |
| 5 | 0.006 | 0.006 | 0.2736 | |
| 6 | 0.009 | 0.009 | 0.6370 | 0.425 |
| 7 | 0.022 | 0.022 | 2.9815 | 0.225 |
| 8 | 0.005 | 0.005 | 3.0980 | 0.377 |
| 9 | -0.018 | -0.018 | 4.6323 | 0.327 |
| 10 | -0.015 | -0.015 | 5.7538 | 0.331 |
| 11 | 0.008 | 0.008 | 6.0651 | 0.416 |
| 12 | 0.024 | 0.024 | 8.7499 | 0.271 |
| 13 | 0.017 | 0.016 | 10.052 | 0.261 |
| 14 | -0.010 | -0.011 | 10.529 | 0.309 |
| 15 | -0.003 | -0.003 | 10.582 | 0.391 |
| 16 | 0.001 | 0.002 | 10.587 | 0.479 |
| 17 | 0.006 | 0.006 | 10.735 | 0.552 |
| 18 | 0.015 | 0.014 | 11.809 | 0.543 |
| 19 | 0.028 | 0.026 | 15.575 | 0.340 |
| 20 | 0.004 | 0.004 | 15.664 | 0.405 |
| 21 | -0.012 | -0.010 | 16.296 | 0.433 |
| 22 | 0.006 | 0.008 | 16.481 | 0.490 |
| 23 | 0.001 | 0.000 | 16.482 | 0.559 |
| 24 | 0.023 | 0.021 | 18.955 | 0.460 |
| 25 | -0.007 | -0.009 | 19.178 | 0.510 |
| 26 | -0.007 | -0.008 | 19.401 | 0.559 |
| 27 | 0.023 | 0.024 | 21.945 | 0.463 |
| 28 | -0.013 | -0.011 | 22.693 | 0.479 |
| 29 | -0.003 | -0.003 | 22.743 | 0.535 |
| 30 | -0.004 | -0.006 | 22.811 | 0.589 |
| 31 | 0.042 | 0.040 | 31.371 | 0.215 |
| 32 | 0.010 | 0.010 | 31.844 | 0.238 |
| 33 | -0.018 | -0.016 | 33.411 | 0.221 |
| 34 | -0.020 | -0.021 | 35.399 | 0.192 |
| 35 | 0.006 | 0.005 | 35.580 | 0.222 |
| 36 | -0.023 | -0.024 | 38.106 | 0.178 |

**Figure 13. Correlogram of Residuals D(USD/MAD)**

*4.4 Coefficients Estimation of the ARIMA Models*

4.4.1 Estimation Equation D(EUR/MAD)

DEURMAD = 0 +

[AR(1)=-0.58278181639,AR(2)=0.205236868879,MA(1)=0.424683530653,MA(2)=-0.303315313536,
]

4.4.2 Estimation Equation D(USD/MAD)

DUSDMAD = 0 +

[AR(1)=-0.701591508377,AR(2)=-0.674835386612,AR(3)=-0.0453531674099,MA(1)=0.6514825193
37,MA(2)=0.655361306839,]

4.4.3 Forecasting

**Table 2. Forecasting Series D(EUR/MAD) & D(USD/MAD) then Calculating EUR/MAD & USD/MAD Using ARIMA Model**

| date | EUR/MAD | D(EUR/MAD) | USD/MAD | D(USD/MAD) |
|---|---|---|---|---|
| 11/03/2018 | 11,3400 | 0.005500000000001393 | 9,1648 | -0.009399999999999409 |
| 12/03/2018 | 11,3455 | 0.0215999999999994 | 9,1418 | -0.03300000000000125 |
| 13/03/2018 | 11,3671 | 0.004699999999999705 | 9,1324 | -0.001299999999998747 |
| 14/03/2018 | 11,3718 | -0.04030000000000023 | 9,0994 | 0.04089999999999883 |

| | | | | |
|---|---|---|---|---|
| 15/03/2018 | 11,3315 | -0.00569999999999915 | 9,0981 | -0.000899999999996789 |
| 16/03/2018 | 11,3258 | 0.002087441845087116 | 9,1390 | 0.0005895270463517412 |
| 17/03/2018 | 11,3279 | 0.0009614052273731249 | 9,1381 | -0.001468107683193019 |
| 18/03/2018 | 11,3288 | -0.000131869456443679 | 9,1387 | 0.0006729926488323415 |
| 19/03/2018 | 11,3287 | 0.0002741669199422857 | 9,1372 | 0.0004918304261723635 |
| 20/03/2018 | 11,3290 | -0.0001868439699393336 | 9,1379 | -0.0007326391646313851 |
| 21/03/2018 | 11,3288 | 0.0001651584283818826 | 9,1384 | 0.0001515845566671418 |
| 22/03/2018 | 11,3290 | -0.0001345986002437702 | 9,1377 | 0.000365755039911644 |
| 23/03/2018 | 11,3288 | 0.0001123382154436759 | 9,1378 | -0.0003256769756746357 |
| 24/03/2018 | 11,3289 | -9.309326451579342e-05 | 9,1382 | -2.520801593100877e-05 |
| 25/03/2018 | 11,3289 | 7.730900538129428e-05 | 9,1378 | 0.0002208760303806963 |
| 26/03/2018 | 11,3289 | -6.416045270235321e-05 | 9,1378 | -0.0001231824807965163 |

*4.5 EUR/MAD Exchange Rate Vasicek Model Estimation*

$\alpha$ =0,17829335 $\mu$ =11,056921 $\sigma$ =0,02

$$dx_t = 0,17829335(11,056921 - xt)dt + 0,02dw$$

**3) USD/MAD exchange rate Vasicek model estimation :**

$\alpha$ =0,09753359 $\mu$ =8,967795 $\sigma$ =0,05

$$dx_t = 0,09753359(8,967795 - xt)dt + 0,05dw$$

**Table 3. Estimation EUR/MAD & USD/MAD Using VASICEK Model**

| date | EUR/MAD | USD/MAD |
|---|---|---|
| 11/03/2018 | 11,6867 | 9,1998 |
| 12/03/2018 | 11,7351 | 9,6985 |
| 13/03/2018 | 11,7814 | 9,7595 |
| 14/03/2018 | 11,6847 | 9,6420 |
| 15/03/2018 | 11,7087 | 9,6088 |
| 16/03/2018 | 11,6694 | 10,0807 |
| 17/03/2018 | 11,8001 | 10,0063 |
| 18/03/2018 | 11,7602 | 9,9375 |
| 19/03/2018 | 11,8304 | 10,2888 |
| 20/03/2018 | 11,8071 | 10,0260 |
| 21/03/2018 | 11,5899 | 9,7898 |
| 22/03/2018 | 11,6712 | 9,3991 |

| | | |
|---|---|---|
| 23/03/2018 | 11,7225 | 10,0347 |
| 24/03/2018 | 11,7145 | 10,3075 |
| 25/03/2018 | 11,4489 | 9,8897 |
| 26/03/2018 | 11,5920 | 9,9114 |

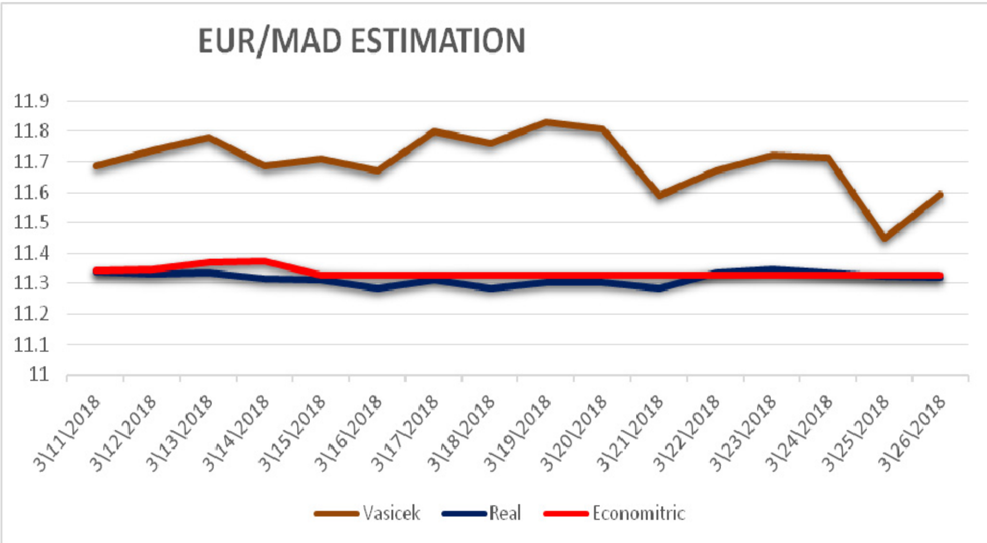



**Figure 14. Comparison between VASICEK and ARIMA(2,1,2) EUR/MAD Predicting**

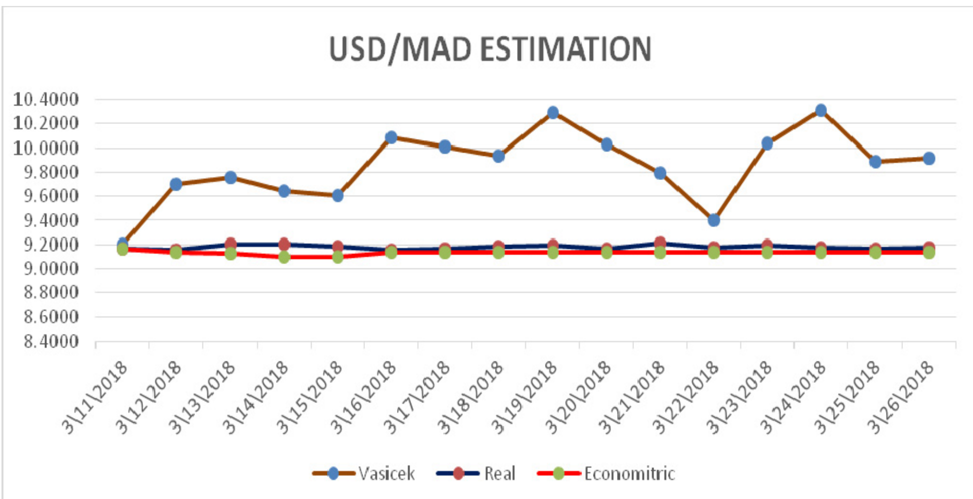



**Figure 15. Comparison between VASICEK and ARIMA(2,1,2) EUR/MAD Predicting**

Both in Figure 14 and 15 the econometric(ARIMA) model gives a best estimation than the Vasicek model (we see that the red line is closer than the blue one)

## 5. Concluding Remarks

First, we have successfully fitted an ARIMA(2,1,2) model to EUR/MAD Moroccan exchange rate and

ARIMA(3,1,2) model to USD/MAD. Its adequacy has been established and, on its basis, we have made forecasts.

Second, we calibrated the Vasicek model and we estimated their parameter and we used it to forecast USD/MAD & EUR/MAD series then we compared the values of each model to the real values and we concluded that the Box-Jenkins model is best and it is more performant to estimate Moroccan exchange rate than the Vasicek model who overestimates values!

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**Notes.**

Note 1. See P. Newbold et al. / Journal of Economics and Business 53 (2001) 85-102 Trend-stationarity, difference-stationarity, or neither: further diagnostic tests with an application to U.S. Real GNP, 1875-1993.

Note 2. For more details between the autocorrelation and the partial autocorrelation function of (PACF) see Appendix 1.

**Appendix 1. (**These definitions are tacked from EVIEWS documentation**)**

**Autocorrelations (AC)**

The autocorrelation of a series where $Y$ at lag $k$ is estimated by:

$$\tau_k = \frac{\sum_{t=k+1}^{T} (Y_t - \overline{Y})(Y_{t-k} - \overline{Y}_{t-k}) / (T-K)}{\sum_{t=1}^{T} (Y_t - \overline{Y})^2 / T}$$

Where $\overline{Y}$ is the sample mean of $Y$. This is the correlation coefficient for values of the series $k$ periods apart. If $\tau_1$ is nonzero, it means that the series is first order serially correlated. If $\tau_k$ dies off more or less geometrically with increasing lag $k$, it is a sign that the series obeys a low-order autoregressive (AR) process. If $\tau_k$ drops to zero after a small number of lags, it is a sign that the series obeys a low-order moving-average (MA) process.

**Partial Autocorrelations (PAC)**

The partial autocorrelation at lag $k$ is the regression coefficient on $Y_{t-k}$ when $Y_t$ is regressed on a constant, $Y_{t-1},\ldots,Y_{t-k}$ . This is a partial correlation since it measures the correlation of $Y$ values that are $k$ periods apart after removing the correlation from the intervening lags. If the pattern of autocorrelation is one that can be captured by an autoregression of order less than $k$, then the partial autocorrelation at lag $k$ will be close to zero.

The PAC of a pure autoregressive process of order p, AR(p), cuts off at lag p, while the PAC of a pure moving average (MA) process asymptotes gradually to zero.

**Appendix 2. Unit Root Test Eur/Mad (Augumented Dukey Fuller)**

**Augmented Dickey-Fuller Unit Root Test on EURMAD**

Null Hypothesis: EURMAD has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=31)

| | | t-Statistic | Prob.* |
|---|---|---|---|
| Augmented Dickey-Fuller test statistic | | -2.049838 | 0.5731 |
| Test critical values: | 1% level | -3.959974 | |
| | 5% level | -3.410753 | |
| | 10% level | -3.127167 | |

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(EURMAD)
Method: Least Squares
Date: 04/10/18 Time: 00:20
Sample (adjusted): 1/05/2000 3/09/2018
Included observations: 4740 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| EURMAD(-1) | -0.001961 | 0.000957 | -2.049838 | 0.0404 |
| D(EURMAD(-1)) | -0.151236 | 0.014353 | -10.53701 | 0.0000 |
| C | 0.021106 | 0.010091 | 2.091579 | 0.0365 |
| @TREND("1/03/2000") | 2.43E-07 | 2.94E-07 | 0.825521 | 0.4091 |

| | | | |
|---|---|---|---|
| R-squared | 0.024072 | Mean dependent var | 0.000226 |
| Adjusted R-squared | 0.023453 | S.D. dependent var | 0.023460 |
| S.E. of regression | 0.023183 | Akaike info criterion | -4.689946 |
| Sum squared resid | 2.545384 | Schwarz criterion | -4.684491 |
| Log likelihood | 11119.17 | Hannan-Quinn criter. | -4.688029 |
| F-statistic | 38.93844 | Durbin-Watson stat | 2.009909 |
| Prob(F-statistic) | 0.000000 | | |

We have a $|T - statistic| = 0.82521$ and the probability p = 0.4091 > 0.05 so we reject the hypothesis of a deterministic non-stationarity, or that the process TS (Trend stationary) of EUR/MAD series and the series is not stationary.

**Augmented Dickey-Fuller Unit Root Test on USDMAD**

Null Hypothesis: USDMAD has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=31)

| | | t-Statistic | Prob.* |
|---|---|---|---|
| Augmented Dickey-Fuller test statistic | | -1.411345 | 0.8579 |
| Test critical values: | 1% level | -3.959974 | |
| | 5% level | -3.410753 | |
| | 10% level | -3.127167 | |

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(USDMAD)
Method: Least Squares
Date: 04/10/18 Time: 17:42
Sample (adjusted): 1/05/2000 3/09/2018
Included observations: 4740 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| USDMAD(-1) | -0.000943 | 0.000668 | -1.411345 | 0.1582 |
| D(USDMAD(-1)) | -0.050876 | 0.014513 | -3.505556 | 0.0005 |
| C | 0.008402 | 0.006602 | 1.270746 | 0.2039 |
| @TREND("1/03/2000") | -2.19E-08 | 5.18E-07 | -0.042303 | 0.9663 |

| | | | |
|---|---|---|---|
| R-squared | 0.003109 | Mean dependent var | -0.000157 |
| Adjusted R-squared | 0.002478 | S.D. dependent var | 0.045368 |
| S.E. of regression | 0.045312 | Akaike info criterion | -3.349666 |
| Sum squared resid | 9.723656 | Schwarz criterion | -3.344211 |
| Log likelihood | 7942.708 | Hannan-Quinn criter. | -3.347749 |
| F-statistic | 4.923945 | Durbin-Watson stat | 1.998644 |
| Prob(F-statistic) | 0.002043 | | |

In the same way we have a $|T - statistic| = 0.042303$ and the probability p = 0.9663 > 0.05 so we reject the hypothesis of a deterministic non-stationarity, or that the process TS (Trend stationary) of USD/MAD series and the series is not stationary.