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*Original Paper*

# Direct Calculating Method for Integral of Multivariate Functions Based on Mathematica

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***Abstract***

*Mathematica is a comprehensive and efficient general mathematical software that integrates numerical and symbolic calculation, graphics and animation, programming and interactive demonstrations. This paper presents the methodology and operational procedures for the direct computation of multiple integrals, curve integrals and surface integrals utilizing Mathematica software. Through the application examples, it can be intuitively seen that this way of calculating multiple integrals with Mathematical software provides a very convenient, fast and effective way to verify the correctness of the integral calculation methods and results.*

***Keywords***

*Mathematica software, Multiple integral, Curve integral, Surface integral*

## 1. Introduction

In general mathematical software, the calculation of multiple integrals, curve integrals and surface integrals is generally to first construct the iterated integral expression, and then get the result by solving definite integral step by step. Such a process requires not only to be familiar with the graph describing the integral region, but also to give the inequality description form of the integral region. Especially for some complex integral regions, it may also be necessary to use the additivity of the integral regions to divide the integral region, construct the repeated integral expression through the inequality description form of the subregion, divide into multiple integrals to calculate the results respectively, and then sum up to get the final result.

Since we have used computer mathematical software to solve the integral of multivariate functions, of course, we hope it is simple, convenient, fast and effective. So is there such a good software that can directly calculate the integral without constructing a repeated integral expression to get the result? Mathematica (Stroyan, 1993; Hollis, 2004) provides a fast and efficient calculation method. In this method, the integral range is directly constrained within the defined region by constructing regions, and the integral of multivariate functions can be calculated directly without constructing iterated integral expression.

## 2. Construction and Geometric Description of Regions

The Wolfram language used in Mathematica provides comprehensive capabilities for creating, analyzing, solving, and visualizing regions. Based on regional equalities, inequalities, and parametric equations, Mathematica can quickly create complex regions. Common function commands are:

**ImplicitRegion**: Describe the region given by equalities and inequalities

**ParametricRegion**: Describe the region given by parameterized functions

**Region**: Display the graph described by the region

**Example 2.1** Graph the space curve bounded by the sphere $x^2+y^2+z^2=9$ and the plane $x+y=1$.

**Solution :** Enter the expression in Mathematica:

A=**ImplicitRegion**[x^2+y^2+z^2==9&&x+y==1,{x,y,z}];

**Region**[A]

The result after execution is shown in Figure 1:

**Figure 1**

**Example 2.2** Construct the regions described by $1\le x^2+y^2\le 2$ and $1\le x^2+y^2+z^2\le 4$, $x\ge 0, y\ge 0, z\ge 0$ separately, and display their graphs.

**Solution :** Enter the expression in Mathematica:

A=**ImplicitRegion**[1<=x^2+y^2<=2,{x,y}];

B=**ImplicitRegion**[1<=x^2+y^2+z^2<=4&&x>=0&&y>=0&&z>=0,{x,y,z}];

{**Region**[A],**Region**[B]}

The results after execution are shown in Figure 2:

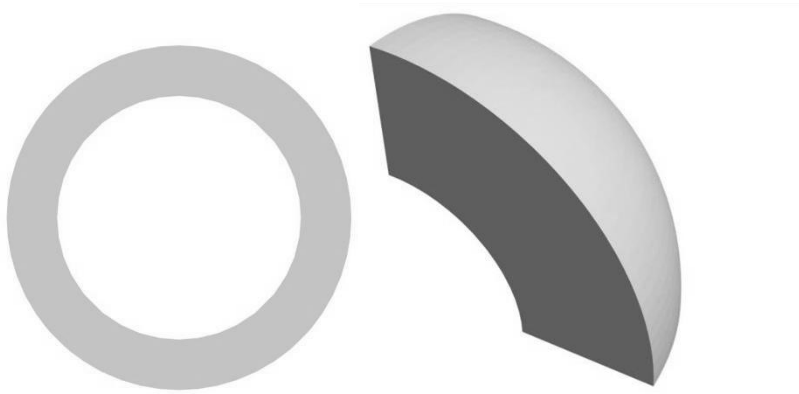

**Figure 2**

**Example 2.3** Graph the conical surface with a base radius of 4 and a height of 4.

**Solution :** Enter the expression in Mathematica:

A=**ParametricRegion**[{h Cos[t], h Sin[t],h},{{t,0,2Pi},{h,0,4}}];

**Region**[A]

The result after execution is shown in Figure 3:

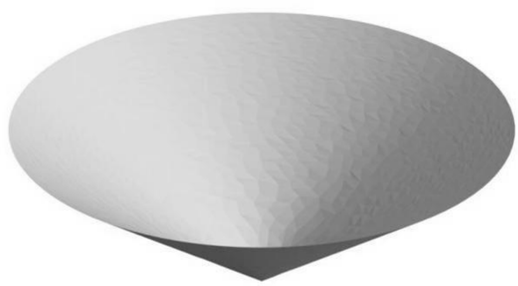

**Figure 3**

### 3. Calculation of Multivariate Function Integrals

In the following, several examples will be used to demonstrate that integrals of multivariate function can be calculated directly without constructing repeated integral expressions.

**Example 3.1 (Curve integral)** Evaluate the curve integral $\int_C \left(x^2+y^2+z^2\right)\mathrm{d}s$, where $C$ is the helical line: $x=\cos t, y=\sin t,\ z=t,\ \left(0 \le t \le 2\pi\right)$.

**Solution:** Enter the expression in Mathematica:

A=**ParametricRegion**[{Cos[t], Sin[t],t},{{t,0,2Pi}}];

**Integrate**[x^2+y^2+z^2,{x,y,z}\[Element]A]

The result after execution is $2\sqrt{2}\pi+\frac{8\sqrt{2}\pi^3}{3}$, that is

$$\int_C\left(x^2+y^2+z^2\right)\mathrm{d}s=2\sqrt{2}\pi+\frac{8\sqrt{2}\pi^3}{3}.$$

**Example 3.2 (Surface integral)** Evaluate the surface integral $\int_D\left(xy+yz+xz\right)\mathrm{d}S$, where $D$ is the finite portion of cone $y=\sqrt{x^2+z^2}$ intercepted by cylinder $x^2+z^2=2ax$.

**Solution:** Enter the expression in Mathematica:

A=**ImplicitRegion**[y==Sqrt[x^2+z^2]&&x^2+z^2<=2 a x,{x,y,z}];

**Assuming**[a>0,**Integrate**[x y+y z+x z,{x,y,z}\[Element]A]]

The result after execution is $\frac{64\sqrt{2}a^4}{15}$, that is

$$\int_D\left(xy+yz+xz\right)\mathrm{d}S=\frac{64\sqrt{2}a^4}{15}.$$

**Example 3.3 (Double integral)** Evaluate the double integral $\iint_D\sqrt{4-x^2-y^2}\,\mathrm{d}x\mathrm{d}y$, where $D$ is the closed region enclosed by half circle $y=\sqrt{2x-x^2}$ and x-axis.

**Solution:** Enter the expression in Mathematica:

A=**ImplicitRegion**[0<=y<=Sqrt[2 x-x^2]&&0<=x<=2,{x,y}];

**Integrate**[Sqrt[4-x^2-y^2],{x,y}\[Element]A]

The result after execution is $\frac{4(3\pi-4)}{9}$, that is

$$\iint_D\sqrt{4-x^2-y^2}\,\mathrm{d}x\mathrm{d}y=\frac{4(3\pi-4)}{9}.$$

**Example 3.4 (Triple integral)** Evaluate the triple integral $\iiint_E z\,\mathrm{d}V$, where $E$ is the region bounded by $x^2+y^2+(z-1)^2\le 1$ and $x^2+y^2\le z^2$.

**Solution:** Enter the expression in Mathematica:

A=**ImplicitRegion**[x^2+y^2+(z-1)^2<=1&&x^2+y^2<=z^2,{x,y,z}];

**Integrate**[z,{x,y,z}\[Element]A]

The result after execution is $\frac{7\pi}{6}$, that is

$$\iiint_E z\mathrm{d}V = \frac{7\pi}{6}.$$

## 4. Conclusion

In the process of incorporating Mathematica software into higher mathematics teaching, it can overcome the lack of explanation of content abstraction, inconspicuous handicraft drawings, and difficulty expanding teaching content in traditional teaching. Especially for some weak students, the application of this software can help them get rid of the dilemma and torture of "listening to the sky", make abstract mathematics vivid and interesting, and help improve students' enthusiasm and enthusiasm for learning mathematics.

This paper presents the methodology and operational procedures for the direct computation of multiple integrals, curve integrals and surface integrals utilizing Mathematica software. Through the application examples, it can be intuitively seen that this way of calculating multiple integrals with Mathematical software provides a very convenient, fast and effective way to verify the correctness of the integral calculation methods and results.

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