Original Paper

Binomial Count Information: How Do the Usual Approximations Fare?

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Abstract

__Focus__ Decision-making is often aided by examining False Positive Error-risk profiles [FPEs]. In this research report, the decision-making jeopardy that one invites by eschewing the Exact factorial-binomial Probability-values used to form the FPEs in favor of: (i) Normal Approximations [NA], or (ii) Continuity-Corrected Normal Approximations [CCNA] is addressed. __Results__ Referencing an audit context where testing sample sizes for Re-Performance & Re-Calculation protocols are, by economic necessity, in the range of 20 to 100 account items, there are indications that audit decisions would benefit by using the Exact Probability-values. Specifically, using a jeopardy-screen of ±2.5% created by benchmarking the NA & the CCNA by the Exact FPEs, it is observed that: (i) for sample sizes of 100 there is little difference between the Exact and the CCNA FPEs, (ii) almost uniformly for both sample extremes of 20 and 100, the FPEs created using the NA are lower and outside the jeopardy screen, finally (iii) for the CCNA-arm for sample sizes of n = 20, only sometimes are the CCNA FPEs interior to the jeopardy screen. These results call into question not using the Exact Factorial Binomial results. Finally, an illustrative example is offered of an A priori FPE-risk Decision-Grid that can be parametrized and used in a decision-making context.

Keywords

FPE, a priori profiling, judgmental screening intervals

1. Introduction

In a decision-making context, it is critical to *have "exact" information to inform the data-analytics so as to arrive at the best decision in the particular context under consideration*. A question of interest is: What <u>exactly</u> does <u>exact</u>-information mean in the decision-making process? Consider the idea of

Exactitude—it establishes a useful context for this research report.

1.1 Exactitude: Universal or Conditional Concept

Following it is important to consider *the* lore of mathematical precision. Experiential streaming feedback over many years instructing Math/Stat courses requires that one clarify what information is actually generated in a computation domain. In this regard, it is most instructive to consider (i) the measurement of **Area** in a flat-horizontal plane, and (ii) the **Rate of Change of Functional Relationships**. For each, most interestingly, there are Exact and Approximate computations that are conditioned on the nature of the need of the information.

1.1.1 Area

Assume that one wants to compute the Area of a Space for a flower garden. In this case, there is a flat-tract rectangle of land that is 4-Meters in Length and 2-Meters in Width. The area of this garden is: Area = Length × Width; thus, 8-Squared-Meters = [2-Meters × 4-Meters]. This is Exact. We have eight (8) Squares each of which is: [1-Meter by 1-Meter] that will <u>exactly</u> fill the space. No approximation; the computation is *exact <u>and</u> so, by definition, useful for the task at hand*.

However, suppose the gardener feels—a rectangle is "boring". He decides to make the flower garden a circle. He sketches a circle with a radius of 2-Meters. In this case, the Area-formula is: Area = $\pi \times [Radius^2]$. The value of π is approximately 3.14. It is part of the historical record that π was well known by the great builders of antiquity: Babylonians, Egyptians and Greeks [Archimedes] *et sans doute Le Nôtre de Versailles*; also part of the historical record is that π is an irrational number—π does NOT have an exact decimal value. Therefore, the gardener computes the Area of the circular flower garden as: Area=12.56 Squared Meters: [3.14 × 2^2]. He asks his Math-friend if this is an exact value; she says:

> *"There is NO exact value for the Area of your garden because you used π in the computation and π only has an approximate decimal-value. However, if you want to be sure, then use 3.15 as π—this approximation will give you is slightly larger estimate of the area and so should serve your purpose of how much topsoil to buy. Actually, no one has ever computed the exact area of a Circle—**ever**. In fact, if a computer started to compute the decimal-value of π at the moment of the Big-Bang creation of the universe, it would still be running today and, in fact, it will never stop! This is why π is called an irrational number".*

In this case, the decision-making information is an ***approximation <u>but</u> useful for the task at hand***.

1.1.2 A Slippery Slop*e*

The Slope of a function is very valuable information; it is the rate of change of a function—the change of the ordinate plot [the y-response] to the change of the driver [the x-value]. To elucidate the concept of slope, consider the function: y ≡ f(x) = x^2. The computational protocol for deriving the slope-function, *also called the first derivative, or $\frac{\Delta y}{\Delta x}$, or the regional rate of change of f(x), or the*

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point-slope-function proceeds as follows:

One needs to form a <u>*limit*</u>ing form to arrive at the derivative. This form in the Cartesian Coordinate [Ordinate[f(x)] & Abscissa[x]] context is: $\lim_{\Delta x \to 0}[[f(x + \Delta x) - f(x)]/\Delta x]$: The derivative is given the alliterative-label: "*Rise [f($x + \Delta x$)] over Run[Δx]*". In the case, where: f(x) = x^2, the slope function is:

$$: \lim_{\Delta x \to 0}[[x + \Delta x]^2 - x^2]/\Delta x] \text{ expanding,}$$

$$: \lim_{\Delta x \to 0}[[[x^2 + 2(x\Delta x) + (\Delta x)^2] - x^2]/\Delta x] \text{ rearranging}$$

$$: \lim_{\Delta x \to 0}[(x^2 - x^2) + 2(x\Delta x) + (\Delta x)^2]/\Delta x], \text{ finally}$$

$$\text{The slope or derivative is: } \lim_{\Delta x \to 0}[2x + \Delta x].$$

As Δx becomes very small $\lim_{\Delta x \to 0}[2x + \Delta x]$ converges to: 2x as a point-limit. This is sometimes referred to as the Point-Process or Instantaneous Slope. This is not an exact slope function as the concept of a point is an intellectual creation and has no measurable value. For example, assume that we want to compute the slope in the right-hand-side range: [12 to 12.00001] for f(x) = x^2. The Point-Process approximation of the slope is: **24.$\overline{0}$**; the ***actual*** slope is:

24.00001 = [[144 + 0.00024 + **0.00001^2**] − 144] / 0.00001 = [[0.00024 + **0.00001^2**]/0.00001]

The ***actual*** slope is sometimes referred to as the Precision Adjusted Slope.

Thus, one says that the slope of f(x) = x^2 IS: 2x. Yes, as a limit; BUT as Δx can NEVER = 0, the correct statement is that the slope of f(x) = x^2 is *approximated* by 2x. However, does this limiting concept or approximation work in practice. The answer is "It depends". As in the flower garden example, if, for the task at hand, one does not need to be concerned with the precision boarder effectively created by Δx, then the slope of 2x will be an ***approximation but useful for the task at hand***. For example, look at any Micro- or Macro-economics textbook. They correctly use the Point-Process slope as it provides conceptual guidance in that illustrative-domain and Δx would only add confusion to the overall idea of the "instructive exercise".

However, if the analyst had a linear function: α + βx, not an OLS-regression estimate, but an actual *a priori* justified function, the slope function would be: $\lim_{\Delta x \to 0}[\beta]$. In this case, there is no Δx term as the Abscissa indicator in the x-range, the rate of change of: α + βx is exactly equal to β. In this case, there is ***no approximation β is the exact slope function and so can be used for the task at hand***.

1.1.3 Summary

The point of the ***Area*** and ***Slope*** discussions was offered to note that sometimes the analytic context is characterized by ***Exactitude***:{The Area of a Rectangle or the Slope of a Linear Function}; sometimes, however, the measurement context is an ***Approximate*** context:{The Area of a Circle or the Slope of most any other function not in the linear class-set}. Thus,

> ***the utility of analytic information is related to the acceptance of variation from exact results***.

With this as the operational mantra, consider the statistical decision making that is very often found in the audit context where the information collected is the Number of Events. In this audit-sampling frame, using the very reasonable audit-protocol requirements that:

(i) the audit InCharge [IC] decides to randomly select *n*-Accounts from a defined collection or population of *N-*Account Events,

(ii) it is possible to randomly select the same Account multiple times and so include it multiple times in the sample of *n* Accounts—this is usually called *sampling with replacement* and is necessary to have valid population estimates,

(iii) there is a protocol for accurately binary-coding: {Yes=1 *or* Not Yes =0} the Account so selected, and

(iv) the IC using experiential judgement specifies the percentage of Accounts scored as Yes that are Expected to be found in the *population* of Accounts under audit examination.

This type of protocol is called a Bernoulli-Selection or -Scoring Protocol; <u>however</u> the Probability-value context that will be used is not formed from a general binary Bernoulli Probability Density function—this will be addressed subsequently. For notation, the probability density function is scripted as:

$$B_{\text{pdf}}[n, \%],$$

Where: *n* is the number of sampled events from the Account under audit, the Account has *N* elements—as such, this is the population from which a random sample of size *n* is taken, and *%* is the a *priori* expectation of the percentage of targets or successes in the population of *N*-individual accounts. In this context,

    (i)   there will be a probability-value computed for each of the *n*-Event[Points]; they are labeled as ***Probability-values.*** [***ALERT: the Probability-Values will be noted herein, for exposition, as P-values. These are NOT the p-values that are the α or Type I constructs used in hypothesis testing.***],

    (ii)   the sum of all *n* such P-values in the sampling-frame, by definition, equals *1.0*,

    (iii)   the sum of the P-values in ***any*** interval [from: Event[i] through: Event[j]; i≠j] is termed the ***False Positive Error*** [***FPE***]-risk or -chance or -gambling-odds under the *a priori* expectation. In the statistical literature, the FPE-risk is sometimes referred to as: The α-Risk or the Type 1 Error, and

    (iv)   for the $B_{\text{pdf}}[n, \%]$, the *a priori* specification of *%* will be variously noted as: the ***Null of the research hypothesis, or the FPE[Null]***, or the ***a priori expectation*** or the ***belief***. ***Important Clarification: The meaning of the FPE-risk is: Given that there is an a priori expectation [%] specified as the "test-against" value, the FPE-risks are the gambling-odds that the a priori expectation is likely the TRUE state of nature GIVEN the ACTUAL results. Thus, if the FPE-risk is a Small-percentage—i.e., the***

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> *gambling-odds are low—this indicates that it is UN-likely that the a priori expectation is TRUE given the large-difference between the a priori expectation and the ACTUAL results. In this case, it is better to opt for rejecting the likelihood that the a priori expectation is the TRUE state of nature in favor of that it is NOT likely to be the TRUE state of nature.*

1.2 Research Plan

This is the point of departure of this research report. Following, the plan is to:

1) Examine the operational context for "exact—& approximate—inferential information" for the $B_{pdf}[n, \%]$,

2) Discuss three computational forms that could be used in creating an inferential FPE-risk from $B_{pdf}[n, \%]$ (i) The ***Exact*** Binomial computation, (ii) The uncorrected Normal ***Approximation*** [NA] of the ***Exact*** Binomial, and (iii) The Continuity Corrected Normal ***Approximation*** [CCNA] of the ***Exact*** Binomial,

3) Offer an illustrative context to motivate the focus of this research report that is: *In an audit context, where statistical parameters are required to form the FPE-risk to guide and inform the IC, what is the jeopardy of using the NA or the CCNA to form the FPE-risk profiles vis-à-vis calculating the Exact Binomial FPE-risk information?*,

4) Offer computational information for two cases: $B_{pdf}[n, \%]$ where n =20 & n =100 for various *%*-realizations to elucidate the sample-size impact re: the possible inferential jeopardy for eschewing the Exact information in favor of either the NA or the CCNA, and

5) Finalize the research report by suggesting an *A priori* FPE-risk Decision-Grid for use in the audit context.

2. The Ubiquitous Binomial Probability Distribution: A Child of a Lessor Statistical God

2.1 The Audit Context

In the audit world, the IC has a vast number of client accounts from which to select in the execution of the audit. The end-game of the audit is to script two opinions:

1) The ***COSO Opinion***: Is management's System of Internal Control over Financial Reporting [ICoFR] adequate to "catch and correct" material-errors that may appear in the financial statements, and

2) The ***Fairness-Opinion***: are the clients' Financial Statements fair representations of the results of operations for the reporting period and thus can be relied upon by interested parties to be relatively free from material error.

Indeed, there are a vast number of ways that the IC can collect client data, vet this data [See Gaber & Lusk (2019)], and then create audit evidence that speaks to the need to conduct subsequent investigations called Extended Procedures Investigations [EPI]; these may be needed if the Expectation

of the IC is not likely to be the case given the collected audit evidence. One of the standard inferential tests in the panoply of the IC is to examine the frequency or the number of binary-Bernoulli occurrences in the Account under audit and to base the EPI-decision on the related inference. The inference engine of choice in the typical case is the FPE-risk.

2.2 The FPE-risk of the Binomial: The Exact Case

The most effective way to introduce this inferential-FPE-risk testing case is by way of an example. This was an actual audit context, except the size of the sample has been reduced for exposition. The IC is in the COSO: ICoFR-interim-phase of the audit and has selected: Accounts Payable for testing. The issue under audit is how many accounts have taken qualified time-related discounts and so reduced the amount that was need to close/satisfy their payable obligations. If *too few discounts* are taken then this could raise ICoFR concerns as to adequate managerial oversight in controlling the resources of the firm and so and may require the IC to consider EPI; also, if *too many discounts* are taken this may strain the Cash Management possibilities need to navigate the economic context and also may require an EPI.

The IC expects that there will a balance between too few and too many APs settled in the audit year. Specifically, the IC downloads the Accounts Payable payment satisfaction protocol of the client [AP-P]. After reading the AP-P and allowing for the usual unavoidable and justifiable reasons for not taking the qualified discount in the COVID-19 era the IC decides that 70% of the time the discount on any account > \$100 should be taken—meaning that if this is the case, then the IC would *not* have evidence that there are issues in sensibly executing the AP-P. The IC randomly samples 20 AP-accounts from the 357 AP > \$100; these 20-APs are investigated by an Audit- Staffer using a Re-Performance & Re-Computation [Rp&Rc] audit protocol to determine if the AP-Protocol was correctly executed by the client. The final determination, after a few discussions with the CFO, who scripted the AP-P, was that of the 20 Account Payable randomly sampled three (3) were not paid to in time to qualify for the discount & 17 were paid in conformity of the AP-P. There is now a choice as to the nature of the Binomial probability density function to be used in forming the FPE-risk inference for the audit. Suppose that the IC selects the number NOT paid. In this case, the *a priori* IC-*expectation under a prudent or desirable Client Cash Disbursement protocol* is that *30%* [100% − 70%] of the APs in the population of N=357 APs *would not be paid on time*. The inference protocol is:

1) Form the B_{pdf}[*n=20*, *30%*] using a *Factorial* Generating Process,

2) Parameterize the range of AP-P Events {0, 1, - - - , 20} as to their EPI impact using the FPE-risk formed from the P-values,

3) Specify an *a priori* FPE-risk interval that would serve to logically reject the expectation of the IC and thus rationalize consideration of an EPI. This creates a B_{pdf}[20, 30%] *A priori* FPE-risk Decision-Grid that will be used to evaluate the results of the testing,

4) Use this B_{pdf}[20, 30%] *A priori* FPE-risk Decision-Grid to evaluate the *actual-results* of the Rp&Rc testing conducted by the Staffer, and

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5) Summarize the inferential test information to be included in the current audit-working papers and so to later appear in the permanent audit file.

2.3 Partial Introductory Illustration: Clarification of the Computational Forms

In what follows, the details of the computations are presented. Subsequently, the full testing using the full $B_{pdf}[20, 30\%]$ EPI-Decision Grid will be detailed.

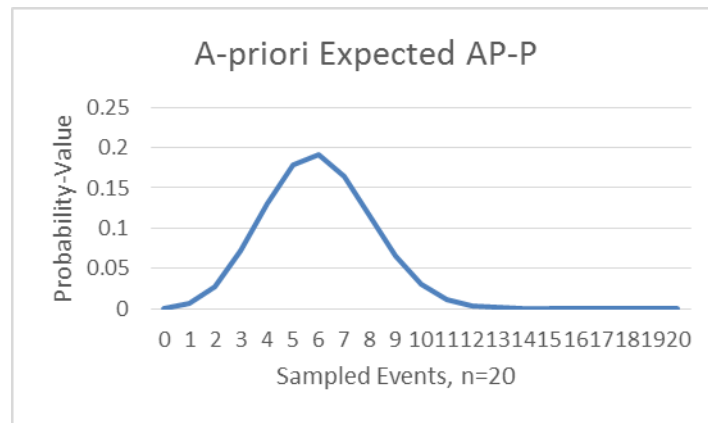In this case, the $B_{pdf}[20, 30\%]$ is presented in Figure 1:

Figure 1. Expectation of $B_{pdf}[20, 30\%]$ for APs NOT Paid Re: AP-P

Figure 1 is the exact binomial probability density function for the $B_{pdf}[20, 30\%]$ that represents the *a priori **Expectation*** of the IC ***under an acceptable execution of the AP-P*** and thus would NOT require an EPI. This will be used to create the $B_{pdf}[20, 30\%]$ EPI-Decision Grid. Assume that the IC is only interested in the cases where too-many APs are paid. In this case, the following $B_{pdf}[20, 30\%]$ EPI-decision Grid is parametrized—i.e., the IC creates the action information is Row 1[Column Headings]:

Table 1. $B_{pdf}[20, 30\%]$ A priori FPE-risk Decision-Grid

EPI Context	Too Many Paid	Too Many Paid	Too Many Paid	Too Many Paid	Ok	Ok	Ok
*P-Value	0.1%	0.7%	2.8%	7.2%	13.0%	17.9%	19.2%
+ΣP-Value	0.1%	0.8%	3.5%	10.7%	23.8%	41.6%	60.8%
NotPaid	0	1	2	3	4	5	6

*P-Value is: The probability-value & +ΣP-Value is: The FPE-risk

2.3.1 Discussion

Recognize that the Mean of the $\boldsymbol{B}_{\text{pdf}}$[20, 30%] is six (6) [20×30%]—shaded in Table 1. Assume that the IC correctly uses the aggregate probability-values in Table 1 as the FPE-risk to screen and evaluate the Actual number of APs-not paid. This Actual information will be forthcoming at some point from the Staffer's Rp&Rc analysis. In this regard, using $\boldsymbol{B}_{\text{pdf}}$[20, 30%], the IC computes the individual Left Hand Side [LHS] P-value aggregations from Event(0=None) to the maximum P-value of Event(6)—these are the FPE-risks at each event. In this case, using a simple VBA-module, the P-values on the LHS away from the Max-value to Event[0] are displayed in Row 2. Using these individual P-values, the relevant FPE-risks are displayed in the Row[3]. For example, for Cell[3,3] shaded the FPE-risk is : 0.8% [0.1% + 0.7%]. Using the information in Table 1, the possibilities for effecting an EPI, as noted in Row 1, is summarized as:

> *IF the actual Rp&Rc results were to fall in the range [from Event[4] through Event[6]] then according to the IC there is not strong or sufficient evidence to indicate that the AP-P is not being correctly executed according to the IC's expectation. Specifically, the lowest FPE-gambling-odds for support of the expectation of 30% in the "**EPI-Ok**" range is 23.8% and occurs for Event [4]. Such a FPE-risk sometimes suggests the wisdom of rejecting the Null of 30%—i.e., the a priori expectation of the IC. However, such a mid-level FPE-risk would usually require other related confirmatory audit indications to rationalize launching an EPI; thus, the IC labeled this as **Ok**—no EPI is contemplated. However, if the Rp&Rc results were that 19 were paid and so one [1] was not paid, the FPE-risk would be 0.8% [< 1%]. This result strongly suggests that there is a "troubling disconnect" between the IC's expectation of 30% not being paid and that fact that 5% [1/20] were not paid. Simply, the actual result of 1 of 20 or 5% is too far from the expectation of 30% [100% − 70%] to be consistent with the IC's expectation; **it could occur, but** the gambling-odds of 30% being the TRUE state of nature is only 0.8%—i.e., 0.8% is the FPE-risk or chance that 30% could be the actual population value given the result as observed. In the cases from Event[0] through Event[3], the IC has coded them as **Too Many Paid** and so this is likely to require an EPI. As for Events [5 or 6], these FPE-risks [46.1 or 60.8%] offer a strong indication that there **is** likely conformity with the a priori expectation of 30% not being paid and so the IC does not expect to launch an EPI and thus they are also labeled as **Ok**.*

The computational basis of the above information will aid in understanding the nature of the technical aspects of using the exact Binomial in a decision-making context. After these details are presented and discussed a more instructive operational context will be possible.

2.3.2 Technical Components Illustrated Given the IC selection of B_{pdf}[20, 30%]

The total number of events in the random sample with replacement of the 357 Client APs is n =20; 30% is the Failure to Execute the AP-P protocol [100% − 70%]. Thus **exact probability value** of Event:(5) computed using a **Factorial Generating Process** is:

$$[20!/[(5!)(20 - 5)!]] \times [[30\%]^{[5]} \times [1 - 30\%]^{[20-5]}]$$

In Excel: = FACT(20)/(FACT(5)*FACT(20-5)) = 15,504

In Excel: = ((0.3)^5)*((0.7)^(20-5)) = 1.155366E-05

Probability-Value of Event(5) = [15,504 × 1.155366E-05] = 17.9%

The Left-Hand-Side [LHS] aggregate value for Event[5] is:

$$\text{FPE-risk:} \sum_{0}^{5} Event[i] = [0.1\% + 0.7\% + 2.8\% + 7.2\% + 13.0 + 17.9\%] = \textbf{41.6\%}$$

The aggregate value, by computational convention, is referred to as the FPE-risk for decision-making purposes. This is the case, as in probability analysis, rarely is the analyst interest in the probability value of only ONE Event; rather, the analysis picks a frontier Event and asks: *What is the probability of this Event or Events to the Left [or Right] of the frontier–event?*

2.3.3 Approximations to the Exact B_{pdf}[20, 30%] Profiles

To be clear, the exact Binomial is just that: The Exact Values that would be relevant for informing the audit decision-making process. However, for practical reasons at the time that the Binomial process were studied, really at the dawn of the electronic computing-age, Normal approximations [NA] to Bernoulli & related Binomial processes were *en vogue* and so offered as "ball-park estimates" that would be relevant in most of the conceivable practical application areas (Note 1). The lack of interest in the "Error" in using the NA *vis-à-vis* the Exact Binomial is that a Continuity Correction [CC] is usually offered as "a correction". This is a **misnomer** as the CC does NOT give the Exact value of the NA of the B_{pdf}[n, %]—it is close but *not exact*. Consider now the approximations to the Exact Binomial.

3. Approximations

As noted above, there are also approximations to the B_{pdf}[n, %] that broach the issue in focus for this research report. Specifically:

> The B_{pdf}[n, %] offers exact information—the FPE-risk Inference Profile—that can be
> readily used in most decision contexts. However, there are also approximations that
> could be used in the stead of the Exact Binomial. These are almost exclusively part of
> **most** all the software platforms commercially available that provide Bernoulli-binary
> inference testing. This begs the question: **What is the jeopardy in eschewing the**
> **Exact inferences in favor of those offered by approximations?**

3.1 The Normal Approximation [NA]

For the NA, one computes or estimates the Mean(μ) and Standard Deviation(σ) and uses these parameters to animate the N(μ,σ) probability density function that can then be used to create

approximate P-values and FPEs. For clarification, assume that the IC wants determine the FPE-profile under the Null that the actual population event has the frequency of 30%. Further, the event of interest is the LHS domain where the actual event measured was Event(5) and thus the FPE-risk is: ***What is the probability that 30% is the TRUE population expectation and one takes a random sample of 20 from that population and finds Event[5] or less as the case?*** The FPE-risk value under the Normal approximation [NA] is:

Normal Approximation: Testing the LHS given that Event (5) was measured:

$$P[(x \leq 5) : N[6: 2.049]] : z_{cal} = \text{Abs}([[5 - 6] / 2.049]) = 0.488$$

Where: $\mu = [20 \times 30\%]$ and $\sigma = 2.049 = \sqrt[2]{20 \times 30\% \times (100\% - 30\%)}$

In Excel (Note 2): = (T.DIST.2T(ABS(0.488),10000))/2 = FPE-risk of **31.3%**

The meaning of 31.3% is that if the true population value were to be 30% and so Event (6) is expected in a sample of 20, the chance of observing Event [5] "or less"—i.e., from Event [5] down through Event [0]—could likely occur 31.3% of the time. These FPE-odds are the FPE-risk of expecting six and observing five. In this case, 31.3% is relatively high and so suggestive that the FPE [Null-belief of 30% in the population of interest] would more or less likely be the case. In this case, there is no <u>strong convincing</u> evidence that 30% is not the likely postulated state of nature in the population under examination and thus the FPE-risk [Null of 30%] is ***not rejected*** and therefore no EPI action would be contemplated.

3.2 "Correction" of the NA

Additionally, it is possible to use a correction to the Normal Approximation. Effectively, the Normal Curve fitted to the binomial probability-blocks "shaves" off a portion of the exact probability in the Left-Hand-Side testing direction. To compensate for this a Continuity Correction is used to belter approximate the binomial exact value. [For an overview see: Bickel and Doksum (2015) and for a detailed protocol: Tamhane and Dunlop (2000, pp.174-175)]. Typically, the magnitude of the continuity correction in the magnitude domain is 0.5. In our context, this will be <u>added</u> to the value of the event frontier of test interest. In this case, the FPE-risk value for Event[5] is:

Normal Approximation using the Continuity Correction [CCNA]

$$P[(x \leq (5 + 0.5) : N[6: 2.049]] : z_{cal} = \text{Abs}([[5.5 - 6] / 2.049]) = 0.244$$

In Excel: = (T.DIST.2T(ABS(0.244),10000))/2 = **40.4%**

Thus, for this example, there are non-trivial proportional ratio differences in the P-values:

For example, the directional ***error*** percentage differences as benchmarked by the Exact Binomial are:

The Error Percentage for the NA is -24.8% [(31.3% − 41.6%) / 41.6%]

The Error Percentage for CC-Normal Approximation is -2.9% [(40.4% − 41.6%) / 41.6%].

Given this illustrative case, it is clear that the CCNA is a ***misnomer***—the CCNA is itself an approximation to the Exact result—it seems prudent to research the jeopardy of eschewing the exact binominal distribution in favor of electing to use software or make computations that use

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approximations. To this end, a dataset was collected to determine preliminary indication information as to the error-risk of using the two usual approximations. This is actually not just an academic investigation, as most of the statistical software that are in use do not offer (i) the continuity correction "option" or (ii) suggest computational alerts that are possible misspecifications in failing to use exact information.

4. Preliminary Investigation of the FPE-Risk

4.1 Study Design

At this juncture, it would be most beneficial to have a *gestalt* regarding the nature of the FPE-risk Profiles for the three computational platforms: The Exact, The CCNA and NA. To this end, consider the three FPE-risk profile variations:

$$B_{pdf}[20, i\%] \text{ v. } B_{pdf}[100, i\%]; \text{ where: } i: 30\% + (5\% \times j); j = 1, - - -, 5.$$

Point of Information: There are n+1 P-values and so there can be n+1 aggregations that give n+1 FPE-risks. Paraphrasing Orwell: "*All FPE-risks are informative; however, some FPE-risks are more informative than others*". In this regard, for a germane inferential focus at the *overview* level, the FPE-risk endpoints selected are the Maximum-P-Value for the $B_{pdf}[n, i\%]$ using **n = 20** & **n = 100** for **% =30%** increased by 5% up to 50%. For example for $B_{pdf}[20, 30\%]$ the FPE-risk at Event [6]: [20 × 30%] will be the Maximum P-value and has a FPE-risk of 60.8%.

As one moves away from the Max-P-value point towards Event [0]—i.e., away on the LHS—the FPE-risk levels are increasingly bereft of practical decision-making impact—i.e., after moving *more than* six-event places from the Max-Event, the subsequent FPE-risks are so low that the expectation could be TRUE that the FPE-risks no longer have decision-making relevance. Thus using experiential intuition an interesting range for the overview of the FPE-risk profile seems to be: The FPE-risk at the Max-P-value and the FPE seven places to the Left on the LHS. For example, for n=20 this will be {Event[6] & Event[0]} and for n=100 {Event[30] & Event[24]}. These FPE-risk profiles will be instructive in probing the differences or jeopardy between the Exact Binomial, the CC-Normal Approximation [CCNA], and the Normal Approximation [NA] pursuant to profiling the various expected differences. As an illustration, consider, the profiles in Table 2.

Table 2. The FPEs for Sample-size Profiles [20 & 100] over the Percentages (%) Expectation Index

	Binomial[P-Value] MaxPoint[100]	Exact [FPE] Max:[20:100]	CCNA [FPE] Max:[20:100]	NA[FPE] Max:[20:100]	Exact [FPE] Max-7[20:100]	CCNA [FPE] Max-7[20:100]	NA[FPE] Max-7[20:100]
30%	Point[30]**8.68%**	60.8%: **54.9%**	40.4%:**45.7%**	50%:**50%**	0.07%:11.4%	0.4%:11.5%	0.2%:9.5%
35%	Point[35]8.34%	60.1%: 54.6%	40.7%:45.8%	50%:50%	0.2%:12.4%	0.5%:12.5%	0.3%:10.4%
40%	Point[40]8.12%	59.6%: 54.3%	41.0%:45.9%	50%:50%	0.4%:13.0%	0.6%:13.1%	0.3%:11.0.5%

| 45% | Point[45]8.00% | 59.1%: 54.1% | 41.1%:46.0% | 50%:50% | 0.5%:13.4% | 0.7%:13.5% | 0.4%:11.4% |
| 50% | Point[50]7.96% | 58.8%: 54.0% | 41.2%:46.1% | 50%:50% | 0.6%:9.7% | 0.7%:9.7% | 0.4%:8.1% |

4.2 Discussion

The codex for this table is: Column 2 presents the ***P-value*** for the Maximum Point for the sample-size 100. The Maximum Point changes over the *i%*-range. This information is provided only as a computation & verification check value. In Columns:{3, 4 & 5} are the ***FPE-risk levels*** for the three Models: {The Exact[Col3], The CCNA[Col4] & NA[Col5]}. Each of these are the ***FPE-risks*** at the Max Point for the two sample-sizes [20 & 100]. In Columns {6, 7 & 8} are the FPE-risks for the Event Point seven positions to the Left of the Maximum-Point, noted as Max-7. To aid in gleaning the information offered in Table 2, the computations in the shaded cells are detailed following using the Excel-script for B_{pdf}[100, *30%*]:

$$P\text{-value: } \textbf{8.68\%}: [[FACT(100) / [FACT(30) * FACT(70)] * [30\%^{30} \times 70\%^{70}]$$

$$\sum_{24}^{30} Event\,[i] = [[\sum_{0}^{24} Event\,[i] = 11.4\%] + 5.0\% + 6.1\% + 7.2\% + 8.0\% + 8.6\% + \textbf{8.68\%}] = \textbf{54.9}\%$$

$$P[\ (x \leq (30 + 0.5) : N[30: 4.58]]: \ z_{cal} = Abs(\ [[30.5 - 30] / 4.58]) = 0.109$$

$$\text{In Excel: } =(T.DIST.2T(ABS(0.109),10000))/2 = \textbf{45.7\%}$$

$$P[\ (x \leq (30) : N[30: 4.58]]: \ z_{cal} = Abs(\ [[30 - 30] / 4.58]) = 0$$

$$\text{In Excel: } =(T.DIST.2T(ABS(0),10000))/2 = \textbf{50.0\%}$$

With these detailed computations as context, the perusal of the FPE-information in Table 2 clearly suggests:

(i) The proportional FPE-risk ratios: Max-values for the Exact Binomial as the benchmark <u>to</u> the CCNA and the NA are often large; for example, for the CCNA at N= 100 for 35%, the ratio is: **−16.1%** [[45.8%-54.6%] / 54.6%] and for the NA is: **−8.4%** [[50%-54.6%] / 54.6%],

(ii) The proportional FPE-risk ratios: at the last P-value point in the P-value profile—this the 7th point to the left of the Max-point—are sometimes large; for example, for CCNA at N= 100 for 35% the ratio is: **0.8%** [[12.5%-12.4%] / 12.4%] not large in any context but for the NA the ratio is: **−16.1%** [[10.4%-12.4%] / 12.4%]—in this case large, and

(iii) the FPE-risk profiles for the sample size of 20 follow the FPE-profile for the sample size N =100.

5. Inferential Results: Statistical Analysis of the Germane FPE-risk Levels

5.1 Context and Rationale

The analysis of the FPE-risk profiles discussed above is interesting to be sure. However, only two (2) boarder cases were displayed in Table 2. In a statistical analysis addressing inferential relevance, it is necessary to enrich the evaluation context. Thus, the evaluation set of FPEs will be those in the Range {Max-2 through Max-7}. The elimination of the Max-point and its LHS-neighbor is reasonable as these FPEs are uniformly in the ***fail to reject the FPE[Null]-point*** and so are not likely to providing useful

inferential profiles of the impact of the CCNA & NA approximations. Therefore, for example, for n=20 for 40% the analytic accrual will be {Event[[20 × 40%] -2] through Event[[20 × 40%] -7] or the five Events: {Event[6] through Event [2]}.

As a further illustrative elaboration, consider Table 3 where the five index points are presented for $B_{pdf}[20, 30\%]$. Also see Table 1:

Table 3. Illustration of the Performance Ratios using the Binomial as the Benchmark

Events in Expectation	Exact Binomial	BNomial FPE-risk	CCNA FPE-risk	NA FPE-risk	P-v Ratio CC/Exact	P-v Ratio NA/Exact
Max-6 Point[0]	0.000798	0.000798	0.003646	0.001711	**357.0%**	114.5%
Max-5 Point[1]	0.006839	0.007637	**0.01407**	**0.00736**	84.2%	-3.7%
Max-4 Point[2]	**0.027846**	0.035483	0.04385	0.02550	23.6%	-28.2%
Max-3 Point[3]	0.071604	0.107087	0.11127	0.07163	3.9%	**-33.1%**
Max-2 Point[4]	0.130421	**0.237508**	0.23212	0.16457	-2.3%	-30.7%

5.2 Discussion

The first column is the selected P-values that will be used to form the dataset for the statistical analysis. To be clear regarding the Index information, Max-2 means that once the maximum P-value is located then the next Event-Point selection accrued is two Events in the LHS-direction from that Max-Point and so on for the next four points. This was achieved by finding the largest Exact Binomial value, this will be a variable given the parameters of $B_{pdf}[n, \%]$. This will always be at the Event: N×%. For example, for $B_{pdf}[20, 30\%]$ the maximum Exact Binomial value will be 6[20×30%] and 7 for [20×35%] and so on. After the maximum value is found, the first two (2) interior-values are passed over and the next five (5) are selected for the dataset. Usually this results in the highest P-value in the range of 25% and progressively lower value as one moves to the last index point. This dynamic will form a reasonable comparison set of points that are in the usual P-value test frontier. For example, for $B_{pdf}[20, 30\%]$ the Max point is: Event[6]: is found as it is the Max:Point as 38,760 × 4.94E-06: 19.2%. Thus, the dataset is: Points: {6-2, 6-3, 6-4, 6-5 & 6-6} OR the five Event Points {4,3,2,1,0}as scripted in Table 3. For example, Event Point[2] is: Max-4 or Point[6-4] = 190 × 0.000147 = **0.027846**. As for the Exact FPE-risk values, they are the aggregation of the previous values. Thus for the Event Point 4 or Max-2: Event Point [6-2] the FPE-risk value is the sum of all the point Binomial-values: **0.237508** [Sum{0.000798, - - -, 0.130421}].

5.3 Approximations CCNA & NA

As for the Approximations, the computations for Event Point Max-5 Point[1] for the two approximations are, as scripted in Excel:

CC-Normal FPE-risk values Approximation[CCNA]

-2.1958 = (1+0.5)-6) / (20*0.3*0.7)^0.5;

0.01407 = (T.DIST.2T(ABS(-2.1957),10000))/2

Standard Normal FPE-risk Approximation[NA]

-2.4398 = (1)-6) / (20*0.3*0.7)^0.5;

0.00736 = (T.DIST.2T(ABS(-2.4398),10000))/2

Finally, the <u>relative</u> benchmarked ratios are:

Point[0] = 3.569791 [(0.003646-0.000798) / 0.000798] or **357.0%**

Point[4] = -0.33107 [(0.071633-0.107087) / 0.107087] or **-33.1%**

In the creation of all the information, the P-values have been limited to LHS-probability values or P(x $<=$ Event[Point(j)]) where: the Event[Point(j) is always never greater than the adjacent lower point to the point where the $B_{pdf}[n, \%]$ has it maximum Exact Binomial probability.

The summary inferential indications are best discussed as the following four profiles.

6. Results: Relative FPE Impact of the Approximations: CCNA & NA

6.1 Analytic Context

In the analysis of the full dataset, so as to not overweight the inference for each of the sample sizes, only the data in the LHS-ranges was used. This produced for each of the two-sample size-arms 75 unique measures [5 × 3 × 5]. This is accounted for as: [5: Percentages{30%, 35%, 40%, 45% & 50%} for each of these there are three Models {Exact, CCNA & NA} and for each of these there are {Five measured Event-Points for the FPEs}. The restriction to the LHS is due to the similarity of the information set for the "flip-side" of the Binomial—recall that at 50% the values on either side of the Max-point are exactly symmetric; thus $B_{pdf}[n, 50\%] \equiv B_{pdf}[n, [1-50\%]]$. Finally, for the ratio information this accrual set for analysis condenses to 25 ratios for each sample size as the ratio computations uses one of the Blocking factors so there are 25-ratios for each of the two (2) sample sizes. With this profile of the statistical accrual set, consider the inferential results.

6.2 For the Sample Size of n=100

6.2.1 Ratio[**CCNA**/Exact]

None of these percentage ratios for the FPE-risks:[CCNA/Exact] were greater than **±2.5%**. The simple profile for these 25 points [five Profile-Points x the five percentage(%)] is: Mean[0.5%]: Median[0.3%]: Standard Deviation [StDev][0.01]: Maximum[1.3%]: Minimum[0.0%]: & Range[1.3%].

6.2.2 Ratio[**NA**/Exact]

All of the percentage ratios for the FPE-risks:[NA/Exact] were greater than ±2.5%. Additionally, all were negative indicating that the Exact FPE-value was greater than that of the Standard construction of the NA FPE-value. The simple profile for these 25 points is: Mean[13.3%]: Median[13.5%]: StDev[]0.02]: Maximum[16.1%]: Minimum[9.8%]: & Range[6.3%].

6.3 For the Sample Size of n=20

6.3.1 Ratio[**CCNA**/Exact]

Some of the percentage ratios for the FPE-risks[CCNA/Exact] were greater than Abs[±2.5%]. There were 15 such points. The simplest profile is the following Chi2 Table:

Table 4. Classification of Serious FPE-risk Departures for the Small Sample Arm

Number	Point[Max-7]	Point[Max-6]	Point[Max-5]	Point[Max-4]	Point[Max-3]
Neutral:<=Abs±2.5%	0	0	1	4	5
Serious: >Abs±2.5%	5	5	4	1	0
Profile[Mean:Med:StD]	122%:67%:1.2	35%:25%:0.3	12%:11%:0.07	2%:1%:0.01	-1%:-1%:0.01

The Pearson Chi2 P-value for Table 4 is 0.01 suggesting that there is a well-defined departure from the Marginals. This indicates that for the Sample size = 20-Arm that as the points move away from the Max-index their departure from the Exact values is increasingly profound.

6.3.2 Ratio[**NA**/Exact]

All of the percentage ratios for the FPE-risks:[NA/Exact] were greater than ±2.5%. Additionally, All, excepting two, were negative. The simple profile for these 25 points is: Mean[−29.3%]: Median[−31.5%]: StDev[0.07]: Maximum[3.7%]: Minimum[−38.7%]: & Range[42.4%].

7. Summary Inferential Indications and Outlook

7.1 Recapitulation & Extension

This research report highlights a lacuna in the statistical decision-making context. It is abhorrent to statistical sensibilities that a simple and pliable model amenable to standard VBA™ or R™ programming that assures inferential exactitude is not *le mode d'emploi* in the audit context. Further, the Factorial Binomial model seems to be ignored by most academic texts, researchers, and developers of most all of the statistical software platforms in current use. It has been clearly demonstrated above that the Normal Approximation [NA] is woefully inappropriate in all of the practical cases of decision-making in the audit context; electing the NA invites significant relative error. The Continuity Correction to the Normal Approximation [CCNA] fares somewhat better, but only for larger sample sizes starting at around 50; but for effecting audit Rp&Rc-testing where sample sizes needed to create audit evidence are often in the range of 20 to 30 to be sensitive to "budgetary control", there are more than a few instances where the variance from the Exact values are in the troublesome-zone. Thus, the impact of this research report is obvious.

> ***Do not use approximations to create relevant decision-making information when the Factorial generating process can be used to form an Exact probability decision space that can be used in the audit decision context for aiding in making the***

Extended Procedures Investigation decision.

7.2 Summary & Extension: A Complete Illustration

In this section, the details of the use of the Exact Factorial Binomial in a typical audit context are developed and summarized. In this case, Table 5 is an expanded, complete and generalizable version of the *A priori* FPE-risk Decision-Grid using, for purposes of illustration, B_{pdf}[20, 30%] that was introduced in Table 1.

Table 5. *A priori* FPE-risk Decision-Grid for B_{pdf}[20, 30%] addressed to Accounts Payable

EPI Context	Too Many Paid	Too Many Paid	Too Many Paid	Too Many Paid	Ok	Ok	Ok	Ok	Ok	Too Few Paid	Too Few Paid
P-Value	0.1 %	0.7 %	2.8 %	7.2%	13.0 %	17.9 %	19.2 %	16.4 %	11.4 %	6.5%	3.1% *
PValueΣ	0.1%	0.8%	3.5%	10.7 %	23.8%	41.6%	60.8%	39.2%	22.8%	11.3 %	4.8%
NotPaid	0	1	2	3	4	5	6	7	8	9	10

* The Probability for Events{11 through 20}= 1.715%; thus Table 5 sums to 100%.

7.2.1 Clarifications

In what follows, the standard construct used to create the Decision-Grid in Table 5 is the False Positive Error [FPE]-risk as derived from the *a priori* specified B_{pdf}[*n=20*, *%=30%*]. *A priori* means that before any data is collected the Decision-Grid that is Table 5 is *a priori* formed and parameterized by the IC.

7.2.2 Details re: Table 5

Assume that there is a *population* expectation [**30%**] or a belief founded on some combination of theory or experiential audit-evidence. This **30%**, posited by the IC, is called the *a priori belief* or the *test expectation* or sometimes the *FPE[Null]—this term is often used; it is linguistically challenged*. Simply, the FPE[Null] is the *a priori* test specification of **30%** offered by the IC that will be tested against [in comparison to] the observed or actual result. Then actual sample evidence is randomly selected from the Account-population, in this case **20** test-cases, that scientifically or objectively speak to or address the population expectation of **30%,** i.e., the *a priori* belief. The FPE-**risk** or -**chance IS** the inferential likelihood that the *a priori* belief [**30%**] is likely to be TRUE <u>given</u> or <u>in</u> the <u>face</u> <u>of</u> the actual evidence collected or observed from the **20**-random trials.

Thus, for B_{pdf}[*n=20*, *%=30%*], also see Figure 1, the IC forms the *A priori* FPE-risk Decision-Grid that is Table 5 in particular—the Column Designations: {*Too Many Paid*, Ok, *Too Few Paid*}. *Then* actual data is collected and processed using the Rp&Rc-audit protocol. Assume that in the sample of 20

Accounts Payable two (2) were actually not paid. In this case, the IC would reference Table 5 and thus observe that this chance or **FPE-[risk or –chance]** *re: the Null is 3.5%*.

7.3 Discussion of the Extended Illustration

IF the *a priori* belief:[**30%**] were, in fact, to be **TRUE**, then in a random sampling context, **n=20**, the likelihood or probability or chance of observing the actual result, produced by the testing, of **two (2) or less** AP-contracts **Not Paid** would likely happen only **3.5%** of the time. As this risk [chance or gambling-odds] is very low or unlikely, a better decision is to reject the *a priori belief* or the FPE[*Null-belief*] that 30% of the time AP-contacts in the Population are Not Paid on time in favor that it more likely that *more* AP-contracts are being paid than expected. Simply, if the FPE-risk or -chance percentage is low—{10%, 5%, 1% or <0.1%}, it is difficult to <u>logically</u> believe that the *a priori belief* of 30% is TRUE in the presence of convincing contradictory <u>actual</u> evidence. The Event-set: [0, 1, 2 & 3] is scored as "***Too Many Paid-range***" by the IC.

Further, if the FPE-risk or -chance is high, {> 25% *or so*}, then the actual results are likely to be consistent with or support the *a priori Null-test belief* and so rejecting the *Null-belief* would make no sense as the *a priori belief* seems likely to be the case—i.e., there is NO inferential difference from the *a priori* expectation. In this case, the IC would act on the *failure to reject the a priori belief* and take No-EPI action on that basis—i.e., the "***Ok-range***": The Event-set: [4 through 8]

7.3.1 Detailed Illustrative Example

Assume that the IC has assigned a Staffer to execute an Rp&Rc audit-protocol for a random sample of 20 Account Payables to: *Determine if the qualified discounts for AP-accounts over $100 are* ***responsibly*** *being taken according to the Client's Cash Disbursement Protocol*. Given the economic impact of the COVID-19 pandemic, the IC **expects** *that 30% of the AP-contracts will not be paid on time so as to take advantage of the qualified discount*. This is the *a priori* probability expectation that is the basis for the FPE-risk used to make the decision IF: *The nature of the execution of the Client's AP-protocol is "in sync" with the 30%-expectation of the IC given the actual evidence*. In this regard, *before* the Staffer's report as to how many of the 20 APs were ***actually*** not paid in a timely manner so as to take the discount is given to the IC, the IC parametrizes the *A priori* FPE-risk Decision-Grid: Table 5.

7.3.2 Partitions of the Inference Profile

In this case, there are ONLY three Decision-Regions:

1) *LHS Context*: According to the parametrization of the IC, IF the Actual Number of APs not paid were to be: {0, 1, 2 or 3} that suggests or indicates that <u>too many</u> APs were paid. This section is thus labeled as "***Too Many Paid***" as is noted in Row 1; this could indicate that the AP-payments were not consistent with "sensible" cash management in the COVID19-era and thus could call into question the adequacy of the Internal Control over Financial Reporting [ICoFR] required in the COSO-context. For example, the P-Values to create the ***FPE of Event***

(3) aggregate to 10.7%. This means: If the <u>population</u> expectation of the IC that 30% of the APs are expected not to be paid were to be TRUE, then the chance of observing in a random sample of 20-APs that three (3) *or less* were not paid would happen only 10.7% of the time. This is a relatively low chance or risk that the *a priori* expectation of 30% is, in fact, likely to be the case; thus, the IC would be justified in ***rejecting*** that the expectation of 30% is likely to be the case. Simply, as the FPE-risk or -chance of being correct re: the *a priori* expectation is only at most 10.7%, this would justify the IC to decide to not accept such low odds of being correct and so rationalize the rejection that 30% is likely to be the case. This rationalizes the "***Too Many Paid***, label thus, suggesting the likelihood that the IC will launch an EPI.

2) ***Right Hand Side[RHS] Context***: If the number of APs not paid were to be *more than eight (8)*, the IC apparently felt that <u>too few</u> were paid. In this case, referencing the *A priori* FPE-risk Decision-Grid for $B_{pdf}[20, 30\%]$, the FPE-chance is at most around 11%. This likelihood of being correct in believing that the *a priori* belief of 30% is TRUE is sufficiently low; and, so would likely suggest that there is little support of the *a priori* expectation. This rationalizes the label "***Too Few Paid***". This rejection of the Null-belief in this RHS-direction also may call into question the adequacy ICoFR and require an EPI, finally.

3) ***Interior Range***: If the number of APs not paid is in the interval: {*4*, **{5, 6, 7}**, *8*}— this is often called the "***Goldilocks Zone***"; not too many & not too few—just the right number ***according to the expectation of the IC***. In the Goldilocks-Range, as the minimal FPE-risk or -chance is about 22% for Events (4 or 8) the label affixed is "***Ok***" over this set of events. Rationale: These two-lower limits are likely to be ***<u>suggestive</u>*** that the *a priori* expectation ***<u>may</u>*** not be TRUE; however, 22% is not likely to call for the rejection of the likelihood that 30% ***<u>could</u>*** be TRUE. Further, any FPE-risk values in the interval **{5, 6, 7}** would be ***<u>strong</u>*** evidence that rejecting the *a priori* belief of 30% would not be consistent with the evidence. Simply, as the FPE-risk or -chance of being correct re: the *a priori* expectation is, in the worst case, around 40% this would likely justify the IC deciding to not reject that 30% is likely to be the case.

The *A priori* FPE-risk Decision-Grid for $B_{pdf}[n, \%]$ is a simple, exact and intuitive decision making tool. Additionally, this model allows the DM to form an "asymmetrical" screening grid. It is the case that there is a prevailing intuition that confidence intervals inherently ***are and need to be symmetric*** around some expectation mid-point. The reason for this erroneous but longstanding impression is that in the NA & the CCNA world, the [1-FPE%]Confidence-Interval is formed as: [%[Expectation] ± $z_{[multiplier]} \times S_e$]. The perpetuating culprit in this drama is the [±]. With the *A priori* FPE-risk Decision-Grid, as it is driven by the experiential judgment of the DM, it is possible to achieve an informed and, if needed, asymmetric screening interval. As is discussed above, there is strong evidence that the NA & CCNA create decision-making jeopardy. The take-away message.

> ***Use the Factor-model and construct an A priori FPE-risk Decision-Grid as was present in Table 5. This is consistent with the best practices execution of the audit***.

7.4 Outlook

Given the benefits of using an *A priori* FPE-risk Decision-Grid for creating audit evidence, it would be productive to program a Decision Support System [DSS] to calculate the FPE-Risk profiles so as to facilitate the execution the audit. In this case, the audit would, in the testing domain, be ***Effective*** as it used Exact decision-making information and, using the DSS, would be ***Efficient***. It is the audit-hallmark of "Best Practices" to have conducted an ***Effective*** and ***Efficient*** audit. Finally, as an extension, it would be an excellent inferential enhancement to benchmark the *A priori* FPE-risk Decision-Grid with a False Negative Error-risk context.

References

Bickel, P., & Doksum, K. (2015). *Mathematical Statistics: Basic Ideas and Selected Topics*. CRC Press[Florida] USA. https://doi.org/10.1201/b18312

Cox, D. (1970). The continuity correction. *Biometrika*, *57*, 217-219. https://doi.org/10.1093/biomet/57.1.217

Feller, W. (1968). *An Introduction to Probability Theory and its Applications* [3rd ed.]. Wiley [New York], USA.

Gaber, M., & Lusk, E. (2019). A Vetting Protocol for the Analytic Procedures Platform for the AP-Phase of PCAOB Audits. *Accounting and Financial Research*, *4*, 43-56. https://doi.org/10.5430/afr.v8n4p43

Hall, P. (1983). A unified approach to the correction of Normal approximations. *J. Applied Mathematics* [Div. SIAM], *43*, 1187-1193. https://doi.org/10.1137/0143077

Tamhane, A., & Dunlop, D. (2000). *Statistics and Data Analysis*. Prentice Hall [New Jersey] USA ISBN: 0-13-744426-5.

Turpin, L., & Jens Jr. W. (2018). On logarithmic approximations for the One-Sided binomial confidence interval. *J Optim Theory Appl*, *177*, 254-260. https://doi.org/10.1007/s10957-018-1257-x

Notes

Note 1. Recall the *Area* & *Slope* discussions above. Feller (1968) and Cox (1970) offered notable initial work in the area of approximations. An excellent math-stat treatment of the history and extensions of the impact of approximations is found in Hall (1983). Interestingly, a search of the *ProQuest™ ABI-INFORM™* search engine using the search terms:{*Abstract[Continuity Correction] & Abstract [Binomial]: no restriction as to publication date*} retrieved only the Hall and Turpin & Jens (2018) papers as being germane to this research report. Hall also points out that there are important approximation effects in terms of the second, third & fourth moments {Variance, Skewness & Kurtosis} that should be considered. However, the impact of these related moments is not an issue entertained herein.

Note 2. The Excel *t*-computation is used to remind the reader that the $N(\mu,\sigma)_{pdf}$ is irrational in that it does not have an exact decimal value in any testing domain. Thus, for most practical purposes, the Excel-precision of the *t*-distribution[*df* =10,000] provides a useful approximation. This is preferable to the archaic two-decimal Tables that clutter-up most statistical-texts.