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Carvalho-Transformations: A Robustness Analysis in the Forecasting Domain

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Abstract

Context *Transformations of Panel-data values are routinely made for qualifying datasets with the intention of enhancing the quality of the decision-making-intel that may be gleaned from inferential-testing. Interestingly, there seems to be a "Spill-Over" of this "Conditional Data-Transformation Imperative" that impacts the development and execution of forecasting*-protocols.

Focus *We offer inferential-tests of Transformations applied to randomly selected S&P$_{500}$ Firm-datasets to address the following research Questions of Interest:*

(1) Is there Transformation-Jeopardy if the wrong Box-Cox-Carvalho-Transformations are selected re: (a) The Capture Rate Profiles for the 95% Forecasting Prediction Internals Or (b) The Relative Absolute Forecasting Error [RAFE] for the Forecasting Predictions?

(2) In a consulting context, when Transformations are correctly used, the client almost always requires that the forecasts be re-transformed to the original measure of the data. Is there a Re-Transformation-Jeopardy re: the forecasting decision-intel needed to inform the decision-making processes of the client?

Results *We found that the theoretical expectations for the 95% Forecasting Prediction Intervals were* *founded even if the transforms were not to have been correctly selected. However, if the wrong transformation was to have been selected, non-trivial RAFEs are the likely result. Finally, if the correct transformation was to have been selected, re-Transformations to the original data-measures likely will inform the decision-making processes.*

Keywords

forecasting model development, Carvalho transformation jeopardy

1. Introduction

1.1 The Nature of the Data

The ***Nature of the Data*** is the initial-driver of ***all*** inferential-analytics. Consider the following musing of Carvalho (2016) who poses the following "semi"-rhetorical-question:

How well have we been teaching arithmetic, harmonic, and geo-metric means to our students? In a recent article by C. R. Rao and colleagues (Rao, Shi, & Wu, 2014), we read:

> "*Although the harmonic mean (HM) is mentioned in textbooks along with the arithmetic mean (AM) and the geometric mean (GM) as three possible ways of summarizing the information in a set of observations, its appropriateness in some statistical applications is not mentioned in textbooks.*"

Yes, in data-analytics, ignoring the likely ***Nature of the Data*** may well compromise the quality of the inferential-information used by decision-makers. Thus, let us consider the transformation taxonomy offered by Carvalho (2016, p. 270) that offers three conditional-screens for selecting the most appropriate context for inferential-testing where samples are required to provide ***population central tendency estimations***.

Table 1. Carvalho Taxonomy for Estimation of Means. *These platforms are available in most versions of *Excel*™ [*Microsoft*™]

Selection Profiles	Preferred Data Measure
Any non-*ratio/rate*-Values in the Real Domain [ℜ], where there is NO likely association among *any* of the point-value segments in the randomly selected data-set.	***Arithmetic Mean*[*AM*]** **Excel[AVERAGE]***
Any Values in the Real Domain [ℜ] > 0, where there IS likely association among some of the point-value segments in the randomly selected data-set.	***Geometric Mean*[*GM*]** **Excel[GEOMEAN]***
Any Values in the Real Domain [ℜ] > 0, where these	***Harmonic Mean*[*HM*]**

values represent Ratios or Rates.	**Excel]HARMEAN]***

1.2 Staged Transformation Selection Triage Protocol

As a user's-guide to Carvalho's taxonomy, we offer the following simple protocol for selecting among the AM, or the GM or the HM as the logical data-context for creating defensible inferential information re: The Maximum Likelihood-sampling Estimate [MLE], $\bar{x}_k$,—i.e., the Mean of a random sample from the k^{th}-population of interest.

Stage 1The initial pivotal triage-question is:

What is the inferential question to be tested in the Forecasting Domain?

Stage II From the specifics of the deconstruction of the test-measures to be used in addressing this inferential question underlying the ***Forecasting study***, the ***Nature of the Data*** decision can be simply gleaned by using the following staged-triage phases:

a. If the intel required is the forecasting-profile of the ***ratio/rate***-measure of the population from which a random sample was taken, then, *independent of the nature of the Coefficient of Determination* [*CoD*]—The standard measure of a multiplicative data-generating process indicating Panel-Point Association—the ***Harmonic Mean*** [**Inverse-Transformation**] is the logical choice***. In this case, following the Carvalho-taxonomy, the inverse-transformation is applied to the data.*** *Carvalho (2016*: *p. 270 [Example 3])****; if not then,***

b. If the intel required is the forecasting-profile of the population from which a random sample was taken ***and*** there **IS** evidence of a meaningful CoD, then the ***Geometric Mean*** [***ln*-Transformation**] is the logical choice. ***In this case, following the Carvalho-taxonomy, the* [*log $_{or}$ ln*]*-transformation is applied to the data.*** *Carvalho (2016*: *p. 270 [Example 2])****; if not then,***

c. If the intel required is the forecasting-profile of the population from which a random sample was taken ***and*** there is **NO** evidence of a meaningful CoD, then the ***Arithmetic Mean*** **[No Transformation]** is the logical choice***. No Nature of Data transformation is needed.*** *Carvalho (2016, p. 270 [Example 1])*

d. **Critical Contextual Note:** We are using the Carvalho Taxonomy ONLY to select the logical transformation for the Forecasting Context. **There is NO GM- or HM-forecasting profile!** However, we find that the Carvalho Taxonomy & *Staged Transformation Selection Triage Protocol* are very useful in the selection of the logical transformation: {***None***, or ***ln*** or

Inverse}to create useful decision-making intel for **Forecasting Studies**. **[End of Staged Triage Protocol]**

e. **Experiential Alert** We find almost exclusively that the AM [No transformation] is considered as the **default-choice** in the forecasting-domain; however, this invites the opportunity to eschew the critical-analytics, that often are time-consuming, needed to make a reasoned and informed choice as to the ***Nature of the Data***.

1.3 Discussion

The intel offered by the Carvalho-taxonomy and the *Staged Transformation Selection Triage Protocol* enable the identification of the "correct" [—most germane in creating useful decision-making intel—] ***parametric estimate of the Central Tendency of the Population from which the random sample was collected.*** In fact, this is the original context that spawned interest in data-transformations. As a vetting-condition, Carvalho assumes [*tacitly*] two important conditions: (i) the Central Limit Theorem [CLT] holds for the sampling-plan, and (ii) a sampled Population that is "*reasonably*" symmetric and ***thus*** usually free from Non-Ergodic segments. This being the case, the Sample Mean, $\bar{x}_k$, is the MLE of the of the Mean of the Population, μ_k [where k: AM or GM or HM]. At this point, an illustrative and instructive profiling is in order.

2. Computational Basics of the Carvalho-Taxonomy

2.1 Computational *Overview*

This is both a pedagogic demonstration as well as a rationale of our construction of a forecasting-test. Using the script offered by Carvalho and assuming the following [Panel: x_i {*i: 1, 2, - - -, n*}, the computations are as follows:

I. Arithmetic Mean [**AM**]: [$n^{-1} \times \sum_1^n x_i$], $\forall x_i \subset \Re$
II. Geometric Mean [**GM**]: Exp[$n^{-1} \times \sum_1^n \ln[x_i]$]], $\forall x_i > 0 \subset \Re$ *ln* is the Natural ***e-base*** logarithm and ***Exp*** the exponentiation of the *ln*-value, and
III. Harmonic Mean [**HM**]: [[n / $\sum_1^n [x^{-1}]$]] $\forall x_i > 0 \subset \Re$. Simply, the HM is the inverse of: [The AM of the inverses].

Using Appendix A [Panel [AI]], we have [using the Carvalho or the *Excel*-script]:

I. **AM**→[[1/12] ×[220,204]] = **18,350.33**
II. **GM**→ Exp[[1/12] ×[117.32]] = **17,621.54**
III. **HM**→ [12] / [7.07E-04]] = **16.968.07**

2.2 Deciding on the Correct Transformation

Mathematically, in the Carvalho-context, it is ***always*** the case that: AM > GM > HM. (See Note 1)
Analytical Note: ***If*** the analyst is dealing with a problem that is impacted by the central-tendency of the

population from which the random-sample was taken, ***then*** a non-trivial error is likely to be created by selecting an incorrect transformation. For example, if the analyst is dealing with a population of Rates/Ratios [A Carvalho [HM-Context]] but elects the Default-option of the AM [i.e., No Transformation] rather than the correct election—the HM-Inverse-Transformation, this ***Relative Absolute Carvalho Error*** [RACE] is:

$$[ABS[\bar{x}_{AM} - \bar{x}_{HM}] / \bar{x}_{HM}] \text{ or:}$$

RACE →**8.2%** [ABS[**18,350.33 – 16.968.07**] / **16.968.07**]

This suggests that electing the logical/correct transformation is required to collect relevant decision-intel useful in informing the decision-making process. ***Subsequently, we shall use a version of the RACE as the measure to inferentially test the jeopardy of selecting the wrong Carvalho-transformation in a forecasting context***—[Relative Absolute ***Forecasting*** Error [***RAFE***]. The simple de-brief is that if the analyst eschews the guidelines offered by the Carvalho-taxonomy and the *Staged Transformation Selection Triage Protocol*, then a non-trivial RACE ***IS*** likely the result.

3. Extension of Transformations to the Forecasting Domain

3.1 Overview

In the preceding section, we have detailed the operational aspects of the Carvalho-transformations and the *Staged Transformation Selection Triage Protocol*. However, the computational profiling is but an aspect of the issue that motivates our research report—to wit, the general question begged by transformations is:

For forecasting studies, what are the inferential consequences or jeopardy of making the wrong choice between {No Transformation: The ln: The Inverse}?

Contextual Alert This seems a reasonable question, as in our experience, rarely does the Forecasting Division undertake an evaluation of the Dataset to determine the ***Nature of the Data*** so as to make a reasoned selection among the ***{No Transformation: The ln: The Inverse}***-transformation-options. Usually, the data-panel, as downloaded, is examined for: (i) serious Box-Plot outliers, or (ii) Early-panel non-Ergodic segments—usually enterprise start-up panel-value-incongruities—but, almost never, for correctly fixing the ***Nature of the Data*** so as to select a logical data-transformation. The experiential-reason for this is, indeed, most interesting. In many ***forecasting*** consulting engagements over the years, where we "correctly" transformed the data and produced the forecasting-profiles ***in the transformed measures***, the forecasting managers resisted using these "Correct, ***but Transformed***" forecasting profiles because they found them confusing; they were not confident that they could understand these forecasting results, ***so presented***, to the extent that they could explain them to the Firm's Planning Committee. ***Behavioral Issue*** We will address this ***mini-conundrum*** as we investigate the issues of forecasting where the Carvalho Taxonomy is in play and suggestions will be offered to address this ***mini-conundrum***.

3.2 The Forecasting Connection

The initial interest of forecasters in transformations seems likely to have been derived from the following Box & Cox [B&C] Transformation *Passpartout*. Box & Cox (1964, p. 213):

> "*First, we can distinguish between analyses in which either (a) the particular transformation, λ, is of direct interest, the detailed study of the factor effects, etc., being of secondary concern; or (b) the main interest is in the factor effects, the choice of λ being only a preliminary step. Type (b) is likely to be the much more common. Nevertheless, (a) can arise, for example, in the analysis of a preliminary set of data.*"

We, as forecasters, are usually interested in aspects of both the (a)- & (b) -versions. However, we are not interested in testing inference-questions re: the vast-multiplicity of the versions of the following transformation-generating-model suggested by B&C [p,216: *Eq(3) Conditioned for y > 0*]:

$$y^{\lambda} = \begin{cases} y^{\lambda}, iff \ (\lambda \neq 0) \\ otherwise \ \log(y) \end{cases} \qquad \text{Eq[1]}$$

Eq[1] can form almost an uncountable number of transformations. We are using Eq[1] in the restricted context where the ***Nature of the Data*** seems best suited to provide inferential-intel for ONLY the following three cases suggested by Carvalho [Table 1]:

I. For the AM where λ = 1 thus, NO Transformation, or

II. For the GM—i.e., the otherwise case; thus, the *ln*[y]- ***or*** the $\log_{10}[y]$-Transformation, or

III. For the HM where λ= [-1] thus, [1/y] the Harmonic-Transformation.

Forecasters are most often interested in B&C-version (b)—the event-impact analysis. In this case, and in particular when linear models are the forecaster's predilection, forecasters often use the B&C-transforms to linearize their Y-Panels—the B&C-(a)-version. For example, if the trajectory of the Response Panel [Y] seems to exhibit marked-concavity [relative to the abscissa] *for the more recent Panel-segments,* then *the inverse- or* the *ln*- or the $\log_{10}$-transformations will often offer a more linear Y-profile than was the case for the downloaded Accounting Information System [***AIS***]-]data. Indeed, most all of the forecasting texts that we have used over the years recommend such transforms, in particular for the OLS-linear two-parameter linear regression [OLSR] Model. As more convincing evidence of the profound impact of the B&C-Carvalho treatment of transformations, we offer the Rule Based Forecasting Model [RBF] of Collopy & Armstrong (1992) aired some three-decades after the B&C (1964) research report.

3.3 The Multiplicative Context for the RBF Model

For their RBF Model C&A have noted the following data-conditioning l*n*[y]-transformation-protocol [***emphasis added***]:

Appendix A Definition of the Features [C&A: p, 1409]

Functional Form Expresses the expected pattern of the trend of the series. This rule base is limited to ***multiplicative*** and additive forms.

Appendix B The Rule Base [C&A, p. 1409]

Functional Form IF the functional form is ***multiplicative***, ***THEN use a log-transformation of the original series.*** (Fitting a log-transformation of the original series assumes a multiplicative growth.)

3.4 Point of Emphasis

It is the case, where there is a multiplicative generating-function—there is likely to be a meaningful CoD, this would likely suggest using the *ln*[y]-Transformation as noted in the Carvalho-taxonomy as the Geometric-Context.

3.5 Summary This discussion establishes the systemic linkages between: [The Box & Cox Transformation function: f_{λ}] and [The Carvalho Taxonomy] that motivated C&A's RBF: rules expressed in their Appendices A & B, noted above. The above presentation is offered as a ***necessary*** preamble to the following protocol focus of this research report:

For forecasting studies, what are the inferential consequences or jeopardy of making the wrong choice between {No Transformation : The ln : The Inverse} Transformations?

4. Research Agenda: Transformation Robustness

4.1 Overview

Recall that we noted that there are ***always*** the following ordered mathematical-differences in the estimation of population-values for the three Carvalho Taxonomy conditions:

$$[\bar{x}_{AM} \rightarrow \mu_{AM}] > [\bar{x}_{GM} \rightarrow \mu_{GM}] > [\bar{x}_{HM} \rightarrow \mu_{HM}]$$

where: $\bar{x}_k$ is the sample Mean that is the MLE-estimate of the k[th] Population the Mean: μ_k

Further, as part of this research report, we will test if this Carvalho Ordering holds also for the RAFE context:

$$\{f_{AIS\,[Y]} > \text{EXP}[f_{\ln\,[Y]}] > 1/[f_{Inverse\,[Y]}]\}$$

where: All three forecasts are presented in their AIS-measures

To decide if there are meaningful inferential indications for these RAFE-differences among these three forecasts, it is necessary to have a test-against False Positive Error [FPE]-Null with a realistic *a priori* p-value in mind as the evaluation- and judgment-screen. Following, are the elements of our experimental-design to arrive at an inferential-conclusion for our question of interest.

4.2 The Accrual of the Dataset-Panels

The issue, in an experiential evaluation of any question that merits inferential-testing, is the ***generalizability*** of the inferential indications—***end of story!*** In this context, care needs to be exercised to select reliable and relevant datasets, and then to conduct ***reasonable-vetting tests*** that ***prima-facia*** address the likely generalizability of the inferential profiles drawn from the selected datasets. Following is a discussion of our selected datasets.

4.3 Data Domain I: Trading-Markets Testing Dataset: The S&P$_{500}$™

We used the *Market Screener*™ Market Profiler (See Note 2) to selected our test-firms. Specifically, we selected, from the Bloomberg Market Trading Platform: BBT[Data-Panels], the following three Groups of Firms from the S&P$_{500}$ as Ranked by *Market Screener*™: {The **Top** 20 [Firms: [1through 20], The **Middle** 20: [We selected a random number from 100 to 400 as the accrual-starting-point. Thus, we selected Firms [141 through 160]] and finally, The **Bottom or Trailing** 20: Firms [481 through 500]]. For the variables to be used in the OLSR-Forecasting only the first 12 Quarters were used, usually starting with 3rd Quarter 2014. We selected this as the starting time index as it was more than five years ***after*** the *Lehman Bros*™ Sub-Prime financial debacle (See Note 3) that almost crashed the world's trading markets ***and*** about two years ***before*** the onset of the COVID-19 pandemic. As for the *Lehman Bros*-event, we judged that this was a sufficient time-lag for the Markets to re-adjust. See LinkedIn™ for a discussion of issues re; The Recovery (See Note 4) from the Lehman-Debacle. We selected 12-quarters as this was a Panel-size of sufficient length to have three-years for fitting an OLSR-Model.

4.3.1 Variables Selection: The S&P$_{500}$

As for the Variables to be used in the{Y←X} OLSR-forecasting, *a priori*, we selected for the Y-Response-variate:

Earnings **B**efore **I**nterest, **T**axes, **D**epreciation & **A**ccruals [*EBITDA*] [BBT:line[53]].

For the X-Variates [The Drivers], we benefited from collegial-discussions regarding logical X-drivers of the Y-variate: [*EBITDA*] as listed on: The BBT [*Income Statement* [GAAP-version]. After such discussions, we selected the following four X-Variables [The Drivers]:

SALES_**R**EV_**T**URN [line[6]]; **G**ROSS_**M**ARGIN[line[57]]; **O**PER_**M**ARGIN[line[58]]; **P**ROF_**M**ARGIN[line[59]]

The technical definitions for all of these five-variables are found on the BBT [*Income Statement* [GAAP-version]]. Scroll over the Variable and Right-Click to access the BBT-definition.

4.3.2 Screening the Initial 60 Firms

As detailed above, we selected 60-S&P$_{500}$ firms. However, not ALL of the S&P$_{500}$-firms were *per se* qualified to address our forecasting-research question of interest. Specifically,

I. As we are intending to take the *ln*- and the *inverse*- transformations, any Y-Panel-variable with a value < 0, will not be possible to use in the forecast protocol. Thus, we ***eliminated*** those Y-Panel-sets,

II. Tamhane & Dunlop (2000, p. 363) [T&D] offer the following inferential-forecasting caution:

   a. *"- - -**extrapolation** beyond the range of the data is a risky business should be avoided."*

   b. In this case, as our X-Variates are the drivers of the OLSR Model, we can ONLY use X-Variates that are **IN** the ordered range of X-variates used in creating the forecasts—i.e., ***Interpolations***. So as to use only X-Drivers that qualify as ***Interpolations***, we selected the X-Drives as follows:

   c. We VBA-screened all the X-Variates to see if the 13^{th} Panel-Point fell ***outside*** of the ordered Range of the 12-Panel X-Drivers used in parameterizing[fitting] the OLSR-Forecasting Model. If so, we ***eliminated*** that Panel-set. Simply, we are only interested in forecasting ***Interpolations***. For example, see Appendix A for the [GROSS_MARGIN n=12] Panel. The Holdback [HB] was **HB[**38.9273] and is in X-variable range:

Min[GROSS_MARGIN**[X]=**38.0054**] : Max**GROSS_MARGIN**[X]**=40.7792**].**

Thus, the forecast is scored as an ***Interpolation***.

III. For the Panels accrued, there were a few instances where there were missing Panel-values. We did not Regression- nor Near-Neighbor-fill these missing-values: rather, we ***eliminated*** that Panel-set, finally

IV. We are screening on the Nature of the Data so as to have sufficient power for the inferential tests. In this regard, we did not select any firm that had an {Y←X} CoD less than 0.05. This will create Firm-Panels that had multiplicative OLSR-profiles that will rationalize the [*ln-transformation*] as the Carvalho-Transform of choice.

After applying these four-screens, we arrived at 43 Firms in total [See Appendix B]:

I. Top Ranked Firms {Y←X} ***Pairs*** qualified for forecasting: [In total: 34 Forecasts],
II. Middle Ranked Firms {Y←X} ***Pairs*** qualified for forecasting: [In total: 33 Forecasts], and
III. Trailing Ranked Firms {Y←X} ***Pairs*** qualified for forecasting: [In total **22:** Forecasts].

4.3.3 Power Vetting

For assurance of the reasonability of the inferential detection, we considered the Power-context of these datasets. In this regard, we used the standard Wang & Chow (2007) [WC] Power: Sample-Size Model to examine the ***minimal*** sample-size for the tests for the smallest number of Panels—i.e., **22**. Setting the WC-Model Parameters at: [Population A(95%); Precision = 25%; (1-α) = 90%; Power= 75%], we arrive at a Sample-Size of 23. As **22** is ≈ 23, there is not an indication of a worrisome detection issue as these reasonable WC-Parameters are sufficient for inferential testing and our minimal sample size is greater than this WC-minimal level.

4.4 Data Domain II The Makridakis M-Competition [M1] A Vetting-Benchmarking Dataset

The M1 dataset [Makridakis (1982)] was selected to provide a vetting ***volatility***-test [measured as the Panel's Coefficient of Variation [CoV]] for benchmarking the ***CoV*** of the S&P_{500} BBT[Data-Panels]. These M1: datasets report, in the main, results of economic activity. For example, The Collective of Automobiles Manufactured: Total Production [France]; Consumer Expenditure OECD: Total Expenditures, Chemical Wood-Pulp Production [Brazil] & Gasoline Production [USA]. These M1: 181-annual series were subsequently used in a forecasting-context by: Collopy and Armstrong (1992) and Adya and Lusk (2016). We are mentioning this as the CoV of these M1 Panels *seems to have been* sufficiently in line with expectations so as to have encouraged numerous research groups to use them in researching aspects of the forecasting-domain. Thus, the M1-Panels serve as time-tested data where the generating-process seems stable and thus are likely indicative of generalizable ***Time-series generating process(es)***. Our final accrual of the M1 vetting-set was 109 Panels—i.e., those matched in Panel length to our S&P_{500} BBT [Data-Panels].

4.5 Forecasting Model

We used the same model form that was used by Makridakis et al. (1982), C&A (1992), and Adya & Lusk (2016):

OLS-Regression : Response-Variate [Y]←X[Driver] + Error[0,1]:

$$f_Y = [\hat{\alpha} + [\ \hat{\beta} \times X_{If}\ ]] \qquad \text{Eq[2]}$$

where: $\hat{\alpha}$ is the estimate of the Intercept; $\hat{\beta}$ is the estimate of the Slope of the Two-Parameter ***Linear*** forecasting model; these population-estimates of the Intercept and Slope parameters are determined by minimizing the **O**rdinary **L**east **S**quares of the Cartesian-Profile of the {Y←X}, n=12 Pairs that are a random sample from a target {Y←X}population, and the $X_{proposed}$ is the X_f is the 13^{th} X-value assuming that it would produce an **I**nterpolation—thus noted as: X_{If}.

In this testing context, we have elected to use four separate regressions for each of the qualifying firms: Response-Variate [Y=$EBITDA^{\lambda}$]←{[X=***SRT***], then [X=***GM***], then [X=***OM***] & then [X=***PM***]}

This suggests that the drivers of Y[*EBITDA*] are: **SRT**[**S**ALES_**R**EV_**T**URN] or **GM** [***G****ROSS*_***M****ARGIN*] or **OM**[***O****PER*_***M****ARGIN* or **PM**[***P****ROF*_***M****ARGIN*]. In addition, there are three Carvalho-transformations [λ : {y=1[AIS], *ln*[y] & 1/y]}]. In this case, there will be: 267 [[34 + 33 + 22] × 3] forecasts used in the inferential analyses. The inferential testing of the *Transformation-jeopardy-effect* will examine:

(i) the ***Capture Rate Profile*** of the forecasts for the respective ***95% Forecasting Prediction Intervals*** **[*95%FPI*]** for each of the Interpolation-forecasts—this will be the FPE-Null testing measure used to determine the relative error capture rates of the transforms of: {AIS *or ln or* Inverse} re: these Interpolation forecasts, and

(ii) the RAFE of the {$f_{Y_{AIS}}$ & $f_{Y_{ln}}$ & $f_{Y_{Inverse}}$} for the Carvalho-transform deemed to the likely candidate v. the other two will be calculated for inferential testing. This will be noted as: The Relative Absolute Forecasting Error [***RAFE***].

Special Note: The Theoretical definition of the [95%FPI] is:

[f_Y] ± [The 95% ***Forecasting Precision*** for the "*Next to be Observed Y-Variate*"]

The meaning of the [95%FPI] is: For this particular 95%FPI there is a 95% chance that the "*Next to be Observed Y-value"* is somewhere ***IN*** this particular 95%FPI, and (ii) also, this particular 95%FPI has a 5% chance of ***NOT*** containing the "*Next to be observed Y-value"* somewhere IN this particular 95%FPI.

At this point, we will illustrate all of the computations for the three transformations: {AIS : *ln* : Inverse} for the creation of the 95%FPIs. The datasets used in the demonstration are the *AAPL*; [Y≡[*EBITDA*]←X[***G****ROSS*_***M****ARGIN*]]. We will detail the Arithmetic Mean [AIS] Test-Panel. The *ln*- & Inverse-analyses will be given in summary notation.

4.5.1 Illustration [AIS]: The AAPL as Downloaded Appendix A [Panel [AI]] The Arithmetic Mean [AIS]

The AM ***holdback*** X-Variable to be used to create the ***Interpolation***-forecast, X_{If} , is: X-[***G****ROSS*_***M****ARGIN* [38.9273]]; Appendix A [Panel [A:IV]] note: This X-Panel-value is the 13th Panel-value and it is NOT Outside the ordered range of the [*GROSS_MARGIN*] Panel-Values used to fit the OLSR Model. The corresponding assumed "*Next Y-EBITDA to be Observed"* is: ***16,429.00—i.e., the 13th Panel-value in the A1-Panel***. The Mean & Median of the *Y*-[*EBITDA*] are respectively: 18,350.33 & 16,815.50.

The OLSR-Forecasting Equation as fitted is: f_{AM} = [–96,369.70 + [2,923.01 × X_{If}]]

The X_{If} is: 38.9273 thus:

The OLSR-forecast is: f_{AIS}= **17,415.21** = [–96,369.70 + [2,923.01 × ***38.9273***]]

The [**95%FPI**] for this Y-*EBITDA*-projection is:

17,415.21 ± [The Precision]

where: The Precision for the above forecast is:

$$\text{Precision} \equiv [\text{T.INV.2T}(5\%,12\text{-}2)] \times [\sqrt{MSE}] \times [f_{Penalty}]$$

where: The MSE is the **M**ean **S**quare **E**rror of the OLSR as fitted, [T.INV.2T(5%,12-2)] is the t-measure for the OLSR model, and the [$f_{Penalty}$] is: (1 + 1/n + [[X_{If} − $\bar{x}_{If}$]^2]/S_{xx})^0.5, S_{xx} = 9.19, and $\bar{X}$= 29.2472.

In this case, the Precision of the 95%FPI is: [2.2814 × 5,169.6569 ×1.0462] =**12,050.52**.

Thus, the 95%FPI is:

17,415.21 ± **12,050.52** or 95%FPI→[5,364.69 : 29,465.74]

Note: The forecast f_{AM}= 17,415.21 is the midpoint of the 95%FPI. The Actual Value of the assumed "*Next Y-EBITDA to be Observed"*—[i.e., that corresponds to the ***GROSS_M****ARGIN* of 38.9263] is **16,429** and this value is ***IN*** the 95%FPI.

Finally, the Precision-benchmarks are: [Precision / Mean] = [**12,050.52**/ 18,350.33] = **0.657**; and the [Precision / Median] is: [**12,050.52**/16,815.50] = **0.717**. These profiles are most useful in detecting possible outliers in the dataset.

AAPL Summary Profile as Recorded for Inferential Testing [Only the **Bolded** Information is Recorded in the Data Set for inferential-testing.] The Forecast was: **17,415.21**. The 95%FPI was created by the X-Interpolation; the 95%FPI was: [5,364.69 : 29,465.74]. The "*Next Y-EBITDA to be Observed"* was: 16,429 and was ***IN*** the 95%FPI. **The Precision/Mean = 65.7%** & **The Precision/Median = 71.7%**. This is **all** the information that is relevant for the Jeopardy-Test re: The Carvalho Transformations.

For the *ln*- and the Inverse-tests we have the following *summary* information:

4.5.2 Illustration[*ln*]: The AAPL The *ln*-Transformation Appendix A [Panel [AII]].

The Essential Conundrum The are never forecasting-profile interpretation issues if, in fact, the AIS-context is the correct ***Nature of Data*** choice for the dataset to be forecasted. In this case, The Data, The Forecast, and the Forecasting 95%FPIs are all in the measured units of the Data as downloaded from the firm's AIS. Thus, decision-makers easily glean the meaning of the inferential forecasting profiles. However, in the preponderance of the cases, for economic data to be forecasted, there are Panel-Point-value-associations that are multiplicative in nature and so the *ln*-Transformation is the logical choice for the correct Nature of Data selection. ***However, here is where the interpretation issues are encountered***. Following, we will offer a proto-typical example that we found useful to introduce the de-construction-Issues and engage the students [in an Intermediate semi-Math/Stat course in Forecasting] in a discussion that seemed to enhance their appreciation for the creation of relevant- and useful- intel from Forecasting in the *ln*-context.

Progenitor Intel It seems that forecasters who are aware of the basic Math/Stat-issues for the generating meaningful inferential profiles from Panels best suited for *ln*-analytics expect that the *ln*[Profile] ***of a single Panel should seamless/homo-morphologically map onto the {Y←X} Forecasting context***. Let us consider this supposition.

The Standard GM-Intel for the ***Single Panel***

In this case, we will use the EBITDA Panel-A1[n=12] of Appendix A. Assume that we test for multipliable-Panel-Point-association so as to select the most likely Carvalho-context. The computations follow:

Phase I ***Nature of the Transformation***

Assume that we have the *a priori* decision-protocol that if the Coefficient of Determination {CoD}: {X←Y}: {Y-*EBITDA*← *GROSS_MARGIN* }> 5%, then there is evidence of a multiplicative generating process. In this case, the CoD is: 0.227, thus, the *ln*-context is deemed the likely selection.

Phase II ***GM Profile for a Single Panel*** EBITDA Panel-A1[n=12] of Appendix A

The usual intel of interest is the Mean[$\bar{x}_{GM}$] & The 95%CI of the $\bar{x}_{GM}$: However, the data-measures are the *ln*-transformed values as downloaded. These are here **Bolded**, for the Panel under investigation.

For the GM-Profile, the analyst will use the *ln*-transformation; we have:

Mean[[$\bar{x}_{\ln[x]}$] = 9.776877377], The Standard Error of the **Mean[[$\bar{x}_{\ln[x]}$]**: s_e[0.084716289], and $t_{95\%,df=11}$[2.20098516]. Thus, the 95%CI[ln[x]] will be: **Lower 95%CI: [9.590418] & the Upper 95%CI: [9.963337]:**

Lower 95%CI[ln[x]] **9.590418083: [9.776877377 –** 0.084716289 × 2.20098516**] &**

Upper 95%CI[ln[x]] **9.963336672: [9.776877377 +** 0.084716289 × 2.20098516**]**

Key Intel: The Mean[[$\bar{x}_{\ln[x]}$] = 9.776877377] is the mid-point of the 95%CI[ln[x]].

If the analyst is requested to offer the Y-*EBITDA*-profile in the Original AIS-Units, the standard re-transformation is to EXP all the above profile-intel. In this case,

Mean[[$\bar{x}_{ln}$] = 9.776877377] & Lower 95%CI[*ln*]: [9.590418083] & Upper 95%CI[*ln*]: [9.963336672] become:

Mean: EXP[9.776877377]→ ***17,621.54≡*** **[$\bar{x}_{GM}$]**

Lower 95%CI: EXP[9.590418083]→ ***14,623.98*** & Upper 95%CI: EXP[9.963336672]→ ***21,233.52***

However, for the Re-transformation or Back-Transformation, the midpoint of this 95%CI is 17,928.75; ***it is not equal to the GM[$\bar{x}_{GM}$] of 17,621.54!***

This usually causes as great deal of consternation for those decision-makers who have been conditioned to expect that the ***GM*[$\bar{x}_{GM}$]** should be the mid-point of the re-transformed 95%CI[GM]; if this is not the case, they are uncertain as to the nature of the intel offered by ***this*** re-transformed ***95%CI***. In this case, to allay any interpretation issues for those who are relying on this re-transformed-forecasting-intel to make decisions for which they WILL be held accountable, we recommend the **Alf & Grossberg (1979)** [A&G] protocol to create the re-transformed-forecasting-intel.

The Correct Re-transformed ***GM[95%CI]*** The A&G-protocol is very simple and intuitive. The computations for the A&G[95%CIs] are most instructive.

The Lower A&G[95%CI] is: **[[$\bar{x}_{GM}$] – ($\bar{x}_{GM}$ × s_e × $t_{95\%,df[n-1]}$)] &**

The Upper A&G[95%CI] is: **[[$\bar{x}_{GM}$] + ($\bar{x}_{GM}$ × s_e × $t_{95\%,df[n-1]}$)]**

Note: The Midpoint of A&G[95%CI] is the Mean: **[$\bar{x}_{GM}$]**.

Computations:

The GM must be created; thus, using the Carvalho-protocol [p.270] we have: EXP[**9.776877**]≡ GM**[**17,621.54**]**. Using the A&G protocol, the Standard Error of the GM: s_e[GM] must be created as follows:

s_e[Panel[ln[x]]] ≡ [The Standard Deviation of the Mean of the *ln*[Panel]] divided by [$\sqrt{n}$]; or

s_e[Panel[ln[x]]] ≡ 0.293465833 / [$\sqrt{12}$] = 0.084716289.

With this, we can form the s_e[GM] as:

s_e[GM] ≡ [[$\bar{x}_{GM}$**]** × s_e[Panel[ln[x]]]×]

A&G[GM[95%CIs]] ≡ EXP[Mean[*ln*[Panel]]] ×[1 ± [s_e[Panel[ln[x]]] × $t_{95\%,df=11}$]

Specifically,

GM[95%CIs]→ EXP[[**9.776877577**] ×[1 ± [0.084716289] ×2.20098516]

The *$Rounded* Lower GM[95%CI] = **14,335.84 &** Upper GM[95%CI] = **20,907.24**

Note: the midpoint of the GM[95%CIs] is

$\bar{x}_{GM}$[17,621.54**] = {[14,335.84 + 20,907.24] / 2}**

The Meaning of the A&G[GM[95%CI]] is: For the population from which the Random Sample, n=12, was taken,

(i) there is a 95% chance that the true population Geometric Mean: [μ_{GM}] will be ***somewhere*** IN the A&G: re-transformed interval: [**[14,335.84] through [20,907.24]**], and

(ii) there is a 5% chance that the true population Geometric Mean: [μ_{GM}] will be NOT be ***somewhere*** IN the A&G: re-transformed interval:[**[14,335.84] through [20,907.24]**].

Forecasting in the ln-context **Contextual Alert** The A&G protocol applies to creating a ***re-transformed 95%CI*** for the GM—***this is the decision-making-intel for the GM***. With this as context, we now have taken-up the discussion of {Y←X}-OLSR ***Forecasting***: where: The Y-Panel is multiplicative in

Nature—*to wit* the Y-Panel is assumed to be *ln*/log Normal. ***We are avoiding labeling this as GM-Forecasting***. The Geometric Mean & Forecasting discussion share ***only*** the assumed Nature of Data Profile—i.e., the *ln*-Normal Distribution. Other than this Nature of Data link, they have ***No Math/Stat theory in common that defines and rationalizes particular aspects of their inferential testing***. With this critical contextual, let us delve into the Forecasting Domain.

Forecasting in the ln-Normal Context Consider the following ***forecasting profile*** of Y-*EBITDA* using the: X:[***G****ROSS*_***M****ARGIN*].

Computations

As indicated above, the CoD for the {Y-*EBITDA* ← ***G****ROSS*_***M****ARGIN* is 0.227 >0.05—this rationalizes the use of the GM-context and so *ln*-transforming of the Y-*EBITDA*-Panel; the $\bar{X}_{Driver}$ = 38.9263 and the corresponding holdback Y-*EBITDA* is: *ln*[16,429] = 9.7068. The Mean & Median of the Y=[*EBITDA*] are respectively: 9.776877 & 9.729837.

The OLSR-Forecast is: $f_{Y[ln]}$= **9.723195** = [3.190664 + [0.167814 × 38.9273]]

The 95% Forecasting Precision Interval [**95%FPI[*ln*]**] for this Y-*ln*[*EBITDA*]-projection of 9.723195 is:

9.723195 ± **0.611662** or the 95%FPI[*ln*] is: [9.111533 : 10.334857]

Note: The mid-point of the [**95%FPI[ln]**] is $f_{Y[ln]}$→ 9.723195 as expected.

AAPL Summary Profile as Recorded for Inferential Testing The Forecast was: **9.723193**. The 95%FPI[*ln*] created by an X-Interpolation; the 95%FPI[*ln*] is: [9.111533 : 10.334857]. The *Next to be observed Y-value* was: *ln*[16,429] = 9.7068, this is the *ln*-transformed Y-value that corresponds to the ***G****ROSS*_***M****ARGIN* of 38.9263; and, this *Next to be observed Y-value* is ***IN*** the 95%FPI*[ln*]. Finally, the Precision-benchmarks are: **Precision/ Mean is 0.611662/ 9.776877 = 6.3%** and the **Precision/ Median is: 0.611662/ 9.729838 = 6.3%**. This is **all** the information that is relevant for the inference-testing re: The selection of the Carvalho Transformations.

Special Note on the ln[Conundrum]: ***Behavioral Un-freezing*** Assume the decision-makers would like ***The Forecast[***$f_{Y[ln[Y]]}$ and ***The 95%FPI[ln[Y]]*** to be ***re***-transformed to the Original Data Measures [Y] as they exist in the firm's AIS. ***Rationale*** Most of the forecasting decision-makers have some university instructional-exposure to Statistical Inference as it relates to FPIs. In this context, they have been ***conditioned*** to believe that the ***Center of the FPIs IS the Forecast***; if this is not the case, this seems to interject ***incertitude*** as to the correctness of these forecasting profiles—the intel of which they will need for decision-making purposes. This is the ***Behavioral Freezing Conditioning*** due to the usual university instruction. ***True, IF the data context is the AIS then, indeed,*** the ***Center of the FPIs IS the Forecast***. However, this is not the case, if: (i) the forecasting data is Multiplicative- or Ratio/Rate-In Nature, and (ii) these FPIs are re-transformed to the original AIS-measures of the data. ***Re***-transformation **dis**lodges the ***re***-transformed Forecast so that it is ***never*** the midpoint of the ***re***-transformed 95%FPIs. Here, *en bref*, is our usual ***Behavioral Unfreezing attempt*** to allay the concerns of the decision-makers re: their concern over the ***dis***lodged Forecast:

> *Please understand: The ln-transformation was used as there was evidence of a multiplicative data-generating process[es] for the Y-EBITDA-Panel. The ONLY forecasting profile, usually of interest to decision-makers, will be* ***The 95%FPI[ln]***→
>
> ***The 95%FPI*****[*****ln*****]: Lower 95%FPI** [9.111533] & **Upper 95%FPI** [10.334857]
>
> ***Note: The*** $f_{Y[\ln[x]]}$ **= 9.723195** ***IS the Midpoint of this 95%FPI*****[*****ln*****].**

Discussion The decision-making meaning of ***The 95%FPI*****[*****ln*****]** is**:**

(i) there is a 95% chance that the "Next to be Observed ln[Y]-value" will be ***somewhere*** IN the FP[*ln*]-Interval: [**[**9.111533**] through [**10.334857**]**], and

(ii) there is a 5% chance that the "Next to be Observed ln[Y]-value" will ***Not*** be ***somewhere*** IN the FP[*ln*] Interval: [**[**9.111533**] through [**10.334857**]**]

This is the only exact technically correct forecasting profile-generated by the OLSR Model that is often used to create decision-making intel. Usually, **Decision-makers ask:**

As we are ONLY interested in the decision-making information conveyed by *The 95%FPI*[*ln*]: This means that the next observed *EBITDA measured as ln[EBITDA] can be expected to be somewhere in the interval* [[9.111533] through **[**10.334857**]**], as indicated above. If this is the case, then it seems that the following ***must also*** be True:

As we only have the measured-values of our AIS, that are values reported as part of the SEC-reporting requirements of the firm, these *EBITDA* values will not be *ln*-transformed but rather in the AIS-measure of our Firm. Thus, it should be the case *that the next AIS reported value for EBITDA should be in the* ***re****-transformed interval:*

*EXP[****95%FPI*****[*****ln*****]***] or*

EXP[9.111533]→ 9,059.15: through EXP[10.334857]→ 30,787.38,

Our response is: Yes, this is exactly correct. The tacit implication is that the re-transformed forecast is of little value; the real decision-intel lies in *The 95%FPI*[*ln*] or The EXP[*The 95%FPI*[*ln*]]. Both give the sane valid forecasting information that can inform the decision-making process of the firm.

Vetting check: Using the endpoints of the *[****95%FPI*****[*****ln*****]***] or the EXP[****95%FPI*****[*****ln*****]***]* is logical and standard practice. To provide vetting intel to test this expected-relationship, we used the 89 *ln*-forecasts that we have created as the test-Panels reported in this research report. We had 89-Interpolation forecasts where we used the mathematically correct [95%FPl[*ln*]s] to evaluate the 89 *ln*-Holdbacks. There were 7 *ln*-[transformed] holdbacks that were ***not in*** the 95%FPl[ln]s. We then created the **re**-transformed end-points, as indicted above, for each of these 89 Panels—i.e., EXP[95%FPl[*ln*]s]. We then tested if the AIS-Holdbacks: [in the original *"AIS"*-measures as downloaded] were in these **re**-transformed "AIS"-EXP[95%FPl[*ln*]s]-end-points. In this vetting-test, there were also 7 "AIS"-Holdbacks ***not in*** the interval of the **re**-transformed end-points. And all of these 7-points were from the same Panels used in this forecasting test. ***Thus, the decision-making jeopardy was zero for this GM-vetting test!*** (See Note 5)

4.5.3 Illustration The AAPL The Inverse-Transformation Appendix A [Panel [A:III]]

The holdback X-Variable to be used to create the forecast, X_f , is: X[[***GROSS_M****ARGIN*[]] = 38.9273; the corresponding holdback Y-*EBITDA*[Actual-Value] is: Inverse[1/16,429] = **6.0868E-05**. The Mean & Median of the Y=[EBITDA] are respectively: **5.8934E-05** & **5.9495E-05**.

The OLSR-Forecast is: $f_{Y[1/x]}$= **6.2119E-05** = [4.50E-04 + [**–**9.95E-06 × 38.9273]]

The 95% Forecasting Prediction Interval for this Y-*EBITDA*-projection is:

6.2119E-05 **± 3.2889E-05** or 95%CI[1/x]s are: [***2.92E-05*** : ***9.50E-05***]

AAPL Summary Profile as Recorded for Inferential Testing The Transformed Forecast was: **6.2119E-05**. The transformed 95%FPI was created by an X-Interpolation and is: [2.92E-05 : 9.50E-05]. The *Next to be observed Y-value* was: [1/*EBITDA*] = **6.0868E-05**; this is the HM-transformed Y-value that corresponds to the ***GROSS_M****ARGIN* of 38.9263; and, this value is ***IN*** the transformed [95%PI[1/x]]. Finally, the Precision/ Mean: **3.2889E-05 / 5.89342E-05 = 55.8%** and the **Precision/ Median is: 3.2889E-05 / 5.94949E-05 = 55.3%.** This is **all** the information that is relevant for the Jeopardy-Test re: The Carvalho Transformations.

Profile Note As expected, but not here detailed, it is also the case that $f_{Y[Inverse]]}$ is in the 95%FPI[1/x] while the re-transformed **forecast [1/$f_{Y[Inverse]]}$] 16,098.13 is not the mid-point of the re-transformed 95%FPI[1/x] of: [10,525.43 : 34,211.43]**

4.6 Summary Discussion

These computations are most useful for those interested in enriching their insights into the effects of the transformations and also, to be sure, on the nature of the 95%PIs re: forecasting Interpolations. Thus, we recommend working through these computations ***before*** going on to the Transformations-Jeopardy testing sections following.

5. The Transformation Jeopardy Testing: Vetting Profiles

5.1 Vetting: To What End?

Vetting is an effective and efficient protocol for providing an indication of the ***generalizability*** of the inferential-results proffered by researchers. Excellent examples of quality vetting are the multi-facetted vetting-tests C&A employed in the development of their RBF-Model. From a technical-perspective, quality vetting usually is an effective way to ensure, to some extent, that the ***dataset*** from which the inferential-results are created generalizes to "*most other datasets of a similar nature*". We have selected the following vetting-tests that we assume offer evidence that that they are likely to address the ***generalizability*** of the Market Dataset that we accrued from the $S\&P_{500}$. Following are three vetting tests that address the generalizability of our $S\&P_{500}$ accrual dataset:

Vetting I The Coefficient of Variation [CoV] : Our S&P$_{500}$ Market-Data *vis-à-vis* The M1 Vetting-Data,
Vetting II The CoV Profiles for The S&P$_{500}$ Market Leaders *vis-à-vis* The Trailing Firms, and
Vetting III The Median-Benchmarked Profiles for The S&P$_{500}$ Market Leaders *vis-à-vis* The Trailing Firms**.**

Discussion In what follows, we offer the rationale to support these vetting-tests as not unreasonable testing-profiles that are likely to provide intel on the generalizability of the S&P$_{500}$ to most any trading Market datasets.

5.1.1 Vetting I CoV**:** The Protocol

As a benchmark for the calibration of the CoV, we will use the C&A Time-series study. C&A note: "*The percentage of Panels that have a Coefficient of Variation [CoV] > 0.2 is 21%*. [Table 1, p.1402]. C&A define the CoV as: *"The standard deviation divided by the mean for the trend adjusted data in the original units.*" [C&A [Appendix A], p.1409]. However, we will be using the *Excel*-standard definition, as we are not able to use their forecast profiles to create their C&A's version of the CoV. However, we will use their measure-context—*the original data* of the Y-[*EBITDA*]-Panels. Thus, all vetting will be done on our S&P$_{500}$ Market-Data as downloaded. Thus, we will use [in *Excel*-script]

CoV ≡ STDVA/ AVERAGE

Note that the Panel-length of the S&P$_{500}$ Market Data, n=12, is much shorter than most of the Panels used by C&A where the median Panel-Length was n =21. C&A imply that shorter-Panels usually have more volatility re: to that of longer-Panels. See C&A (p. 1407) also see [Schnaars & Bavuso (1986: p. 27)] where it is shown that "*movements in series collected at such short intervals are dominated by random fluctuations*". Also, C&A used only annual-Panels, usually reports of country production. The CoV-profile of these sort of Panels usually have central-tendencies that are lower than are found for CoV-profiles of the relatively highly volatile Quarterly market-trading S&P$_{500}$ [Data-Panels] that we have accrued. Finally, the percentage of the C&A series that were multiplicative was 83% [p, 1401]; this aligned with our S&P500 Panels that had a CoD average of 50.1%. *Expectation* In this case, it seems inferentially reasonable to benchmark our S&P$_{500}$ Market Data of the Y-[*EBITDA*]-Panels, using C&A's CoV profile of 21% for their 126 M-Competition Panels. ***We expect that our trimmed- S&P$_{500}$ [Y-[EBITDA]-Panels] would have higher volatility and so their CoVs-Profile would have a higher Mean[CoV].***

5.1.2 Vetting II Blocked Inferential Testing: The Protocol

As a second directional-inferential test, we will test the CoV by the S&P$_{500}$ Firm-Groupings {Top v. Trailing}. *Expectation* In this case, as volatility is usually an important measure for investors ***we proffer, that the Top-Firms are more likely to have lower Volatility [CoV] than that the firms at the Trailing-end of the S&P500 rankings.***

5.1.3 Vetting III The Precision Protocol: The Protocol

Precision is a measure of the ***width*** of the Forecasting Prediction Interval. The magnitude of the width of the 95%FPI, of course, depends on a number of features of the dataset—effectively: (i) The relative magnitude of the measured data, (ii) The volatility of the Panel, and (iii) The third-moment: [The Skewness-Profile] of the Panel-values. To make inferential comparisons over various-datasets, often Forecasting Precisions are benchmarked by the Panel's Y-Mean or the Panel's Y-Median. This usually, will "unitize" the Precision-values and so facilitate inferential-comparisons. We prefer the Median as the benchmark as it is much less prone to asymmetrical-effects than is the Mean. ***Expectation*** In this case, as we have rationalized that the Top Firms are more likely to have lower Volatility [CoV] than that of the Trailing-$S\&P_{500}$ Firms, thus, ***our expectation is that the Trailing-$S\&P_{500}$ Firms will have wider-95%FPIs vis-à-vis the width of the 95%FPIs of the Top-$S\&P_{500}$ Firms.***

5.2 Vetting Results

Given the above overview, we present the test-results of the three Vetting tests profiled in Table 2 following:

Table 2. Vetting Tests for the $S\&P_{500}$ Panels

Vetting I	
Coefficient of Variation [CoV]	
M1 Cov, n=126	**CoV > 0.2 [21.0%}**
BBT[Data] n=89	**CoV > 0.2 [70.0%]**
Directional p-value	**ToP: <0.0001**
Vetting II	
Coefficient of Variation [CoV] by Firm	
Top Firms n=34	**Median [0.306]**
Trail-Firms n=22	**Median [0.331]**
Directional p-value	**W/K-W[0.145]**
Vetting III	
Benchmarked Precision by Firm	
Top Prec/Md n=34	**Median [0.404]**
Trail-Prec/Md n=22	**Median [0.441]**
Directional p-value	**W/K-W[0.23]**

5.2.1 Results: Vetting I

In this case, we used the directional two-sample Test of Proportions as follows:

$$z_{cal} = [70.0\% - 21.0\%] / \sqrt{0.00236 + 0.001317} = 8.08$$

This p-value is < 0.0001; thus, there is ***conclusive*** evidence that: (i) The $S\&P_{500}$ Panels and the M1 Data-Panel come from different volatility-populations [Measured as the CoV], and (ii) The percentage of Panels for which the CoV was > 0.2 was significantly > for the $S\&P_{500}$ Panels than for the M1 Data-Panels. ***Summary Vetting I is founded.***

5.2.2 Results: Vetting II

In this case, we selected the ***Median*** as the inferential measure; thus, the test was the **W**ilcoxon / **K**ruskal-**W**allis Tests (Rank-Sums)[Chi^2-version][**W/K-W**]. The Directional p-value is 0.145; thus, there is ***reasonable*** [not conclusive] evidence that the volatility [Measured as the CoV] of the Top $S\&P_{500}$ Firms is < than that of the Trailing $S\&P_{500}$ Firms. ***Summary Vetting II is suggested.***

5.2.3 Results: Vetting III

In this case, , we selected the ***Median*** as the inferential measure; thus, the test was the **W**ilcoxon / **K**ruskal-**W**allis Tests (Rank Sums)[Chi^2-version]. The Directional p-value is 0.23*;* thus, there is ***reasonable*** [not conclusive] evidence that the width of the Median-Benchmarked Precision of the Top $S\&P_{500}$ Firms is < than that of the Trailing $S\&P_{500}$ Firms. ***Summary Vetting III is suggested.***

5.3 Vetting Summary

These three-p-values are consistent with our expectations as noted above and overall suggest that the $S\&P_{500}$ is not likely to be an anomalous accrual set that would not generalize to Market Trade Firms on major Stock Exchanges. ***This being the likely overall vetting-profile, we offer that the $S\&P_{500}$ will likely provide inferential indications that generalize to Market Traded Firms.***

6. Final Indications & Principal Results: The Capture Rate Profile & The RAE-Profiles

6.1 Overview: The Capture Rate

We used the 43 Firms listed in Appendix B to provide performance of the 95%FPIs for the three Carvalho-Transformations: {AIS & *ln* & Inverse}. "*Question Rhétorique*": If we only selected from the 60 $S\&P_{500}$ those firms that could qualify for the Carvalho-Taxonomy transformations, and then, only used X-Variates that were ***Interpolations***, is it not reasonable to ***expect*** that the 95%FPI would test out to be founded for the three Carvalho-transformations? These results are presented in Table 3 following:

Table 3. Test of the *Interpolation Expectation* re: The 95%FPI Fail to Capture Rates of the Carvalho-transformations

Carvalho Transformation	f_YCapture Profile	Mean 95%CI		Tukey HSD Pairs	Wilcoxon Method Pairs
	IN : **f_Y[95%CI]**	Lower	Upper	p-values	p-values
None [*AIS*] n = 89	92.1%	86,4%	97.8%	**[0.48]→**	**Inverse v. *ln* ← [0.23]**
***ln* n = 89**	91.0%	85.0%	97.1%	**[0.66]→**	**Inverse v. AIS ←[0.35]**
Inverse n = 89	95.5%	91.1%	99.9%	**[0.96]→**	**AIS v. *ln* ←[0.79]**
p-value	**Welch[0.43] & W/K-W[0.48]**			N/A	N/A

6.2 Testing the ***Interpolation Expectation*** *of the 95%FPI*

The 95%CIs of the **Mean**-values for ***The Capture Rates*** are: {AIS[**92.1%**] : *ln*[**91.0%**] : Inverse[**95.5%**]]}. In Table 3, the 95%**CIs** [a non-directional-test] are presented for the three transformations results. For all three Carvalho-transformations, ***95%*** is **IN** each of respective 95%CIs***; this conforms to the Math/Stat-expectation underlying the OLSR Model for the three Carvalho-transformations***. In addition, the Welch(1951)-ANOVA-parametric [non-directional p-value [***0.43***]] and the W/K-W Median-test [non-directional p-value [***0.48***]] indicate that given the Central Tendencies of the AM[92.1%] & GM[91.0%] & HM[95.5%] of these 89-trials, the FPE[Null] is the likely State of Nature—***to wit, there is no evidence overall that there are differences among the three transformations*** relative to the Capture Rate. See the Shaded Cells. Further, for the Pairs-Comparisons: {[**Inverse v. *ln*] & [Inverse v. AIS] &** [***AIS* v. *ln***]} using the Tukey-HSD Profiles as well as the Wilcoxon-Pair comparisons-test provide related confirmatory-intel that there is no overall evidence that would suggest that the Null of the Paired-Population-comparisons for the three Carvalho-transformations is not the State of Nature. For example, for the Tukey HSD Profiles, the test for: **[AIS:[92.1%] v. *ln*:[91.0%]]** gives [1.1%]. The non-directional p-value for this difference re: The FPE[Null]-evaluation is 0.96. This strongly suggests that there is no evidence that given the Sampling Percentage Means of AIS:[92.1%] & *ln*:[91.0%] there is likely a population difference in the central tendencies between the HM-Population-profile and that of the AM-Population-profile for the Fail to Capture Rates of the respective 95%FPIs. *Implication* ***The three Carvalho-transformations do not seem to produce 95%FPI Capture Profiles that are inconsistent with the Theoretical Expectation of the OLSR Forecasting Model where the X-drivers were Interpolations. Thus, there is no jeopardy regarding the performance of the 95%FPIs in this forecasting-context. This is another way of saying that for market-trading datasets where: the likely Carvalho-transformation would have been the ln-transformation due to the CoD-profile for the S&P500-Panels, the Nature of the Data does not interact with the***

OLSR-Interpolation-forecasting FPIs in such a manner so as to affect the expected performance of the OLSR Forecasting Model.

*6.3 Testing the **Inferential Nature of the Mathematical Order** AM > GM > HM. The Benchmarked RAE*

Recall that mathematically for the Carvalho Taxonomy the AM > GM > HM. In the forecasting context, the same order-montage is expected to be the case. ***Rationale*** The ***Forecast[*** $f_{\lambda[[Y]}$ where: λ is the transform-set: {The AIS or The [*ln*]] or The [Inverse]}will be a sampling-projection of the population's {Y_{λ} ← X} generating process, the sampled forecast of which, will be the sample-central-tendency estimator. Thus, these sample projections, when ***re***-transformed, must follow the order: AM > GM > HM. However, these projections are NOT GM- or HM-Forecasts. They are merely the ***re***-transformation of the $f_{\lambda[Y]}$ = [$\hat{\alpha}$ + [$\hat{\beta}$ ×X_{If}]] where: $f_{AIS[Y]}$ > $f_{\ln[Y]}$ > $f_{Inverse[Y]}$. Therefore, the ordering montage will be perfectly suited to examine the Relative Absolute Forecasting Error [RAFE]. ***Protocol Test*** For the test-dataset of 60 S&P_{500} firms, there is a ***clear*** single correct choice among the Carvalho-transformations {AM : GM : HM}; given that the Mean of the CoDs for the screened 89 firms was 0.501. The clear choice would have been the GM[*ln*]-context. Thus, relabeling the [AM-profile] as the [AIS-profile] we can benchmark the re-transformed [AIS-profile] & the Inverse-profile by the *ln*-profile *the likely correct transformation*. In this case, a logical measure for creating the ***Relative Absolute Forecasting Error*** **[*RAFE*]**-intel will be: {$f_{AIS[Y]}$ v, $f_{\ln[Y]}$} & {$f_{Inverse[Y]}$ v. $f_{\ln[Y]}$}. However, this will be a ***heuristic RAFE-measure*** as the ***re***-transformation will create an f_Y that is, as discussed above, off-center re: the re-transformed 95%FPIs. However, as an overall approximation relative to the RAFE for the transformed series it is useful. The initial logical vetting-test will be: (i) the directional orientation of the these transformed f_Y-values using the following Excel-Transitivity-screen:

[IF(AND) {$f_{AIS[Y]}$ > $f_{\ln[Y]}$}, {$f_{\ln[Y]}$ > $f_{Inverse[Y]}$),"Transitive", "Transitivity Issue"].

(ii) as well as directional-bias for the RAFE-Measures. Even though Transitivity is mathematically expected, perhaps there are mathematical conditioning features of the RAFEs to suggest that:

[[ABS [$f_{AIS[Y]}$ − $f_{\ln[Y]}$]] / $f_{\ln[Y]}$] > [[[$f_{Inverse[Y]}$ − $f_{\ln[Y]}$]] / $f_{\ln[Y]}$]

Results For the sampled 267-forecasts, transitivity was found 100% of the time as expected. Further, for the directional-bias, we tested the following directional percentage Null H_o:

[[ABS [$f_{Ais[Y]}$ − $f_{\ln[Y]}$]] / $f_{\ln[Y]}$] ≤ [[[$f_{Inverse[Y]}$ − $f_{\ln[Y]}$]] / $f_{\ln[Y]}$]

Results There were 41.6% [37/89] comparative instances where

[[ABS [$f_{AIS[Y]}$ − $f_{\ln[Y]}$]] / $f_{\ln[Y]}$] > [[[$f_{Inverse[Y]}$ − $f_{\ln[Y]}$]] / $f_{\ln[Y]}$]

Indication: There is no evidence that would suggest that the Null H_o should be rejected in favor of the H_a of inequality—the testing p-value for rejection of $H_o$was > 0.5 ***Suggested Implication***: There is ***no evidence*** that the creation of the RAFE discussed above is biased in a directional disposition of the magnitude of the RAE in favor of the AM/GM-test.

Given the nature of this vetting indication and accepting that these re-transformation results are approximations, we offer the following RAFE-profiles.

6.3.1 RAE Computational Illustration

Assume that we are creating the RAFE for the $f_{AM\,[Y]}$ and the $f_{Inverse\,[Y]}$ re: the $f_{ln\,[Y]}$. For example, consider the *AAPL* Panels. The forecasts were [See *Section 4.5*]

The [AM] forecast is: f_Y = 17,415.21

The [GM] forecast is: f_Y = 9.723193 Transformed to the "AIS"-measure is: EXP[9.723193] = *16,700.50*

The [HM] forecast is: f_Y = 6.2119E-05 Transformed to the "AIS"-measure is: [1/6.2119E-05] = 16,098.13

In this case, we can form the *Relative Absolute Forecasting Error* **[*RAFE*]** assuming that the *ln*-transformation is the logical transformation as follows:

Using **AIS** Rather than *ln*→ noted as [AIS/[*ln*]]:

RAFE[AIS/*ln*] is: ABS[**17,415.21 –** *16,700.50*] / *16,700.50* = ***4.28%***

Using **Inverse** Rather than *ln*→ noted as [Inverse/*ln*]:

RAFE[Inverse/*ln*] is: ABS[**16,098.13 –** *16,700.50*] / *16,700.50* = ***3.61%***

In this case, we have made these calculations of all of the 89 Trials.

The results are presented following:

Table 4. Relative Absolute Forecasting Error results for the *ln: Gold Standard* relative to the AIS & Inverse *IQR is the Inter-Quartile Range

RAFE	**Mean**	**95%CI of the Mean**	**Median**	**IQR*[25th : 75th] Percentiles**
[AIS/[*ln*]], n=89	12.1%	6.1% : 18.0%	3.63%	0.1% : 9.9%
[*Inverse*/[*ln*]] n=89	10.8%	7.1% : 14.5%	3.90%	0.1%: 11.3%
p-value	Welch **0.72**	Fail to Reject FPE[Null]	W/K-W **0.76**	Fail to Reject FPE[Null]

6.3.2 Discussion

In this Relative Absolute Forecasting Error Profile, we see the true focus of the test-results. Given the p-values of the Means & the Medians, there is ***no evidence*** that there is a difference in failing to use the correct Geometric-Context **as between** using the Arithmetic- **or** the Harmonic-Context. Thus, the Relative Absolute Error referencing only the respective forecasts [f_Y] for making ***either*** of these "incorrect" ***Nature of the Data*** elections will be:

In the RAFE[***Mean Profile],*** the Relative Absolute Forecasting Error is: *around*_11.5% [(12.1% + 10.8%)/2] &

In the RAFE[*Median-Profile]*, the Relative Absolute Forecasting Error is *around* 3.8% [(3.63% + 3.90%)/2]—***neither of which is trivial.***

6.4 Summary Implication

The important inferential-intel is in the 95%CIs and the IQRs of the RAFE-measures for the[AIS/[*ln*]] & [Inverse/[*ln*]]. For all four Pairs, zero [**0**] is NOT in the 95%CIs nor in the IQRs, suggesting that the averages reported: Mean[RAFE[11.5%]] and the Median[RAFE[3.8%]] of Table 4 are likely indications of the inferential-jeopardy of failing to select the most likely Carvalho-Transformations in a forecasting-context. *Indication* ***The RAFEs are not likely to be judged as trivial; this suggests that failure to select the likely/correct transformation, in our case, the ln. will likely compromise the decision-making relevance of the forecasting intel so generated.***

7. Summary and Outlook

7.1 Summary

We set out to investigate the effect of the usual transformations of data in a forecasting-context. We were interested in this topic as a research project, as a *ProQuest*™[*ABI/INFORM*®]-*Global* subject-search, with the *Title* or *Abstract* screening-terms [in ***Bold/Italic***], found no peer-reviewed articles that offered indications of: [***Failing*** to select the ***Correct***-t***ransformation*** from the ***Carvalho Taxonomy*** for ***Forecasting Studies***].

Initially, referencing the research report of Alf & Grossberg (1979), we offered a reminder of the proper creation of the 95%CI of the GM-context where: the $\bar{x}_{GM}$ is the midpoint of the 95%CI. We used A&G's-protocol to motivate a discussion of a troublesome issue that arises in a forecasting-context. To wit: often when the *ln* or Inverse are the correct Transformations, the decision-making users of the forecasting-intel "***require***" that the forecasting profiles, reported for the transformed values, be re-transformed ***back*** to the original data-measures—presumably to avoid mis-interpretation or confusion on their part. ***This is reasonable***. However, such re-transforms interject another contentious issue! As there is ***NO*** A&G-version of this re-transformation ***for forecasts***, most analysts simply re-transform the end-points of the 95%FPIs ***back*** to the original AIS-data-measures. However, this ***will*** result in the transformed-forecast to not be the Mid-point of the re-transformed end-points of the 95%FPIs. Often in our experience, this peculiar dislodgement of the re-transformed forecast so that it is not the midpoint of the re-transformed end-points of the 95%FPIs, causes confusion on the part of the decision-makers. A not atypical reaction by decision-makers is: ***Mistakes are afoot***! Tacitly indicating that we, as forecasting analysts, do not know what we are doing! And thus, having a lack of confidence in the decision-relevance of the re-transformed end-points of the 95%FPIs, the decision-makers are again confused as to a logical course of action. ***Be Alert, this sometimes results in a conundrum of a lack of confidence!***

Another issue that we addressed was making the wrong selection from the Carvalho Taxonomy. We illustrated this only for the datasets that we accrued where the correct selection was the [*ln*]-transformation. For this case, there was a non-trivial RAFE in making the wrong selection that we measured only for the respective forecasts: {$f_{Y_{AIS}}$ & $f_{Y_{ln}}$ & $f_{Y_{Inverse}}$}.

7.2 The take-aways from our study are:

I. ***Recommendation*** In forecasting, it is necessary to understand the Nature of the Data that will be used to generate forecasts by using both: ***The Carvalho Taxonomy*: **Table 1 & **The *Staged Transformation Selection Triage Protocol*** to make a reasoned choice of the transformation that will best generate a relevant estimation of:

(i) the ***correct central tendency***: [AM or GM or HM]] of the population from which the random-sample was taken, or

(ii) aid in selecting among: [The AIS-data, or The *ln*-Transformation or The Inverse-Transformation] to create ***Forecasting Intel*** that can be expected to inform the decision-makers engaged in planning the future of the organization.

II. In testing the ***Interpolation Expectation*** for the 95%FPIs [Table 3], using our sample from the $S\&P_{500}$ for the three Carvalho Nature of Data Cases {AIS & *ln* & Inverse}, we found that the mathematical expectation for the respective 95%FPIs created by Interpolations was founded for the forecasts in the transformed measures: {AIS & *ln* & Inverse}—*to wit*, the inferential testing found that it is likely that 95% of the Holdback-cases were IN the respective 95%FPIs. ***Implication*** Failing to select the correct Carvalho-transformation did not affect the performance of the 95%FPIs in that in all three test-cases conformed to the Mathematical expectation underlying the 95%FPIs.

III. In inference testing the profiles of the ***RAFE***, where the correct transformation was the [*ln*] which was thus was used ***as the benchmark*** for the AIS- & the Inverse-transformations, we found that it is likely that the two FPE[Nulls] should be rejected thus indicating that there are non-trivial differences among the RAFE-measures for the re-transformed *ln vis-à-vis* those of the AIS- & -Inverse-transforms. ***Implication*** Using the wrong transformation likely will compromise the quality of the intel produced by the Forecasting Model.

IV. Un-Freezing It is certainly ***not unreasonable for decision-makers to request that the results of the Forecasts of the transformed data be re-cast into the natural AIS-measures of the data***. However, there is ***NO A&G-version*** of a protocol to create re-transformed 95%FPIs where: the re-transformed forecast will be the mid-point of the re-transformed 95%FPIs; thus, we recommend—***Do not be concerned that the transformed forecast will NOT be the mid-point of the re-transformed 95%FPIs***. This is a feature of the re-transformation that does not affect

the reasonability and utility of the re-transformed 95%FPI endpoints to inform the decision-making process. ***Implication*** It is not illogical to use the end-points of the re-transformed 95%FPIs—they are valid-intel and so are useful Planning Indications re: The Forecasting "*Problématique*.". Further, ***experientially***, we have found the re-transformed 95%FPI-endpoints to be valuable- and relevant-intel that can be expected to inform the decision-makers of the organization.

7.3 Outlook

Our study seems to beg the following related question:

For forecasting studies, where: ***Tamhane & Dunlop Extrapolations*** *are the usual forecasting-fare, what is the inferential consequence or jeopardy of making the wrong choice between the {AIS : ln : Inverse} transformations?*

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Appendix A

Table AI *AAPL*: EBITDA [Y]Panel, n=12] No Transformations are applied to this panel. Holdback[HB]

EBITDA[Y]	**15,480**	**12,228**	**13,134**	**26,821**	**20,757**	**17,167**
17,742	27,125	16,464	12,631	14,309	26,346	**HB[**16,429**]**

Table AII *AAPL*: EBITDA [Y]Panel, n=12] The ln-Transformations are applied to Table A1

***ln*-EBITDA[Y]**	**9.647304**	**9.411484**	**9.48296**	**10.19694**	**9.940639**	**9.750744**
9.78369	10.20821	9.708931	9.443909	9.568644	10.17907	**HB[9.706803]**

Table AIII *AAPL*: EBITDA [Y]Panel, n=12] The Inverse-Transformations are applied to Table A1

***Inverse*-EBITDA[Y]**	**6.46E-05**	**8.18E-05**	**7.61E-05**	**3.73E-05**	**4.82E-05**	**5.83E-05**
5.64E-05	3.69E-05	6.07E-05	7.92E-05	6.99E-05	3.8E-05	**HB[6.09E-05]**

Table AIV *AAPL*: GROSS_MARGIN [X, n=12] Panel Holdback [HB] where the **HB[**38.9273] is in range: **Min**[GROSS_MARGIN**[X]=**38.0054**] : Max**GROSS_MARGIN**[X]**=40.7792**].** Thus, the forecast is an Interpolation.

GROSS_MARGIN[X]	**39.3178**	**39.3647**	**38.0054**	**39.8678**	**40.7792**	**39.6754**
39.898	40.0978	39.4031	38.0235	38.0197	38.5139	**HB[**38.9273]

Appendix B

Table B These are the Final Firms after the Screening Protocol was applied, The BBT Bloomberg Industrial Classification System [BICS] are Noted. The [trailing number] is the Market Screener Rank Number Accessed [29 Sept 2023 9h53[AM]] https://www.marketscreener.com/quote/index/S-P-500-4985/components/

AJG[159]	**AMZN[3]**	**APH15[2]**	**AAPL[1]**	**AVGO[15]**	**BBWI[483]**	**BRK/A[8]**	**DVA[495]**
DXC[494]	ECL[160]	EMR[146]	FTNF[148]	GD[156]	GM[141]	GOOGL[5]	HAS[481]
HD[19]	JCI[158]	JNJ[12]	LLY[16]	MAR[151]	MCHP[154]	META[7]	MHK[496]
MPC[143]	MRK[20]	MSFT[2]	MSI[153]	NVDA[4]	NWSA[487]	PARA[485]	PG[17]
PH[147]	PSX[155]	PXD[149]	ROP[145]	SEE[490]	TSLA[6]	UNH[10]	VFC[493]
WHR[482]	WRK[486]	XOM[13]					

Notes

Note 1. This ordered-relationship assumes that the Panel values are: (i) not all the same, and (ii) all > 0; for condition (ii) otherwise the GM and the HM are not defined.

Note 2. See https://www.marketscreener.com/quote/index/S-P-500-4985/components/

Note 3. https://www.investopedia.com/articles/economics/09/lehman-brothers-collapse.asp

Note 4.https://www.linkedin.com/advice/0/how-did-lehman-brothers-collapse-impact-global-economy-bsyac#what-experts-are-saying

Note 5. For more details on *ln*-transforming the Y-response Variable and Back-Transforming the results of the OLSR Model to the AIS-measures, we recommend the following Duke University presentation found at: https://people.duke.edu/~rnau/regex3.htm. Scroll to the bottom of this page and click onto: part 3 of the beer sales regression example where there is an application of the *ln*-transformation in modeling the effect of price on demand, including how to use the EXP-function to "un-log" the forecasts and confidence limits to convert them back into the units of the original data. However, as expected the re-**transformation dis-loges** the Forecast so that is not the mid-point of the 95%FPIs. In addition, for an example using the log-Normal population distribution—that is the reference population associated with the GM, see Tamhane & Dunlop (2000, Example 2.39) where the log-Normal inference-results are re-transformed to their normal decision-context. Finally, as a summary indication here are three transformation realities:

For a population with the profile: Mean: [μ_Y], Median: [$\dot{m}_Y$] & Standard Deviation: [σ_Y] for a general

Transform: {T≡ λ_Y}, and the re-transformation noted as: [λ^{-1}], the operational reality for the

Population Parameters as well as for their sampling Estimators is:

μ_Y ≠ [λ^{-1}→[λ_Y[μ_Y]]] –i.e., the back-transform of the [Mean of the ln-transformed data] is ≠ [to the [μ_Y]] &

$\sigma_Y \neq [\lambda^{-1} \rightarrow [\lambda_Y[\sigma_Y]]]$; however, for strictly monotonically increasing transformation, it is the case that:

$\dot{m}_Y = [\lambda^{-1} \rightarrow [\lambda_Y[\dot{m}_Y]]]$.