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Original Paper

Judgement Aggregation of Preference-Evaluation

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Abstract

In response to an Arrovian impossibility in combining ranking and evaluation, it is often asked whether we could embed this impossibility result into judgment aggregation. I.e. preference-evaluation aggregation is a special case of judgment aggregation. We argue for this claim, after proving this result as a corollary of Dietrich's (2015) work. We thereby provide a new proof of the impossibility result in preference-evaluation aggregation and clarify the relation between judgment and preference-evaluation aggregation, and to illustrate the generality of the judgment aggregation model.

Keywords

social choice, judgement aggregation, impossibility results, generality

1. Introduction

The traditional method in social choice theory involves aggregating individuals' preferences over alternatives. Contrasted with the former approach, Brams and Fishburn (1978) proposed approve voting which aggregates individuals' evaluations over alternatives. Preference ranking and evaluation are fundamentally different kinds of information. But some researchers consider using both of them. Brams and Sanver (2009) think that individuals not only have a preference ranking over alternatives but also would draw a line between the acceptable and unacceptable alternatives. i.e. the preference-evaluation model. But unfortunately, Kruger and Sanver (2021) proved an impossibility in combining preference ranking and evaluation approaches which is similar to the Arrovian impossibility theorem. This means that there exists incompatibility between the two. Then we have a question: Can we embed preference-evaluation aggregation into a more general framework? Such as judgment aggregation.

It is relatively easy to express preference-evaluations with linear rankings in the judgment aggregation framework. We can assign a two-place predicate representing strict preference, and a one-place predicate representing evaluations, and add rationality conditions for asymmetry, transitivity, connectedness, and consistency of evaluations with preference. To illustrate the possibility of embedding this work into judgment aggregation, let us suppose a three-committee has to make a collective judgment (True or False) on three connected propositions (preference rankings and evaluations):

$a \in A$: individual approves alternative a. A is the set of alternatives approved by the individual.

$b \in D$: individual disapproves alternative b. D is the set of alternatives disapproved by the individual.

aPb: The individual strictly prefers a to b.

This is an example of preference-evaluation which contains two levels of evaluation (approve or disprove). As shown in Table 1, the Voter1 accepts $a \in A$ and aPb but rejects $b \in D$; the Voter2 accepts $b \in D$ but rejects $a \in A$ and aPb; the Voter3 accepts aPb but rejects $a \in A$ and $b \in D$. Then by the consistency of evaluations with preference (if individual accepts aRb and $b \in A$, then $a \in A$), the judgments of each voter are individually consistent, and yet the majority of judgments on the propositions are inconsistent: a majority rejects $a \in A$, a majority rejects $b \in D$, but a majority accepts aPb. So, this is a version of the discursive paradox in preference-evaluation aggregation. Because it resembles Condorcet paradox of cyclical majority preferences and the various recent impossibility theorems in judgment aggregation resemble Arrows and other theorems on preference aggregation, there are reasons to believe that we can find some impossibility theorems in judgment aggregation resembling the impossibility result in preference-evaluation aggregation.

Table 1. Discursive Paradox in Preference-evaluation

	$a \in A$	$b \in D$	aPb
Voter1	T	F	T
Voter2	F	T	F
Voter3	F	F	T
Majority	F	F	T

As we know, after the initial discursive paradox (Pettit, 2001) was put forward, there is literature on finding the possibility of consistent judgment aggregation under various conditions. List and Pettit (2002) have provided a first model of judgment aggregation based on propositional logic and proved that no aggregation rule generating consistent collective judgments can satisfy some conditions inspired by (but not equivalent to) Arrows conditions on preference aggregation. This impossibility result has been extended and strengthened by using multi-value logic (Pauly & van Hees, 2006). Many excellent works about the Arrow-like impossibility theorems have been done by Dietrich (2006a), Grdenfors (2006), Mongin (2008), Nehring and Puppe (2002, 2008, 2010), Dietrich and List (2007a, 2008, 2013), Dokow and Holzman (2010), Nehring (2005) and Dietrich and Mongin (2010).

But in this growing literature on the impossibility theorems in judgment aggregation, we found that the axiom of proposition-wise independence in the judgment aggregation sense is often used. This axiom is often criticized (e.g., Chapman, 2002; Mongin, 2008), but rarely weakened. It is a much stronger restriction than the independence conditions that Kruger and Sanver (2021) apply in their work. So in our paper, we are ready to use the axiom of Independence of irrelevant propositions (Dietrich 2015) (a weakened version of proposition-wise independence) to correspond to the two independence conditions in Kruger and Sanver's work.

To give a brief overview of the rest of the paper: in Section2 we define the judgment aggregation framework (in the version of List and Pettit (2002), and more precisely Dietrich (2007, 2014, 2015)). In section 3 based on two versions of independence conditions, we show our central impossibility concerning the aggregation of preference-approval and the aggregation of preference-evaluation which contains more than two evaluation levels. Final remarks are provided in Section 4.

2. The Judgment Aggregation Framework

The basic building blocks of our model is a group of individuals $1,2,3,\dots,n (n \geq 2)$. The group of individuals has to make collective judgments on some logically connected propositions.

Formal logic. (Dietrich 2007) A logic is an ordered pair $(\boldsymbol{L}, \vdash)$. $\boldsymbol{L}$ is a non-empty set of formal expressions (propositions) closed under negation (i.e., if $p \in \boldsymbol{L}$, then $\neg p \in \boldsymbol{L}$). $\vdash$ is an entailment relation, where, for each set $S \subseteq \boldsymbol{L}$ and each proposition $p \in \boldsymbol{L}$, $S \vdash p$ means that S entails proposition p. Then a set of propositions S is consistent, if there does not exist any $p \in \boldsymbol{L}$ for which $S \vdash p$ and $S \vdash \neg p$, and inconsistent otherwise. The Formal logic need to satisfy the following three minimal conditions:

1) Self-entailment: For all $p \in \boldsymbol{L}$, $p \vdash p$.

2) Monotonicity: For all $p \in \boldsymbol{L}$ and $A \subseteq B \subseteq \boldsymbol{L}$, if $A \vdash p$, then $B \vdash p$.

3) Completability: $\emptyset$ is consistent and each consistent set $A \subseteq \boldsymbol{L}$ has a consistent superset $B \subseteq \boldsymbol{L}$ which contains an element of each pair $p, \neg p \in \boldsymbol{L}$.

It is easy to check that many kinds of logic satisfy these three conditions containing standard propositional logic, standard modal logic, conditional logic and predicate logic which is used to represent preference-evaluation.

The agenda. The agenda is a non-empty subset $X \subseteq \boldsymbol{L}$ which is a set of propositions on which each individual of group has to make judgments. X is also closed under negation. For the sake of simplicity, we assume that $\neg\neg p$ means p. In the discursive paradox example of Introduction, the agenda is

$X = \{a \in A, \neg a \in A, b \in D, \neg b \in D, aPb, \neg aPb\}$. If $p \in X$, then we often write the pair $(p, \neg p)$ as $\pm p$. So, for the example before, we also can write the agenda as $X = \{\pm a \in A, \pm b \in D, \pm aPb\}$.

Individual judgement set. An individual judgment set is a set $J \subseteq X$ of propositions which the individual accepts. The individual judgement set is rational if it is consistent and complete. $\boldsymbol{J}$ $(\boldsymbol{J} \neq \emptyset)$ is the set of all rational judgment set. The profile of the rational individual judgment set is $(J_1, \ldots, J_n) \in \boldsymbol{J}^n$.

Aggregation rule. An aggregation rule is a function F that assigns to every profile of the rational individual judgment set a collective judgment set $F(J_1, \ldots, J_n) \subseteq X$. In our paper, the domain of aggregation rule is $\boldsymbol{J}^n$ which is also called Universal domain (i.e. The domain of F is the set of all possible profiles of rational individual judgment sets.) If F is a function $F: \boldsymbol{J}^n \longrightarrow \boldsymbol{J}$, then F also satisfies Collective rationality which means F often generates rational collective judgment sets. But not all aggregation rules satisfy it, for example, majority rule generates inconsistent outputs. So more generally, F is a function $F: \boldsymbol{J}^n \longrightarrow 2^X$ which possibly generates inconsistent outputs.

Before giving the agenda, we would use in our embedding work, we must define a simple predicate logic for representing preference-evaluations. We consider a standard preference-evaluation aggregation model, where there is a set of individuals $N = \{1, \ldots, n\}$ and each individual has a strict (asymmetrical, transitive and connected) or general(reflexive, transitive and connected) preference ordering $(P \text{ or } R)$ over a set of alternatives $A = \{a_1, \ldots, a_m\}$ with $m \geq 2$. Then denote by $E = \{e_1, \ldots e_t\} \in (2^A)^t$ a set of possible evaluations with $t \geq 2$.(i.e. partition A into t parts and $e^1(e^t)$ is the best (worst) evaluative category) Each individual evaluate all the alternatives to get his or her E. The general (strict) preference-evaluation is a pair $(R, E)((P, E))$ and is consistent if xRy and $y \in e_j$, then $x \in e_i$ with $i \leq j$. (i.e., It is impossible to prefer a worse category to a better category.) When $t = 2$, the preference-evaluation is equivalent to preference-approval.

A simple predicate logic for representing general preference-evaluations. We consider a predicate logic with constants $a_1, a_2, a_3 \in A$ representing the alternatives, variables v, w, v_1, v_2, ..., identity symbol $=$, one-place predicates $\in e_i$ for all $e_i \in E$ (representing the evaluation), two-place predicate R and P (representing the general and strict preference), logical connectives $\neg$(not), $\vee$(or), $\wedge$(and), $\rightarrow$(if-then) and $\forall$(universal quantifier). Then $\boldsymbol{L}$ is the smallest set such that

$\boldsymbol{L}$ contains all propositions of the forms $\alpha P \beta$, $\alpha R \beta$, $\alpha \in e_i$ for all $e_i \in E$ and $\alpha = \beta$, where α and β are constants or variables, and whenever $\boldsymbol{L}$ contains two propositions p and q, then $\boldsymbol{L}$ also contains $\neg p, \neg q$, $(p \vee q)$, $(p \wedge q)$, $(p \rightarrow q)$ and $(\forall v)p$, where v is any variable.

Then we give the definition of entailment relation $\vdash$ such that

For any set $S \subseteq \boldsymbol{L}$ and any proposition $p \in \boldsymbol{L}$, $S \vdash p$ if and only if $S \cup U \vdash p$ in the sense of standard predicate logic where U is the set of rationality conditions on general preference, relation between general preference and strict preference and the extra consistency condition between preference and evaluation:

(reflexivity) $(\forall v_1), v_1 R v_1$

(transitivity) $(\forall v_1)(\forall v_2)(\forall v_3), (v_1 R v_2 \wedge v_2 R v_3) \rightarrow v_1 R v_3$

(connectedness) $(\forall v_1)(\forall v_2), (\neg v_1 = v_2) \rightarrow (v_1 R v_2 \vee v_2 R v_1)$

(relation between R and P) $(\forall v_1)(\forall v_2), v_1 R v_2 \rightarrow \neg v_2 P v_1$

(extra consistency) $(\forall v_1)(\forall v_2), (v_1 R v_2 \wedge v_2 \in e_j) \rightarrow v_1 \in e_i (i \leq j)$

It is easy to represent the strict preference-evaluations by deleting reflexivity from U and adding the asymmetry condition.

The general preference-evaluation agenda. The general preference-evaluation agenda for the set of alternatives $A = \{a_1, \ldots, a_m\}$ is the set of propositions

$X = \{\pm xRy, \pm z \in e_i, \pm(\neg z \in e_i \wedge \neg z \in e_{i+1} \wedge \ldots \wedge \neg z \in e_{i+k}) : x, y, z \in A, x \neq y, e_i \in E, 0 < k \leq t - 3\}$

.

Maybe you feel strange why we add the propositions $\pm(\neg z \in e_i \wedge \neg z \in e_{i+1} \wedge \ldots \wedge \neg z \in e_{i+k})$ in to the general preference-evaluation agenda. Firstly, it is easy to check that $\pm(\neg z \in e_i \wedge \neg z \in e_{i+1} \wedge \ldots \wedge \neg z \in e_{i+k}) \in \boldsymbol{L}$. Secondly, when individual evaluate some alternative z, it is easy for him to make judgement on this proposition at the same time and this proposition would play an important role in proving a lemma in the next section of this paper. We need to notice that a rational judgement set $J \subseteq X$ corresponds with a general preference-evaluation which satisfies rationality conditions for the preference and the extra consistency between the preference and evaluation.

The strict preference-evaluation agenda. The strict preference- evaluation agenda for the set of alternatives $A = \{a_1, \ldots, a_m\}$ is the set of propositions $X = \{\pm xPy, \pm z \in e_i, \pm(\neg z \in e_i \wedge \neg z \in e_{i+1} \wedge \ldots \wedge \neg z \in e_{i+k}) : x, y, z \in A, x \neq y, e_i \in E, 0 < k \leq t - 3\}$.

It is worth noting that the size of the strict preference-evaluation agenda is smaller than that of the general preference-evaluation agenda. The reason is that for the strict preference, the judgment on $\neg xPy$ is the same as the judgment on yPx, whereas for the general preference, the judgments on $\neg xRy$ and yRx are sometimes different.

When $|E| = 2$, $E = \{e_1, e_2\}$ such that e_1 is the set of approved alternatives and e_2 is the set of disapproved alternatives. So, in this case preference-evaluation agenda corresponds to preference-approval agenda.

The general preference-approval agenda. The general preference-approval agenda for the set of alternatives $A = \{a_1, \ldots, a_m\}$ is the set of propositions $X = \{\pm xRy, \pm z \in e_i : x, y, z \in A, x \neq y, e_i \in E\}$.

The strict preference-approval agenda. The strict preference-approval agenda for the set of alternatives $A = \{a_1, \ldots, a_m\}$ is the set of propositions $X = \{\pm xPy, \pm z \in e_i : x, y, z \in A, x \neq y, e_i \in E\}$.

We would emphasize that when $|E| \leq 3$, the form of propositions $\pm(\neg z \in e_i \wedge \neg z \in e_{i+1} \wedge \ldots \wedge \neg z \in e_{i+k})$ does not exist in the agenda, because there is not some k in the range $0 < k \leq t - 3$. The reason why we give this setting is that the proof of lemma2 in the next section does not use this form of propositions when $|E| \leq 3$.

Then let us discuss the independence condition. In the literature, the axiom of proposition-wise independence which corresponds Arrow's independence of irrelevant alternatives was often criticized.

Proposition-wise independence: For all proposition $p \in X$ and all profiles $(J_1, \ldots, J_n)$ and $(J_1^*, \ldots, J_n^*)$ in J^n, if $p \in J_i \Leftrightarrow p \in J_i^*$, then $p \in F(J_1, \ldots, J_n) \Leftrightarrow p \in F(J_1^*, \ldots, J_n^*)$.

This axiom says that collective judgment on one proposition only depends on individuals' judgment on this proposition, not on others. This is too strong to complete our embedding work. Remembering Kruger and Sanver's (2021) paper, the independence condition they use to achieve the impossibility theorem has two versions.

(a) Binary independence is applied to the preference ranking part of the aggregation, which requires that the output ranking of each pair of alternatives only depends upon the input rankings of these pairs. The evaluations of each alternative can be computed independently of any other alternative's information.

(b) The preference ranking part of aggregation only depends upon the preference part of the profile (i.e., the preference ranking aggregation is independent of evaluation). The evaluations of each alternative can be computed independently of any other alternative's information.

As we know, each of these two versions is weaker than proposition-wise independence. In other words, the axiom we could use for embedding work cannot forbids that the collective judgment on one proposition depends on peoples judgments on other. For example, from the definition of independence condition, the collective judgements on proposition $z \in e_1$ and $\neg z \in e_1$ are both seem to be affected by the propositions $\pm z \in e_2, \pm z \in e_3, \ldots, \pm z \in e_t$. We cannot ignore individuals' judgement on these propositions and thinks that the propositions $\pm z \in e_2, \pm z \in e_3, \ldots, \pm z \in e_t$ are relevant to $z \in e_1$ and $\neg z \in e_1$. Then we are ready to define a relevance relation R on the agenda X, where for all $q, p \in X$, qRp means that q is relevant to p. The set of propositions relevant to $p \in X$ is denoted by

$R(p) = \{q \in X : qRp\}$

Sometime relevance relation R does not distinguish between a proposition and its negation. This is called negation-invariant:

$qRp \Leftrightarrow q'Rp'\ if\ q' \in \{\pm q\}\ and\ p' \in \{\pm p\}$

In this setting, $R(\pm p) = R(p) = R(\neg p)$.

The new independence axiom in the judgment sense for our embedding work is

Independence of irrelevant propositions (IIP): (Dietrich 2015) For all proposition $p \in X$ and all profiles

$(J_1,\ldots,J_n)$ and $(J_1^*,\ldots,J_n^*)$ in $\boldsymbol{J}^n$, if $J_i \cap R(p) = J_i^* \cap R(p)$ for all individuals, then $p \in F(J_1,\ldots,J_n) \Leftrightarrow p \in F(J_1^*,\ldots,J_n^*)$.

This axiom means that collective judgment on propositions only depends on the individuals' judgments on its relevant propositions. It is easy to know that proposition-wise independence is a special case of **IIP** when $R(p) = p, \forall p \in X$.

Dietrich (2015) assume a plausible conditions for relevance relation which is called for **non-underdetermination**: every proposition is settled by the judgments on the relevant propositions. i.e. For every $p \in X$ and every consistent set S of the form $\{q^* : q \in R(p)\}$, where $q^* \in \{q, \neg q\}$,

either $S \vdash p$ (S is an $(R-)$ explanation of p)

or $S \vdash \neg p$ (S is an $(R-)$ explanation of $\neg p$)

Relevance relation. A relevance relation is a binary relation R on the agenda X satisfying **non-underdetermination**.

Then we would give two versions of relevance relation which is implicit in two versions independence conditions of preference-evaluation aggregations we have introduced before.

(a) The relevance relation on preference-evaluation agenda X is defined by $R(\pm xRy) = \{\pm xRy, \pm yRx\}$ for all $xRy \in X$, $R(\pm xPy) = \{\pm xPy\}$ for all $xPy \in X$ and $R(\pm z \in e_i) = \{\pm z \in e_j : \forall e_j \in E\}$ for all $z \in e_i \in X$.

(b) The relevance relation on preference-evaluation agenda X is defined by $R(\pm xRy) = \{\pm xRy : x, y \in A\}$ for all $xRy \in X, R(\pm xPy) = \{xPy : x, y \in A\}$ for all $xPy \in X$ and $R(\pm z \in e_i) = \{\pm z \in e_j : \forall e_j \in E\}$ for all $z \in e_i \in X$.

In order to conveniently describing the theorem in the next section, we will write the preference-evaluation agenda equipped with relevance relation of **(a)** as

the preference-evaluation agenda (a). (same thing for **(b)**)

Then let us choose one suitable unanimity axiom for our embedding work. As we know, the unanimity axiom used in Kruger and Sanver's (2021) work is weaker than the traditional unanimity principle. It works only for the propositions in the form of evaluation part. Even though in their proof eventually it also works for the preference part, we cannot directly use the traditional unanimity axiom which is defined below,

Unanimity: For all profiles $(J_1,\ldots,J_n)$ in $\boldsymbol{J}^n$ and all propositions $p \in X$, if $p \in J_i$ for all individual i, then $p \in F(J_1,\ldots,J_n)$.

Because this axiom is not natural for relevance-based aggregation which satisfies **IIP**. I.e., People's judgments on propositions relevant to p should not do nothing. Even if p gets all acceptions of people, there can be many rejections on its relevant propositions. Nehring (2005) think that such agreements on p and disagreements on its relevant propositions often lack normative force. But such situation would not happen for those propositions which can be explained in only one way. Dietrich (2015) calls such propositions **unambiguous** propositions.

Unambiguous proposition: Given a relevance R on X, a proposition $p \in X$ is **unambiguous** if it has only one explanation, and **ambiguous** otherwise. In order to easily give the subsequent definition and proof, the set of unambiguous propositions is denoted by U_R.

In our **preference-evaluation agenda (a)** example, $U_R = \{xPy, z \in e_i : x, y, z \in A, e_i \in E\}$. The proposition xPy and $z \in e_i$ are unambiguous as both of them have only one explanation: $\{xPy, \neg yPx\}$ and $\{z \in e_i, \neg z \in e_{-i} : e_{-i} \in E \setminus \{e_i\}\}$. The proposition xRy is ambigious as it has two explanations $\{xRy, yRx\}$ and $\{xRy, \neg yRx\}$. When $t > 2$, the proposition $\neg z \in e_i$ is ambiguous as it has $t-1$ explanations of the form $\{z \in e_i, \neg z \in e_{-i} : e_{-i} \in E \setminus \{e_i\}\}$. If $t = 2$, then $z \in e_1 \Leftrightarrow \neg z \in e_2$. So, the proposition $\neg z \in e_{-i}$ is also unambiguous when $t = 2$.

In our **preference-evaluation agenda (b)** example, $U_R = \{z \in e_i : z \in A, e_i \in E\}$. If we use one unanimity principle for the unambigious propositions, it will help us completing the embedding work.

Unambiguous agreement preservation (UAP) (Dietrich 2015): For all profiles $(J_1, \ldots, J_n)$ in J^n and all unambigious propositions $p \in U_R$, if $p \in J_i$ for all individuals i, then $p \in F(J_1, \ldots, J_n)$.

3. Arrow's Impossibility Theorem in both Preference-evaluation and Preference-approval Agenda

Remembering the definition of dictatorship in Kruger and Sanver's (2021) paper, the preference-evaluation aggregator is dictatorial if there is a individual d whose strict preference ranking and evaluation over alternatives are reproduced in the collective preference-evaluation. As we know, this corresponds to the form of propositions xPy and $z \in e_i$ which are both unambiguous propositions. So, we decide to use the axiom of weak dictatorship which is only for unambiguous propositions.

Weakly dictatorial: The judgment aggregation F is weak dictatorial, if there is an individual d such that for all unambiguous propositions $p \in U_R$, $p \in J_d \Leftrightarrow p \in F(J_1, \ldots, J_n)$, $\forall (J_1, \ldots, J_n) \in J^n$.

As we know, in order to get a standard impossibility theorem in judgment aggregation (under classical relevance i.e. $R(p) = p$ for all $p \in X$), the restrictions of pair-negatability and pathconnected (proposed by Dietrich (2007, 2015)) must be added to agenda.

Pair-negatability: Pair-negatability means that the agenda X has an inconsistent subset Y such that $(Y \setminus \{p\}) \cup \{\neg p\}$, $(Y \setminus \{q\}) \cup \{\neg q\}$ and $(Y \setminus \{p, q\}) \cup \{\neg p, \neg q\}$ are each consistent for some pair of distinct propositions $p, q \in Y$.

Pathconnected is defined by conditional entailments between propositions. (proposed by Nehring and Puppe, 2002).

Conditional entailment: The proposition $p \in X$ conditionally entails $q \in X$ (written $p \vdash^* q$) if $p \cup Y \vdash q$ for some set $Y \subseteq X$(possibly $Y = \emptyset$) that is consistent with p and $\neg q$. p properly conditionally entails q if $p \nvdash q$, i.e. p is consistent with $\neg q$. (p cannot unconditionally entails q)

For any propositions $p, q \in X$, if there exists propositions $p^1, p^2, \dots, p^l \in X (l \geq 2)$ such that $p = p^1 \vdash^* p^2 \vdash^* \dots \vdash^* p^l = q$, we write $p \vdash\vdash q$.

Pathconnected: X is pathconnected if $p \vdash\vdash q$ for all $p, q \in X$.

Theorem 1. (Dietrich and List 2007b) For a pair-negatable and pathconnected agenda X under classical relevance, a judgment aggregation F satisfies universal domain, collective rationality, proposition-wise independence, and unanimity principle if and only if it is weakly dictatorial.

This theorem only generalizes Arrow's impossibility theorem in its strict preference versions. In order to correspond to relevance relation between propositions in agenda, Dietrich (2015) define a strong variant of conditional entailment.

Constrained entailment: The proposition $p \in X$ constrainedly entails $q \in X$ (written $p \vdash R\, q$) if $p \cup Y \vdash q$ for some set $Y \subseteq U_R$(possibly $Y = \emptyset$) that is strongly consistent with p and $\neg q$ (i.e., consistent with all R −explanations of p and all ones of $\neg q$). p constrainedly entails q in virtue of Y. (written $p \vdash_{R,Y} q$)

For $p, q \in X$, p **properly** constrainedly entails q, if $p \vdash_R q$ and every $R-$ explanation of q is consistent with every R −explanation of $\neg q$. p **irreversibly** constrainedly entails q, if $p \vdash_{R,Y} q$ for a set Y such that $\{q\} \cup Y \nvdash p$.

Dietrich (2015)'s impossibility theorem under non-classical relevance also needs paths of constrained entailment.

For any propositions $p, q \in X$, if there exists propositions $p^1, p^2, \dots, p^l \in X (l \geq 2)$ such that $p = p^1 \vdash_R p^2 \vdash_R \dots \vdash_R p^l = q$, we write $p \vdash\vdash_R q$. If moreover one of these constrained entailments is proper(irreversible), we write $p \vdash\vdash_R^{proper} q$ ($p \vdash\vdash_R^{irrev} q$).

Pathlinked: A set $W \in X$ is pathlinked, if $p \vdash\vdash_R q$ for all $p, q \in W$, and is properly(irreversibly) pathlinked if moreover $p \vdash\vdash_R^{proper} q$ ($p \vdash\vdash_R^{irrev} q$) for some(hence all) $p, q \in W$.

Theorem 2 (Dietrich 2015). If the set $\{p, \neg p : p \in U_R\}$ of unambiguous or negated unambiguous propositions is properly and irreversibly pathlinked, every judgment aggregation rule F satisfies universal domain, collective rationality, IIP, UAP is weakly dictatorial.

As we know, this theorem generalizes Arrow's impossibility theorem in its general (strict) versions. In our paper, we proved this theorem also generalizes Kruger and Sanver's (2021) impossibility theorem in its general (strict) preference-evaluation versions. To see why, note the following results.

Readers can check easily that constrained entailment satisfies contraposition as the definition of constrained entailment is symmetric in p and $\neg q$.

Lemma1. (Contraposition) For all $p, q \in X$ and all $Y \in U_R$, $p \vdash_{R,Y} q \Leftrightarrow \neg q \vdash_{R,Y} \neg p$.

Lemma2. The general or strict preference-evaluation agenda **(a)** for a set of at least two alternatives satisfies the assumptions of Theorem 2, i.e. the set $\{p, \neg p : p \in U_R\}$ is properly and irreversibly pathlinked.

Proof. Let X be the general preference-evaluation agenda **(a)** (the proof for the strict preference-evaluation agenda is left to the reader). Recall that in the case of the general or strict preference-evaluation agenda **(a)**, $U_R = \{xPy, z \in e_i : x, y, z \in A, x \neq y, e_i \in E\}$, where $xPy := \neg yRx$. I show that

(i) U_R is pathlinked.

(ii) There are $r, s \in U_R$ with proper and irreversible constrained entailments $r \vdash_R \neg s \vdash_R r$.

By (i) and Lemma 1, $\{\neg p : p \in U_R\}$ is also pathlinked, which together with (ii) implies that $\{p, \neg p : p \in U_R\}$ is properly and irreversibly pathlinked, then we complete the proof.

(i): Consider any $p, q \in U_R$. I show that $p \vdash\vdash_R q$.

(1) p and q are the form of proposition of strict preference. Then consider $p = xPy$, $q = x'Py'$, x, y, x', $y' \in A$, $x \neq y$, $x' \neq y'$. We show that $xPy \vdash\vdash_R x'Py'$.

Case $x \neq x', y'$ and $y \neq x', y'$: $xPy \vdash_{R,\{x'Px, yPy'\}} x'Py'$.

Case $y = y'$ and $x \neq x', y'$: $xPy \vdash_{R,\{x'Px\}} x'Py = x'Py'$.

Case $y = x'$ and $x \neq x', y'$: $xPy \vdash_{R,\{yPy'\}} xPy' \vdash_{R,\{x'Px\}} x'Py'$.

Case $x = x'$ and $y \neq x', y'$: $xPy \vdash_{R,\{yPy'\}} xPy' = x'Py'$.

Case $x = y'$ and $y \neq x', y'$:$xPy \vdash_{R,\{x'Px\}} x'Py \vdash_{R,\{yPx\}} x'Px = x'Py'$.

Case $x = x'$ and $= y'$: $xPy \vdash_{R,\emptyset} xPy = x'Py'$.

Case $x = y'$and $y = x'$: $xPy \vdash_{R,\{x\in e_t\}} y \in e_t \vdash_{R,\{yPx\}} x \in e_t \vdash_{R,\{y \in e_{t-1}\}} yPx = x'Py'$.

(2) Without loss of generality, let p be the form of proposition of strict preference and q be the form of proposition of evaluation. Then consider $p = xPy$ and $q = z \in e_i$, $x, y, z \in A$, $x \neq y$, $e_i \in E$. We show that $xPy \vdash\vdash_R z \in e_i$ and $z \in e_i \vdash\vdash_R xPy$.

For $xPy \vdash\vdash_R z \in e_i$

Case $x = z$

$i = 1$: $xPy \vdash_{R,\{y \in e_1\}} x \in e_1 = z \in e_i$.

$2 \leq i < t$: $xPy \vdash_{R,\{y \in e_i\}} (\neg x \in e_{i+1} \wedge \neg x \in e_{i+2} \wedge \ldots \wedge \neg x \in e_t) \vdash_{R,\{yPx, y \in e_i\}} x \in e_i = z \in e_i$

.

$i = t$: $xPy \vdash_{R,\{x \in e_t\}} y \in e_t \vdash_{R,\{yPx\}} x \in e_t = z \in e_i$.

Case $y = z$

$i = 1$: $xPy \vdash_{R,\{y\in e_1\}} x \in e_1 \vdash_{R,\{yPx\}} y \in e_1 = z \in e_i$.

$2 \le i < t$: $xPy \vdash_{R,\{x\in e_i\}} (\neg y \in e_1 \wedge \neg y \in e_2 \wedge \ldots \wedge \neg y \in e_{i-1}) \vdash_{R,\{yPx,x\in e_i\}} y \in e_i = z \in e_i$

.

$i = t$: $xPy \vdash_{R,\{x\in e_t\}} y \in e_t = z \in e_i$.

Case $x \ne y \ne z$

$i = 1$: $xPy \vdash_{R,\{zPx,y\in e_1\}} z \in e_1$.

$2 \le i < t$: $xPy \vdash_{R,\{zPx,y\in e_i\}} (\neg z \in e_{i+1} \wedge \neg z \in e_{i+2} \wedge \ldots \wedge \neg z \in e_t) \vdash_{R,\{yPz,y\in e_i\}} z \in e_i$.

$i = t$: $xPy \vdash_{R,\{zPx,z\in e_t\}} y \in e_t \vdash R, \{yPz\}\ z \in e_t$.

For $z \in e_i \vdash\vdash_R xPy$

Case $x = z$

$i < t$: $x \in e_i \vdash_{R,\{y\in e_{i+1}\}} xPy$.

$i = t$: $x \in e_t \vdash_{R,\{xPy\}} y \in e_t \vdash_{R,\{x\in e_{t-1}\}} xPy$.

Case $y = z$

$i = 1$: $y \in e_1 \vdash_{R,\{xPy\}} x \in e_1 \vdash_{R,\{y\in e_2\}} xPy$.

$i > 1$: $y \in e_i \vdash_{R,\{x\in e_{i-1}\}} xPy$.

Case $x \ne y \ne z$

$i < t$: $z \in e_i \vdash_{R,\{xPz,y\in e_{i+1}\}} xPy$.

$i = t$: $z \in e_t \vdash_{R,\{zPy\}} y \in e_t \vdash_{R,\{x\in e_{t-1}\}} xPy$.

(3) p and q are the form of proposition of evaluation. Then consider $p = x \in e_i$ and $q = y \in e_j$, $e_i, e_j \in E$, $x, y \in A$. We only show that $x \in e_i \vdash\vdash_R y \in e_j$ (the process of proof of

$y \in e_j \vdash\vdash_R x \in e_i$ is the same as $x \in e_i \vdash\vdash_R y \in e_j$).

Case $x = y$ (note that $\exists z \in A$, $z \neq x$, as $|A| \geq 2$)

$i < t, j = 1$: $x \in e_i \vdash_{R,\{z \in e_{i+1}\}} xPz \vdash_{R,\{z \in e_1\}} x \in e_1$.

$i < t, 2 \leq j < t$: $x \in e_i \vdash_{R,\{z \in e_{i+1}\}} xPz \vdash_{R,\{z \in e_j\}} (\neg x \in e_{j+1} \wedge \neg x \in e_{j+2} \wedge \ldots \wedge \neg x \in e_t) \vdash_{R,\{zPx, z \in e_j\}} x \in e_j$

.

$i < t, j = t$: $x \in e_i \vdash_{R,\{z \in e_{i+1}\}} xPz \vdash_{R,\{x \in e_t\}} z \in e_t \vdash_{R,\{zPx\}} x \in e_t$.

$i = t, j = 1$: $x \in e_t \vdash_{R,\{xPz\}} z \in e_t \vdash_{R,\{x \in e_{t-1}\}} xPz \vdash_{R,\{z \in e_1\}} x \in e_1$.

$i = t, 2 \leq j < t$: $x \in e_t \vdash_{R,\{xPz\}} z \in e_t \vdash_{R,\{x \in e_{t-1}\}} xPz \vdash_{R,\{z \in e_j\}} (\neg x \in e_{j+1} \wedge \neg x \in e_{j+2} \wedge \ldots \wedge \neg x \in e_t) \vdash_{R,\{zPx, z \in e_j\}} x \in e_j$

.

$i = t, j = t$: $x \in e_t \vdash_{R,\emptyset} x \in e_t$.

Case $x \neq y$

$i < t, j = 1$: $x \in e_i \vdash_{R,\{y \in e_{i+1}\}} xPy \vdash_{R,\{y \in e_1\}} x \in e_1 \vdash_{R,\{yPx\}} y \in e_1$.

$i < t, 2 \leq j < t$: $x \in e_i \vdash_{R,\{y \in e_{i+1}\}} xPy \vdash_{R,\{x \in e_i\}} (\neg y \in e_1 \wedge \neg y \in e_2 \wedge \ldots \wedge \neg y \in e_{i-1}) \vdash_{R,\{yPx, x \in e_j\}} y \in e_j$

.

$i < t, j = t$: $x \in e_i \vdash_{R,\{y \in e_{i+1}\}} xPy \vdash_{R,\{x \in e_t\}} y \in e_t$.

$i = t, j = 1$: $x \in e_t \vdash_{R,\{xPy\}} y \in e_t \vdash_{R,\{x \in e_{t-1}\}} xPy \vdash_{R,\{y \in e_1\}} x \in e_1 \vdash_{R,\{yPx\}} y \in e_1$.

$i = t, 2 \leq j < t$: $x \in e_t \vdash_{R,\{xPy\}} y \in e_t \vdash_{R,\{x \in e_{t-1}\}} xPy \vdash_{R,\{x \in e_i\}} (\neg y \in e_1 \wedge \neg y \in e_2 \wedge \ldots \wedge \neg y \in e_{i-1}) \vdash_{R,\{yPx, x \in e_j\}} y \in e_j$

.

$i = t, j = t$: $x \in e_t \vdash_{R,\{xPy\}} y \in e_t$.

(ii): For $x, y \in A$, $e_1, e_2 \in E$, we have $x \in e_1 \vdash_{R,\{y \in e_2\}} xRy (= \neg yPx)$, and $xRy \vdash_{R,\{y \in e_1\}} x \in e_1$, in each case properly and irreversibly.

■

By the Theorem 2 and Lemma 2, we get

Theorem 3. Given the general or strict preference-evaluation agenda **(a)** for a set of at least two alternatives, every judgment aggregation F satisfying universal domain, collective rationality, IIP and UAP is weakly dictatorial.

Because the preference-approval agenda is a special case of the preference- evaluation agenda, we get

Corollary 1. Given the general or strict preference-approval agenda **(a)** for a set of at least two alternatives, every judgment aggregation F satisfying universal domain, collective rationality, IIP and UAP is weakly dictatorial.

Then we found that if some conditions are satisfied, **the strict preference- evaluation agenda (b)** can be changed into **the strict preference-evaluation agenda (a).**

Lemma 3. Given the strict preference-evaluation agenda **(b)**, if judgement aggregation F satisfies IIP and UAP, $R(\pm xPy) = \{\pm xPy\}$ for all xPy in this agenda.

$Proof$. Suppose to the contrary that there is some $xPy \in X$ such that

$R(\pm xPy)\backslash\{\pm xPy\} \neq \emptyset$.

This means that for this $xPy \in X$ there must exist two judgement profiles $(J_1,\ldots,J_n), (J'_1,\ldots,J'_n) \in \boldsymbol{J}^n$ such that

$(J_1,\ldots,J_n)|(\pm xPy) = (J'_1,\ldots,J'_n)|(\pm xPy)$ and

$(J_1,\ldots,J_n)|R(\pm xPy) = (J'_1,\ldots,J'_n)|R(\pm xPy)$

but F yields different outcome. i.e. $xPy \in F(J_1,\ldots,J_n)$ and $\neg xPy \in F(J'_1,\ldots,J'_n)$.

Construct a profile $(J^*_1,\ldots,J^*_n)$ from $(J_1,\ldots,J_n)$ and a profile $(J''_1,\ldots,J''_n)$ from $(J'_1,\ldots,J'_n)$ as shown in the Table 2. N^* is the largest coalition in N such that the members in N^* strictly prefer x to y. The judgment on the agenda in the picture above are consistent and readers can check it easily.

Table 2. New Profile Constructed from $(J_1,\ldots,J_n)$ and $(J'_1,\ldots,J'_n)$

	N^*	$N \setminus N^*$
$(J_1,\ldots,J_n)$	xPy	$\neg xPy$
$(J'_1,\ldots,J'_n)$	xPy	$\neg xPy$
$(J^*_1,\ldots,J^*_n)$	$xPy, y \in e_1, x \in e_1$	$\neg xPy, y \in e_1, x \in e_2$
$(J''_1,\ldots,J''_n)$	$xPy, x \in e_1, y \in e_2$	$\neg xPy, y \in e_2, x \in e_2$

Since $y \in e_1$, $x \in e_1$, $y \in e_2$ and $x \in e_2$ are not in $R(\pm xPy)$,

then $(J_1,\ldots,J_n)|R(\pm xPy) = (J^*_1,\ldots,J^*_n)|R(\pm xPy)$ and

$(J'_1,\ldots,J'_n)|R(\pm xPy) = (J''_1,\ldots,J''_n)|R(\pm xPy)$.

By IIP, we have $xPy \in F(J^*_1,\ldots,J^*_n)$ and $\neg xPy \in F(J''_1,\ldots,J''_n)$.

By UAP, we have $y \in e_1 \in F(J^*_1,\ldots,J^*_n)$ and $y \in e_2 \in F(J''_1,\ldots,J''_n)$.

By the consistency between preference and evaluation, we have $x \in e_1 \in F(J^*_1,\ldots,J^*_n)$ and $\neg x \in e \in F(J''_1,\ldots,J''_n)$. But $(J^*_1,\ldots,J^*_n)|\{R(\pm x \in e_1)\} = (J''_1,\ldots,J''_n)|\{R(\pm x \in e_1)\}$. By IIP, there is a contradiction.

■

Then by Theorem2, Lemma 2 and Lemma 3, we have

Theorem 4. Given the strict preference-evaluation agenda **(b)** for a set of at least two alternatives, every judgment aggregation F satisfying universal domain, collective rationality, IIP and UAP is weakly dictatorial.

By Lemma 3, For strict preference-evaluation agenda (b), $UR = \{xPy, z \in ei : x, y, z \in A, ei \in E\}$. This means that Weak Dictatorship is equivalent to the definition of dictatorship which is defined by Kruger and Sanver (2021).

Because the preference-approval agenda is a special case of the preference- evaluation agenda, then we also have

Corollary 2. Given the strict preference-approval agenda (b) for a set of at least two alternatives, every judgment aggregation F satisfying universal domain, collective rationality, IIP and UAP is weakly dictatorial.

4. Discussion

In our paper, we use the predicate logic to represent the preference- evaluation. After proving the Lemma 2, then the preference-evaluation agenda **(a)** satisfies some assumptions of Dietrich (2015)'s Theorem 2 which play the most important role in our embedding work. Then by Lemma 3, we By Lemma2, we found that if judgment aggregation satisfies IIP and UAP, the strict preference-evaluation agenda **(a)** will be changed into the strict preference-evaluation **(b)**. So, by Lemma 2 and Lemma 3, Theorem 2 can generalize preference-evaluation(approval) impossibility theorems based on two independence conditions **(a)** and **(b)**.

Table 3. The Embeddings of Concepts

Preference-evaluation aggregation	Judgment aggregation
Preference ranking and evaluation over a set of alternatives of which the number is no less than 2	Judgment set in the preference-evaluation agenda which satisfies the assumptions of Theorem 2
Asymmetry(reflexivity), transitivity and connectedness of preference ranking. The extra consistency between preference ranking and evaluation	Consistency and completeness of the judgment set
Universal domain	Universal domain
Collective rationality	Collective rationality
Independence condition **(a)** and **(b)**	Independence of irrelevant propositions
Unanimity for evaluation part	Unambiguous agreement preservation
Preference-evaluation dictator	Weak dictator

After this generalization, we construct an explicit embedding of preference- evaluation aggregation into judgment aggregation. The correspondence between preference-evaluation and judgment aggregation concepts under the constructed embedding is summarized in Table 3.

These findings not only propose a new proof of Arrowian impossibility theorems in combining preference and evaluation but also show the generality of judgement aggregation. The Theorem 2 (Dietrich, 2015) applies to a large class of aggregation formulated in formal logic of which a predicate logic for representing preference-evaluation is a special case.

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