Original Paper

Teaching about Waves with Applications in Lenses

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Abstract

The paper examines ways of teaching wave pulses traveling on strings and wave fronts in two dimensional waves. The phenomena examined for pulses are: reflection, transmission and superposition. Two methods of finding the refracted wave fronts, the corresponding rays and the refraction angle are presented. Then the gradual change of the speed of propagation of a wave is presented. In the last part is examined the application of the concept of wave fronts in lenses and Huygens' method of finding the shape of a perfect lens. The students' difficulties with connection with all these subjects are presented.

Keywords

reflection, transmission, Refraction, wave fronts, angle of refraction

1. Introduction

An introductory course of Physics has to deal with particle and wave notions. Teaching about waves presents special difficulties (Whitman, 2002). Lessons on wave phenomena follow lessons on particle kinematics and dynamics and the students have the tendency to associate their knowledge on particles to deal with wave phenomena. Another difficulty appears when students are introduced to two – dimensional waves. When they face questions on the wave length and the speed of wave they tend to consider as faster the shorter waves (emitted from a source and passing from one medium to another). Waves are frequently being taught through sinusoidal waves, which is some times misleading because the archetypal waves (sea waves) are of finite duration. According to research on student reasoning (Whitman, 2002), students tend to focus on explanation that are based on finite length (and not infinite) wave-trains.

2. Teaching Model

The teaching of wave phenomena was based on the "tutorials of Introductory Physics" (McDermot et al., 1998). In these lessons the waves are taught first by using pulses (Arons, 1990). This is similar to the way wave are introduced in PSSC (Haber Schaim et al., 1991).



Figure 1. Percentage of Students Taking Part in Tests

In the department of Elementary Education the teaching of waves is done in the third semester in the lesson "Principles of Physics". The students attend interactive lectures and then they take part in tests, where they also have to hand their home assignment. The attendance is voluntary and also the participation on tests. The attendance in the first years was small, but eventually it became a kind of tradition and most of the students attend all the tests.

The subjects that were taught are the following

- a) Reflection of one-dimensional pulses.
- b) Reflection and transmission of a pulse as they pass from one medium to another.
- c) Superposition of pulses.
- d) Two dimensional pulses: reflection and refraction.
- e) Transmission of pulses in a medium where the wave speed changes gradually.
- f) Application of wave phenomena to lenses.

The teaching was developed in 5 academic years (2003-2008). The lectures were done with the help of PowerPoint presentations, by software developed by one of the authors and some demonstrations.

Both the presentations and the software were uploaded in the web so even students who did not want or was not possible for them to attend the lectures could download them. It was found that the use of the PowerPoint presentation greatly enhanced the ability of the students to learn the "wave models".

For the teaching of pulses the use of Excel or Visual Basic was considered necessary because the observation of demonstrations using strings was not possible in the case of lectures. Some of the students attended laboratories and they could use these experiments. Also a video was used for the superposition of pulses. The software permitted the change of parameters and the change of conditions in the limits. In Figure 2 we have an instance of the case of a pulse traveling from a medium of high speed on the right to a medium of lower speed on the left. We can see the transmitted pulse and the

reflected pulse. We can see that the amplitude of the wave does not change the speed of the wave. Simpler demonstrations were developed by using Excel.



Figure 2. Pulses by Excel File "Pulse Motion"

3. Student Difficulties with Waves

To examine the understanding of the students we used:

A) The homework they returned each week.

B) The responses in the weekly tests.

C) The responses in exam questions.

The first source is not very reliable because many students who come to take part in the weekly tests just copy the work of other students, but students who are honest they show the same difficulties as with test.

a) **Reflection of pulses:** Students were asked on the two kinds of ends: fixed and free. To check on the understanding of the students, questions were used similar to the following:

A wave pulse travels towards the right (Figure 3). What is the shape after reflection on a fixed and what is the shape after reflection on a fixed end.



Figure 3. Simple Pulses by Using EXCEL



Figure 4. Wave Pulse Travelling to the Right

By analyzing the tests on this and similar cases for 146 students we have:

	Compost	Leading part on the same	Reflected on the	Wrong	No
	Correct	side as the incident	wrong side	Distance	answer
Fixed end	30	28	27	36	53
Free end	26	35	11	24	68

Table 1. Analysis of Responses of Students about Reflected Pulses

We can guess that many students consider the pulses like "solid bodies" colliding with the end of the string, since many of them have drawn the leading part of the reflected pulse on the same side as the incident pulse (Whitman, 2002).

b) **Transmission of pulses:** The students were asked to give the shape and distance of pulses after they pass through the point where two different strings join. As for this subject, the data collected from tests and exams show that the students face difficulties a) with the distance of the pulse, the shape of the pulse and its direction. One usual error is to draw the trailing part of the reflected pulse as the leading part. This is more easily seen when two pulses of different shape are considered. Many students draw as first reflected pulse the second.

There were several questions of predicting the shape of the pulses for several cases.

The incident pulse could be either on the left or the right side, and there were different combinations of media.



Figure 5. A Wave Pulse Travelling to the Left

In the example depicted in the Figure 5 we have the following results:

Table 2.	Student	Responses	to the	Test	of Figure 6	,

Total Correct	Leading part on the same	Reflected on	Wrong	No answer	
Iotai	Iotal Collect	side as the incident	the wrong side	distance	NO allswei
74	17	3	6	11	43



Figure 6. Two Pulses Travelling to the Left

After the tests and the feedback, similar questions were asked two months later in the exams. On such example is depicted in Figure 6: 2 pulses travel to the left: The students were asked to draw the reflected and transmitted pulses. We have the following results for correct answers:

Total	transmitted: shape	transmitted distance	reflected: shape	Reflected: distance
43	13	19	10	19

Students were also asked about errors in pictures (Figure 7) showing reflected and transmitted pulse. One such picture was found in a Greek textbook (Joannou et al., 2001, p. 66).



Figure 7. Erroneous Figure in a Textbook. The Upper Figure Shows a Pulse Traveling to the Right, and the Lower Figure the Result of Reflection and Transmission

The students were asked what errors they can find in the picture and to draw the correct picture in the case the pulse travels from a medium 1 to a medium 2 which has: a) higher speed or b) lower speed. In case (a) there are two errors: the width of the transmitted pulse should be greater and the reflected pulse should be on the same side of the spring as the incident. In case (b) the distance of the transmitted pulse should be smaller than the reflected.

In case (a) among 33 students the 20 found the first error and 11 the second error. In a repetition of the test 15 days later 52% answered correctly. This schema was observed in 2 academic years (2007, 2008). It can be seen that the results for 2008 were much better (Figure 8). This reflects the better participation of students in tests.



Figure 8. Percentages of Student Answers c1<c2 Case (a) , c1>c2 Case (b) (Figure 7). (I) is the First Test, (II) is the Repetition 15 Days Later

In case (b) the error was found by 8 students and 20 students considered as erroneous the width of the reflected.

On the question of drawing the correct pulses in case (a) 10 students of the 11 who found the errors drew correctly the reflected and transmitted pulses and in case (b) 5 students drew correctly the two pulses.

Similar tests were taken out of McDermott (McDermott, 1998). Two pulses are shown on two strings after the incident has reached the mid point. The hands of two students are shown (Figure 9). One student produced the pulse. By examining the side of the pulse the student can find which of the two students produced the pulse and also determine which string is the heavier.



Figure 9. Question about "Who Produced the Pulse?"

In the table are shown the number of students answering the question: Which student produced the pulse?

Total	Correct with	Correct with	Correct (no	Wrong	We do not have	No
number	wrong reasoning	correct reasoning	reason given)	wrong	enough information	answer
72	12	12	11	15	12	10

Table 4. Analysis of Results for Figure 9

c) Superposition of pulses

It was examined how the students could apply the principle of superposition for the moment when two pulses were on the same place (Figure 10). This subject was considered by the students very difficult. With the help of Excel files (Figures 10, 11) and PowerPoint presentation many students managed to answer. Among 79 students 72 answered correctly. But only 39 of them gave the correct reasoning. Here we got only few students who answered using the idea that the pulse is a solid body.



Figure 10. Two Pulses Approaching Each Other

The superposition of waves is used to predict the shape of the pulse during the reflection.



Figure 11. EXCEL Graphs Showing the Model of Reflection Using Superposition of Pulses

The results of homework assignments show that the students could apply the superposition for given wave shapes (about 95% success), but when they were asked to apply the same idea to predict the shape during the time of reflection their results were much poorer. Among 71 students only 36% answered correctly for the moment when the pulse was not yet completely reflected and about 46% for the moment when the pulse was completely reflected. Only 24% gave a satisfactory reasoning. Later during the exams about 25% of the students are able to apply the notions of superposition to the reflection of pulses.

d) Two dimensional pulses: reflection and refraction

After examining the basic notions of periodical waves the students were introduced to two dimensional waves.

One difficulty that students face is the relation between wave length and wave speed. Usually students think that the smaller wave length corresponds to faster movement. This is considered as p- prim

according to diSessa and Sherin (1998).

The notion of the wave front was introduced using waves in water. The students could see that the waves in a basin could travel faster in deeper water. By using the wave fronts it was introduced the ray concept as the direction of movement of wave fronts. In the cases which were studied the wave fronts were considered as perpendicular to the rays.



Figure 12. Software for Showing the Wave Fronts in Conjunction with Rays

For the case of two dimensional waves the software (Figure 12) that was used helped the observation of wave-fronts in connection with rays and showing roughly the proportion of reflected wave. Presentations with the "ripple tank" were of limited value because it was difficult to observe the refraction of the pulses.

4. Wave Treatment of Refraction

As was mentioned in the beginning the wave fronts offer a better approach for students to understand refraction than a "dogmatic" teaching of rays. Hobbes method (Shapiro, 1973) of turning the wave fronts is quite similar to what introductory texts present (McDermott, 1998). The student has to turn a transparency with periodic wave fronts until they fit with wave fronts lying in another medium (Figure 13). Of course Hobbes did not present periodic wave fronts but something that reminds more of the two connected wheels (who turn independently of one another) mentioned by Harrison and Treagust (1993). As can be seen in Figure 14 about 40% of the student could use the transparencies to find the refracted wave fronts, the rays and the angles of refraction.

Wave fronts can be used in treating lenses. This can help students understand that all points of a wave front have the same phase.



Figure 13. Use of Transparencies to Find the Refracted Wave Fronts



Figure 14. Student's Response to the Task of Finding the Refracted Wave Fronts

The students had also to use Huygens' method of constructing wave fronts (Huygens, 1694). To students were given different sets of media. In a test with 3 media (35 students) the students had to show that there was total reflection in the third medium.

Table 5. Use of Huygens method for finding the refracted wave fronts and rays with 3 media

	correct	Only rays	No answer	various errors
2nd medium	14	3	16	2
3rd medium	1	5	21	8



Figure 15. Applying Huygens' Principle to Find the Refracted Wave Fronts and Errors by Students

In Figure 15 we can see one simple way of finding the refracted wave front. As it is seen in this application of Huygens' Principle, a semicircle is drawn from the end of one wave front. From the next wave front a tangent is drawn to the semicircle. The other wave fronts are drawn as parallels to this one. In the example the 3^{rd} medium does not permit the formation of a wave front. It can be seen that most of the students did not realize that this is happening (only one succeeded in 35) and ignored the fact that no tangent can be drawn. In case there is a refraction students succeeded much better in drawing the wave fronts. There was a question of a wave with an angle of incidence of 10° traveling from one medium to another in which the speed of wave propagation is double. The students had to draw the incident and refracted wave fronts and find the angle of refraction. From 44 students there were the following successful answers:

Table 6. A simple Case for Huygens' Method

Incident	refracted	Angle
33	24	22

In all cases examined the results) were the following:

Table 7. Results for Huygens Method

Number of students	INCIDENT	REFRACTED	ANGLE
143	97	63	57

At first the Huygens' method was more difficult but later the students showed that at least in more simple case they could apply it.

As it can be seen both methods (using transparencies and Huygens' method) give approximately the same rate of success (Tables 6 & 7).

e) Gradual change of the speed of wave propagation

In the case of gradual change of the speed of the waves, such as in the case of a tank of water with a bottom that has not a constant depth (Prifti et al., 2003) or seismic waves in the interior of the earth or gradual change of the refractive index in the air, software presented to the students the case of a gradual change. The speed

could either increase or decrease with depth and the direction of first movement could be chosen. The use of this software helped the students to see that a 'ray' is a term that does not have the same meaning as "straight line" but is the path (ray – path) that a wave pulse follows. It should be emphasized that rays should be seen only as a mathematical tool and not as the most basic entity. Students who attend our lectures expressed their idea that waves are more understandable than rays. Students who had a better background in high school physics (where they are not taught about wave pulses or wave fronts) preferred to describe the different refraction phenomena with rays. When presented with a ray that starts horizontally in a medium where the speed increases with depth they proposed that the ray will continue as a straight line (In Figure 16, it can be seen that the ray goes up).

In a test students were presented with wave fronts and were asked to draw the rays. From 41 students only 6 could draw the correct rays. On a question asking to put in a series of increasing speed several points only 10 out of 40 could give a correct reasoning.



Figure 16. Gradual Change of Speed: Wave Fronts and Rays

It was asked to draw the bottom of the tank (Figure 16) where drops fall on the left side, so that the wave fronts take the form on the right. 14 out of 43 students gave the correct bottom. As for the rays only 9 students drew correctly the rays (one common mistake is to consider as rays only straight lines).



Figure 17. Question about Gradual Change of Speed

In another form of the question (Figure 17) there were three sets of rays and the students had to draw the bottom. Of 51 students 13 found the correct shape of bottom and 8 the correct rays.

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Another question used with another group of students was: given the shape of the bottom to find the shape of the wave fronts. This was more difficult: From 49 students 11 gave correct wave fronts and 3 correct rays.

Another question asked to complete the wave fronts if the rays are given and the points of the wave fronts on the y axis (Figure 18). 18 out of 84 managed to draw correct wave fronts.



Figure 18. Question about Drawing Wave Fronts

f) Application of wave phenomena to lenses

These notions are applied to Optics. The model of waves and the model of rays are applied to explain the phenomena of reflection and refraction. This subject of using both models in Optics was examined by Colin and Viennot (2001).

One subject that was studied was the concept of optical length. Students were asked to draw some rays starting from one point and to find were they approximately meet after passing a lens. The students had to draw rays perpendicular to the wave fronts (Figures 19, 20). The students had to count the number of periods (which is equivalent to finding the time) between the source and the point where most of the rays seem to pass near it. Out of 34 students 14 of them had drawn the emergent rays as passing through one point. This was their expectation (from the thin lens approximation which is taught) and the rest found that the rays did not pass through one point, but most of them found that they pass near this point. As for the number of periods, 18 found between 17 and 23 number of periods. So students could see that the usual theory was not sufficient for predicting what happens with a thick lens and for rays that are not paraxial. This helped the teacher to introduce the ideas of Huygens for constructing a "perfect" lens which focuses exactly to one point (Huygens, 2005, Figure 20).



Figure 19. Students' Response on the Task of Drawing Rays with Given Wave Fronts



Figure 20. Wave Fronts Produced by Software Fakoi.exe

Huygens proposed a method which extends the above ideas. If there is a surface separating two media and in front of it is a source point then a surface can be found so that the time of propagation of light from the source to the focus will be equal for all paths. This can be seen in a spherical or a hyperbolical surface (Figure 21).



Figure 21. Perfect Focusing with Huygens' Method: HUYGENS_LENS.EXE

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Special software (Huygens_Lens_Construction.exe) permitted to the teacher to present sheets of paper with wave fronts from the source and inside the lens. The student has to find the shape of the lens that produces a given focus point by drawing homocentric circles (with center at the focus point) starting from a point on the optical axis where the lens is thickest and proceeding by adding one period at a time.

Students were more successful in producing the shape of the lens than in the above problem as can be seen in Figure 22.



Figure 22. Students Responses to the Task of Drawing a Perfect Lens

As can be seen in the figure most of the students found almost the same shape which is very similar to fig 119 of Huygens book.

5. Conclusion

By using suitable software students showed much improvement in their responses. Using PowerPoint presentations enhanced much the teaching of waves. We can discern different kinds of difficulties. There was a difficulty in relation to the reflection of a pulse because students could not apply the model, according to which the shape of the string during the reflection is calculated by using an imaginary pulse coming from the other side. Also a generalized notion of the ray in case of a gradual change of the speed was found to be very difficult.

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