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Original Paper

The Advantages of SPC Models in Quality Control Processes

Gross Jonathan[1], Elalouf Amir[1*], Levin Avraham[1] and Neeman Dror[1]

[1] Department of Management, Bar-Ilan University, Israel

[*] Elalouf Amir, E-mail: Amir.Elalouf@biu.ac.il

Abstract

Product quality, which is positively associated both with firms' income and with their expenditures, is a crucial factor in competition among firms. Accordingly, the field of quality control and process management has developed substantially over the last decades. In the past, the leading approach in quality control was sampling across the product line and classifying samples as either "good" or "bad". If a sample was classified as "bad", the entire batch would be thrown out. This binary approach is problematic, as it might lead to wastage. This article will attempt to show, using a mathematical control model, that process management based on statistical data can improve quality control processes by saving time and money, and can ultimately improve the quality of the final product.

In what follows we first review several common approaches to quality and process control; these approaches are not based on mathematical modeling. We then introduce our approach, called statistical process control. On the basis of this approach, we develop a mathematical model with the objective of minimizing the expected cost per unit of time in the manufacturing process. We then present a numerical example of our method.

Keywords

SPC, quality control, manufacturing, product quality, Mathematical control model

1. Introduction

There are numerous approaches to defining "quality", and each approach is associated with specific processes for quality control. Four popular approaches include the following:

a. *The performance based approach, the implementary-operative approach*.

This approach, proposed by Crosby (1979), defines quality as conformance to requirements. That is, a product, service or prototype is considered to be of good quality only if it fulfills a set of specifications or requirements dictated by the customer, manufacturer, or management. For example, a service provided in a fast food restaurant such as McDonald's is considered to be of good quality if it meets the specifications for waiting time, serving etiquette and product preparation that have been formulated by the company. According to Crosby's definition, there is no difference in quality between a new

Mercedes that meets its customers' expectations and an old Volkswagen Beetle that does the same.

Crosby suggests that the first step in creating quality is defining the requirements. In fact, in many cases, redundancies and delays in the development of products, services and prototypes can be attributed to a lack of sufficiently well-defined performance requirements.

b. *The economic approach*.

This approach, proposed by Schonberger (1986), defines quality according to the following two dimensions, which are intertwined:

- Minimizing "wasted production"
- Successes on the first attempt

"Wasted production" refers to activities and expenditures that add no value to the customer, the product, the service, the prototype or the process. Wasted production includes so-called non-conformance quality costs, such as hours of work and material wasted on repairs, mistakes, compensating customers for bad service or damaged goods, and meetings aimed at addressing problems. Activities associated with non-conformance can take up tens of percentage points of employees' and managers' time. The term "wasted production" also refers to the ineffective time of knowledge workers and sales and development personnel.

Examples of wasted production (Schonberger, 1986) include the following:

- In an organization for the management of temporary manpower, it was found that employees in the payroll department spent most of their time correcting errors that had been made during calculation of the salaries.
- Sales representatives report that 40-60% of their work hours are wasted on multiple meetings with the same clients, rescheduling meetings that did not take place, etc.
- In the customer support department of a credit card company, hundreds of workers were employed to answer calls about problems that were caused by a lack of clarity in the company's billing statement.

Successes from the first attempt – this is our optimal scenario. Every process we will try to succeeded on the first production because it will be the minimum cost and success in this process will minimize the entire process cost..

c. *The customer-centric approach*.

This approach is based on Juran's (1989) definition of quality. Juran, one of the founders of the quality doctrine, claims that one of the most important aspects of quality is "fitness to use". This definition emphasizes the customer's centrality. The customer sets the standards, and all components of the system—the products, the services, the prototypes and the employees—are subject to the customer's requirements. Juran sees the work process as a chain of internal suppliers who work to fulfill the needs and requirements of internal customers. In addition, each member of this chain must bear in mind that he or she is also serving the end customer, who is external to the chain.

***d.** The uniformity approach*.

Taguchi, a Japanese engineer and one of the founders of the modern quality doctrine, emphasized uniformity and minimization of differences as critical components of quality (Taguchi, 1986). According to this approach, product or service uniformity is indicative of processes that are well controlled and well managed, i.e., uniformity indicates that the organization's processes are able to yield the same output time after time, despite the many factors that create "noise" or differences in the system. These factors include, among others:

- suppliers of raw materials and components;
- workers of the organization;
- production specifications;
- computational systems and equipment in the company.

In product development, product uniformity is dependent on structured and uniform processes that, on one hand, facilitate planning, design review and information-sharing among all participants in the process and, on the other hand, provide developers with opportunities to engage in creative design. In most cases, problems in development do not result from a lack of creativity or a lack of ability to overcome technological knowledge gaps; rather, they stem from unclear requirements, a lack of communication between marketing and development teams, problems with over-specification and over-design, and problems with deadline estimation, control and surveillance. Structured and uniform processes can resolve these issues. Such processes do not necessarily all adhere to an identical format or management style. Rather, the various processes can be classified into different families, and an appropriate structure can be determined for each family of processes.

In service organizations, uniformity means relying on repeatable processes that provide all customers with the same level of service. McDonald's, for example, seeks to provide high-quality, uniform service that meets pre-specified time and performance standards. All McDonald's customers worldwide should receive similar quality of service and similar quality of food, produced using the same methods. The ability to ensure uniform service constitutes a value catalyst for McDonald's, because this reputation causes customers to continue patronizing the chain, no matter where they are.

We have presented here four of the most popular approaches to defining the term "quality". Additional approaches define quality as meeting and exceeding customers' expectations, and other approaches break down quality into several aspects (such as reliability, performance, additions, perceived quality, reaction time, life expectancy) (for example Trkman,2010, Vergidis et al 2008).

2. Statistical Process Control

A method that has recently been assimilated into the world of quality management is statistical process control (SPC). This approach was first developed by Shewhart (1931) and was subsequently improved upon by Duncan (1956), who adapted it for the context of quality control. According to Duncan, the main principle in the statistical process model is identifying variance that is meaningful to the process and that can be reduced to improve results. Moreover, Duncan emphasized that variance that is associated with secondary processes and/or is not significant should be ignored. This distinction between different types of variance is necessary for defining the control values of the process.

The $\overline{X}$ model (Shewart 1931) is an SPC method that has received a great deal of research attention. In this model, products are sampled and subsequently measured according to dimensions that are indicative of product quality (e.g., weight, length, etc.). These measurements are then compared to predefined standards. Specifically, upper and lower bounds are defined for the various quality measurements. When the measurements of the sampled products fall within those bounds, the process is considered to be under control; if they exceed the bounds, the process is considered to be out of control. Appropriate responses are defined for the occurrence of deviations from those bounds.

This approach diverges from the common view that a product's deviation from a given reference point is necessarily detrimental to the production process. It suggests that even if such a deviation occurs, the product should not necessarily be rejected, and that there may be alternative, more appropriate responses. In contrast, in the sampling methods discussed above, there is no procedure in place for evaluating individual deviations or differences, such that when a certain deviation occurs, there is no choice but to classify the process as "bad" and to discard the associated products.

The SPC provides a means of achieving maximal productivity with minimal operating costs and has been shown to yield economic benefits in several industry settings. For example, Chase (1977) reported that Erco Papers, Inc. achieved a 1000% return on investment within six months of implementing SPC.

2.1 The Mathematical Model

In this article we seek to show how it is possible to control a process by evaluating statistical data and by comparing these data with bounds that have been determined according to the specifications and goals of the process. We further show the benefits associated with this approach.

Our model is based on the mathematical model $\overline{X}$ $N(\mu 0, \alpha / \sqrt{n})$, as presented by Linderman and Choo (2002).

2.1.1 Basic Assumptions

As noted above, the $\overline{X}$ SPC model is based on measuring a set of product dimensions and comparing these measurements with predefined standards. At any given time, the process under evaluation is assumed to be in one of several possible states: A process is considered to be *in-control* if the quality of its products meets the defined standards; otherwise, the process is *out of control*. Yet the sampling and measurement system may not always indicate accurately whether the process is in-control or out-of-control. Specifically, two types of errors are possible: type 1 errors (the system incorrectly indicates that the process is out of control) and type 2 errors (the system incorrectly indicates that the process is in-control). Thus, the "state" of the process is made up of the process's actual state and the state that is identified by the evaluation system.

In cases in which a process is identified as being out of control, intervention is required in order to identify the factor causing the deviation and to restore the process to an in-control state. It is important to emphasize that a deviation does not always indicate that the process must be altered or repaired: It could be that the deviation is simply a "false alarm" that does not indicate the actual state of the process, such that there is no need to disqualify the products. Thus, when a deviation from predefined standards is identified, it is necessary to seek out the reason for the deviation.

The basic assumptions of the $\overline{X}$ model proposed by Linderman and Choo (2002) are as follows:

- The measured value for the purpose of quality control has a normal distribution, $N(\mu, \frac{\sigma}{\sqrt{n}})$, with μ being the average value, σ being the standard deviation and *n* being the size of sample.
- The time during which the manufacturing process is in an in-control state has an exponential distribution, $f(t) = \lambda e^{-\lambda t}$, with λ being the number of deviations from the in-control state in a given unit of time.

In order to construct an SPC $\overline{X}$ model, we propose a statistical-economic approach that relies on the following three parameters: The size of the sample, *n* (number of units tested in each sample), the interval between samples, *h*, and the distance *L* between the upper and lower bound, measured in units of standard deviations from average.

The choice of parameter values is based on economic and statistical properties. Parameter values should take into account the costs of inspecting the sample, searching for problems in the process, and resolving them, as well as the cost of a false alarm. An excessively high rate of sampling or too many units in each sample could significantly lengthen the manufacturing time and increase costs. Furthermore, if the upper and lower bound are too close, there will be too many false alarms. Therefore, it is important to find a method for determining the vector of parameter values that minimizes costs. Duncan (1956) proposed the first economic $\overline{X}$ model, and the idea developed over the years. The model proposed herein is one of its implementations.

2.1.2 Model Formulation

- At its initiation the process is in-control; the average quality value is μ, and the standard deviation is σ.
- Samples are evaluated in control cycles separated by time intervals of *h*. The process ceases when the system is identified as being out of control.
- After *i* samples, the process enters an out-of-control state, and after (*i* + *j*) samples the SPC system identifies the deviation and triggers a signal.
- When a signal is triggered, the manufacturing process stops so that the problem can be identified. The process then resumes from an in-control state, beginning a new control cycle.
- The time between signals has an exponential deviation $f(t) = \lambda e^{-\lambda t}$.
- At the end of every cycle one of 4 states is possible:
    - S_{00} - The process is in-control and there is no signal (compatible)
    - S_{01} - The process is in-control and there is a signal (incompatible)

- S_{10} - The process is out-of-control and there is no signal (incompatible)
- S_{11} - The process is out-of-control and there is a signal (compatible)

The probability of each state is presented below in Table 1.

Let $F(t)$ be the cumulative distribution function that represents the probability of the process to be out-of-control after time *t*. Thus, the probability for the process being in-control at the end of the sampling cycle is $F(h) = 1 - e^{-\lambda h}$.

As indicated above, two types of errors are possible: signaling an out-of-control state when the process is in-control (false positive; type 1 error), and not identifying that the process is out-of-control when it is (false negative; type 2 error). The probability of a type 1 error is denoted α, and the probability of a type 2 error is denoted β. Clearly, the objective is to minimize the number of errors and to ensure that the out-of-control states are recognized as early as possible. In order to achieve this it is necessary to correctly define the distance between the upper and lower bound.

Table 1. States

State	Probability
The process is in-control and there is no signal, S_{00}	$(1-F(h))(1-\alpha)$
The process is in-control and there is a signal (type 1 error); S_{01}	$(1-F(h))\alpha$
The process is out-of-control and there is no signal (type 2 error); S_{10}	$F(h)\beta$
The process is out-of-control and there is a signal; S_{11}	$F(h)(1-\beta)$

Another important component of the control process is the cost. This factor greatly influences the feasibility of performing process management in the first place. The cost of a control cycle is calculated as its cumulative costs from beginning to end. The cost of a time unit is the ratio E(*C*)/E(*T*), where E(*C*) is the expected cost per cycle, and E(*T*) is the expected duration of the cycle. The objective of our model is to minimize this ratio, i.e., to obtain $z = \min(E(C)/E(T))$.

2.2 The Model's Solution

In what follows we formulate a mathematical expression for $E(T)$ .In state S_{00} there are no occurrences that could prolong the cycle time; therefore, the expectancy for the cycle's duration is $E(T)$.

In state S_{01} the manufacturing process stops because of a false alarm, and therefore the expected cycle time is increased by Z_0, time wasted on searching for the nonexistent problem in the process. Therefore, the cycle duration expectancy is $E(T) + Z_0$.

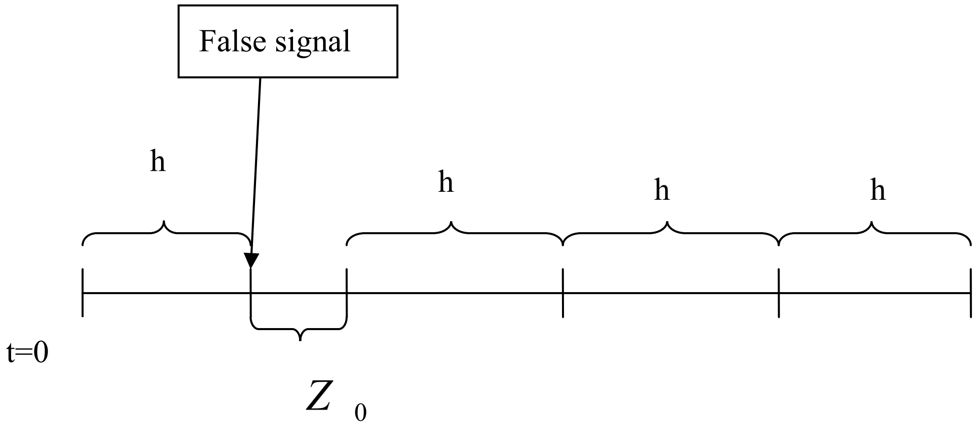

Figure 1. False signal

In state S_{10} the system is out-of-control, but no deviation from the norm is identified. The average amount of time until an error is identified is as follows: $ATS = h(ARL) = \frac{h}{1-\beta}$, (ARL – Average Run Length, ATS – Average Time to Signal). In addition, once the error is identified, it takes time Z_1 to locate the malfunction and to repair it; therefore, the duration of the control cycle in this case is $\frac{h}{1-\beta} + Z_1$.

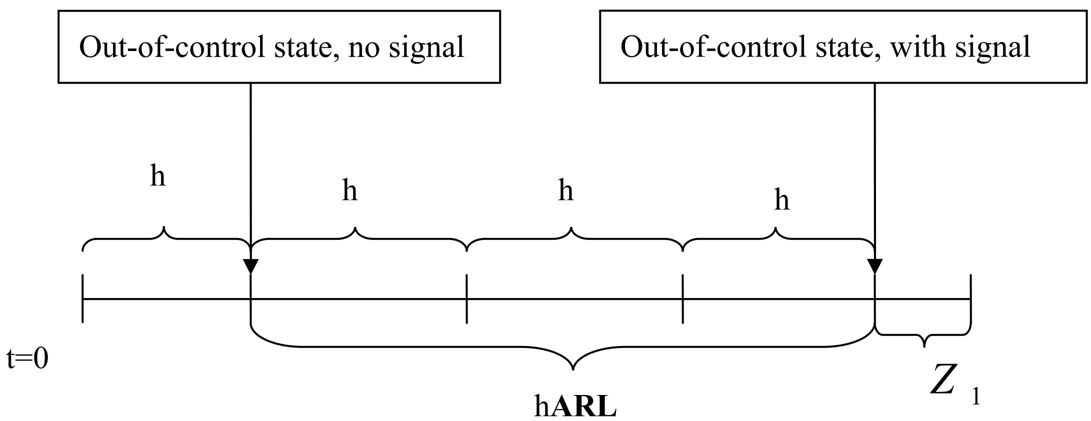

Figure 2. Out of Control

In state S_{11} an out-of-control state is correctly identified, and the process is stopped and repaired. Thus, the duration of the cycle in this case is Z_1.

The expectancy of the cycle duration is the sum of the products of the individual cycle duration expectancies and their respective probabilities:

$$E(T) = h(\frac{1}{F(h)} + \frac{\beta}{1-\beta}) + \alpha Z_0(\frac{1-F(h)}{F(h)}) + Z_1 = h(\frac{1}{1-e^{-\lambda h}} + \frac{\beta}{1-\beta}) + \alpha Z_0(\frac{e^{-\lambda h}}{1-e^{-\lambda h}}) + Z_1$$

We will now present the mathematical expression of $E(C)$. The process has added costs such as:

- The sampling cost $a + bn$, with a being a fixed cost and b being a variable cost of sampling a production unit.
- The cost of manufacturing in an in-control state, C_0, and the cost of manufacturing in an out-of-control state, C_1 ($C_1 > C_0$).
- The cost of a false positive, Y.
- The cost of identifying the cause of a problem and fixing it, W.

In state S_{00} all costs result solely from manufacturing and sampling, and they add up to $a + bn + C_0 h + E(C)$, with $E(C)$ being the expectancy of a cycle cost.

In state S_{01} the cost of a false positive is added, and therefore the cost expectancy in this case is $a + bn + C_0 h + Y + E(C)$.

In state S_{10} the process is out-of-control, but the SPC system does not recognize this immediately, and production continues in an uncontrolled state (cost C_1). During a period of $\tau = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})}$ (the time until the transition into an out-of-control state) the cost of manufacturing is C_0, and after the transition the production continues for (h-t) periods of time at a cost of C_1. As described above, the deviation is identified after $ARL = \frac{1}{1-\beta}$ samples; therefore the cost of manufacturing in an out-of-control state is $ARL(a + bn + C_1 h)$. After the process stops, the manufacturer looks for the cause of the malfunction and repairs it at a cost of W. In total, the cost of the cycle in this state is $a + bn + C_0\tau + C_1(h-\tau) + \frac{1}{1-\beta}(a + bn + C_1 h) + W$.

In state S_{11} an out-of-control process is identified as such immediately after the first sample, i.e., after h units of time; therefore, the cost expectancy for a single cycle in this case is $a + bn + C_0\tau + C_1(h-\tau) + W$.

The sum of the products of the individual expectancies and their probabilities gives:

$$E(C) = C_0 h(\frac{1-F(h)}{F(h)}) + C_1 h(1 + \frac{\beta}{1-\beta}) + Y\alpha(\frac{1-F(h)}{F(h)}) + (a+bn)(\frac{1}{F(h)} + \frac{\beta}{1-\beta}) + \tau(C_0 - C_1) + W$$

$$= C_0 h(\frac{e^{-\lambda h}}{1-e^{-\lambda h}}) + C_1 h(1 + \frac{\beta}{1-\beta}) + Y\alpha(\frac{e^{-\lambda h}}{1-e^{-\lambda h}}) + (a+bn)(\frac{1}{1-e^{-\lambda h}} + \frac{\beta}{1-\beta}) + \tau(C_0 - C_1) + W$$

From these formulations, we can deduce the function of cost expectancy per unit of time $E(C)/E(T)$.

2.3 Numerical Example for the SPC Method

In the following numerical example we demonstrate the SPC approach and show that the latter achieves lower costs and better efficiency.

We assume that the interval between samples is 1 hour, the size of the sample is 5 units, and the distance between the upper and lower bounds is 3 standard deviations. Table 2 shows the expected cost per hour achieved with the economic statistical model and that achieved through the use of rules of thumb, while all the rest of the data remain constant.

Table 2. Parameters

	Parameter	$\overline{X}$ model
Parameter vector	H	2.96
	N	9
	L	2.25
Parameters of the manufacturing process	λ	0.505
	α	0.045
	β	0.036
	Z_0	0.25
	Z_1	1
	C_0	50
	C_1	100
	Y	25
	W	550
	a	0.5
	b	0.1
Cost (in USD)	$E(C)/E(T)$	171.576

In particular, Table 2 shows that the expected manufacturing cost per hour under the $\overline{X}$ model is 25% lower. In addition, the ability to recognize errors in the $\overline{X}$ model is higher because the distance between the upper and lower bound is smaller. Thus, in this example, the model we propose performs better in terms of reducing costs and thereby maximizing profits.

3. Conclusions

This paper proposes an SPC-based mathematical model aimed at identifying whether a given process is in-control or out-of-control. A numerical example suggests that our approach can potentially reduce costs and thereby increase profits. We further suggest that it is crucial to clearly define the goals of the organization in order to achieve maximal benefit from the SPC approach.

Yet another insight we can propose is that a mathematical model is not always the best means of achieving optimal control of a process. The SPC model is not easy to calculate; it requires highly accurate parameters, which may be costly to obtain. As a result, organizations that do not require high accuracy in control processes are unlikely to use the SPC method. Rather, they are more likely to use the traditional sampling method discussed in the beginning of this article.

Most importantly, if a firm does decide to control its processes using the SPC method, it is necessary to examine the process at hand and to choose the control model that is best suited to it. A potentially interesting direction for future research is to analyze which statistical control techniques fit specific types of organizations. Every organization has different goals and different methods. To achieve the best possible outcome, it is necessary to define models that are specifically tailored to the organizations in which they will be applied.

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