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Application of the Logistic Growth Model: Estimation of Livestock Population in Mongolia

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Abstract

We propose the logistic growth model to predict number of livestock in Mongolia. For this purpose, we estimate its parameters based on data retrieved from National Statistical Office of Mongolia for period of 2004-2013. Once parameter estimates are made, we apply those parameters into the model.

Keywords

logistic growth model, function, livestock, mathematical model, parameter estimation, population estimation

1. Introduction

Due to rapid advances in science and technology, dramatic changes are occurring in agriculture that has crucial implications for human health, livelihoods, and the environment. Growing population, increasing income and dramatic urbanization in developing countries are causing a massive increase in demand for food. Both public and private sectors must prepare for these changes with long-run policies and investments that will meet consumer demand, improve health benefits, create income opportunities, and decrease environmental stress.

Number of livestock is one of the core indicators for stakeholders for above purposes. In fact, reliable estimation on the number of animals in the country is demanded for the Ministry responsible for livestock to develop, implement and monitor long-run policies and for the National Statistical Office to estimate livestock value added. According to National Statistical Office (NSO) of Mongolia, agricultural sector accounts for more than 14.6% of Gross Domestic Product (GDP) of Mongolia in 2012. The livestock sector dominates, contributing more than 80% of total agricultural production (D. Baterdene, 2010). In contrast, the private sector is interested in investment that will satisfy consumer demand created by increased population in the coming decades. The population of Mongolia "can be expected to grow by about one third over the next 30 years, adding approximately 1.4 million people to the 2010 population" (UNFPA, 2012).

The most populations are constrained by limitations on resources and none is unconstrained forever (R. Enkhbat, 2005). Therefore, we apply the logistic growth model to estimate the number of animals more accurately. The model postulates that the relative growth rate decreases when population approaches the carrying capacity of the environment. In other words, the initial stage of growth is approximately exponential; then as saturation begins, the growth slows, and at maturity, growth stops (Verhulst, Pierre-François, 1845).

2. Method

2.1 Logistic Growth Model (Verhulst Model)

We use the following so-called logistic growth model which is given by a differential equation:

$$\begin{cases} y' = ky(M-y) \\ y(0) = y_0 \end{cases} \tag{1}$$

A solution of this differential equation can be easily obtained as:

$$y(t) = \frac{My_0}{y_0 + (M - y_0)e^{-Mkt}}, \tag{2}$$

Where:

$y(t)$: Number of livestock at a moment,

y_0: Initial value at moment $t = 0$,

$k \text{ and } M$: Parameters of the model.

In order to estimate parameters k and M by least square method, we need to use the following type of statistical data for each livestock species: camel, horse, cattle, sheep and goat.

Year (t)	t_1	t_2	…	t_i	…	t_n
Number of livestock (y)	y_1	y_2	…	y_i	…	y_n

where, $y(t_i) = y_i$, $i = 1, 2, \ldots, n-1$.

2.2 Least Square Method (LSM)

The least square method (LSM) requires solving the following problem (D. Bertsekas, 1999):

$$E(M,k) = \sum_{i=1}^{n-1}\left[\frac{My_0}{y_0 + (M-y_0)e^{-Mkt_i}} - y_i\right]^2 \to min_{M,k>0}, \tag{3}$$

Problem (3) in a general is non-convex hard optimization problem. For finding global solutions k and M for this problem, we can use a stochastic method (2) or a global search method (1) but it is difficult to solve problem (3) globally. In order to simplify global search methods, we use a difference scheme for y'. In other words, we replace y' with its finite difference scheme $y(t+1) - y(t)$ for each t. Then model (1) becomes as:

$$y(t+1) - y(t) = ky(M-y), \; t = 0,1,2,\ldots,n-1; \tag{4}$$

Now we introduce a function $F(M,k)$ and apply the least square method to (4). Then we have

$$F(M,k) = \sum_{i=1}^{n-1} [ky_i(M - y_i) + y_i - y_{i+1}]^2 \to min_{M,k>0}$$

For obtaining k and M, we write down optimality conditions by taking partial derivatives of the function $F(M,k)$ with respect to k and M.

$$\begin{cases} \dfrac{\partial F}{\partial k} = 2\sum_{i=1}^{n-1} [ky_i(M - y_i) + y_i - y_{i+1}]y_i(M - y_i) = 0 \\ \dfrac{\partial F}{\partial M} = 2\sum_{i=1}^{n-1} [ky_i(M - y_i) + y_i - y_{i+1}]ky_i = 0 \end{cases}$$

If we simplify the above system, then we obtain following:

$$\begin{cases} k\sum_{i=1}^{n-1} y_i^2(M - y_i)^2 \sum_{i=1}^{n-1} (y_i - y_{i+1})y_i(M - y_i) = 0 \\ k\sum_{i=1}^{n-1} y_i^2(M - y_i) + \sum_{i=1}^{n-1} (y_i - y_{i+1})y_i = 0 \end{cases}$$

From the first problem, we can easily find k as follows:

$$k = \frac{\sum_{i=1}^{n-1} y_i(y_{i+1} - y_i)(M - y_i)}{\sum_{i=1}^{n-1} y_i^2(M - y_i)^2} \qquad (3)$$

If we substitute k in the second equation, we find M. In other words, a value of M can be found by solving the following quadratic equation.

$$AM^2 + BM + C = 0,$$

where

$$\sum_{i=1}^{n-1} y_i(y_{i+1} - y_i) = a,$$

$$(y_{i+1} - y_i)y_i = b_j, j = 0,1,2, \ldots, n-1;$$

$$A = a\sum_{i=1}^{n-1} y_i^2 - \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} y_i^2 b_j,$$

$$B = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} y_i^2 b_j(y_i + y_j) - 2a\sum_{i=1}^{n-1} y_i^3,$$

$$C = a\sum_{i=1}^{n-1} y_i^4 - \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} b_j y_i^2 y_j.$$

3. Result

Using the statistical data of livestock in Table 1.

Table 1. Livestock by Type of Species 2004-2013

Type (000s)	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Camel	256.6	254.2	253.5	260.6	266.4	277.1	269.6	280.1	305.8	321.5
Horse	2,005.3	2,029.1	2,114.8	2,239.5	2,186.9	2,221.3	1,920.3	2,112.9	2,330.4	2,619.4
Cattle	1,841.6	1,963.6	2,167.9	2,425.8	2,503.4	2,599.3	2,176.0	2,339.7	2,584.6	2,909.5
Sheep	11,686.4	12,884.5	14,815.1	16,990.1	18,362.3	19,274.7	14,480.4	15,668.5	18,141.4	20,066.4
Goat	12,238.0	13,267.4	15,451.7	18,347.8	19,969.4	19,651.5	13,883.2	15,934.6	17,558.7	19,227.6
Total	28,027.9	30,398.8	34,803.0	40,263.8	43,288.4	44,023.9	32,729.5	36,335.8	40,920.9	45,144.4

Source: National Statistical Office, Agricultural Sector in 2012.

We obtained estimates of parameters k and M for each type of livestock species in Table 2.

Table 2. Estimates of Parameters for *k* and *M* each Type of Livestock

Type (000s)	*M*	*k*
Camel	600	0.000109
Horse	5,930	0.00000103
Cattle	10,220	0.000000609
Sheep	104,897	0.0000000611
Goat	96,378	0.0000000512

Then, we applied estimates in Table 2 into the logistic growth model formula (4) to obtain results in Table 3. We computed this problem on Microsoft Excel application.

Table 3. Projected Number of Livestock by Type of Species 2014-2030

Type (000s)	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Camel	331.2	340.8	350.4	359.9	369.2	378.4	387.4	396.3	405.0	413.4
Horse	2,709.3	2,799.6	2,890.2	2,981.0	3,071.8	3,162.3	3,252.5	3,342.2	3431.1	3,519.2
Cattle	3,060.5	3,215.8	3,375.3	3,538.8	3,705.8	3,876.2	4,049.5	4,225.4	4,403.6	4,583.4
Sheep	21,127.3	22,229.6	23,373.3	24,558.5	25,784.7	27,051.5	28,358.2	29,704.0	31,087.7	32,508.0
Goat	19,998.8	20,792.5	21,608.9	22,447.7	23,309.0	24,192.6	25,098.1	26,025.3	26,973.8	27,943.1
Total	47,227.0	49,378.4	51,598.2	53,885.8	56,240.5	58,660.9	61,145.7	63,693.2	6,6301.1	68,967.2

Type (000s)	2024	2025	2026	2027	2028	2029	2030
Camel	421.7	429.8	437.6	445.2	452.6	459.7	466.6
Horse	3,606.4	3,692.3	3,777.0	3,860.3	3,942.0	4,022.1	4,100.5
Cattle	4,764.6	4,946.7	5,129.2	5,311.7	5,493.6	5,674.6	58,54.1
Sheep	33,963.3	35,451.9	36,971.7	38,520.5	40,095.9	41,695.2	43,315.8
Goat	28,932.7	29,942.0	30,970.4	32,017.1	33,081.3	3,4162.0	35,258.4
Total	71,688.7	74,462.7	77,285.9	80,154.7	83,065.4	86,013.7	88,995.4

Based on forecasting in Table 3 we constructed following growth trend by type of species in Figure 1.

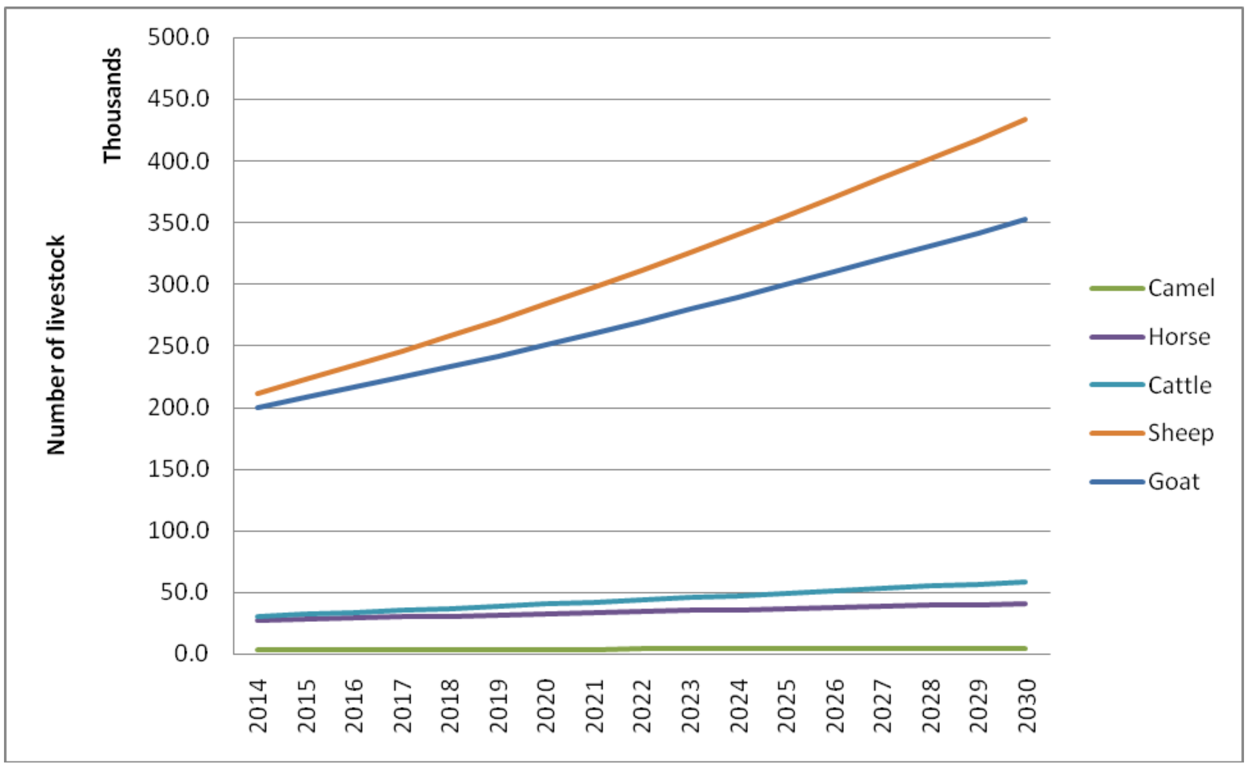

Figure 1. Number of Livestock by Type of Species 2014-2030

Source: Table 3.

4. Discussion

Valid sources of information and appropriate methods of forecasting data from these sources are crucial to both public and private sectors. Reliable information and estimates are necessary for organizations in public sector to develop, implement and monitor policies. In contrast, forecasting is used as one of the most critical components of decision making in private sector. Both sectors must have substantial consideration of the circumstances surrounding the decision, available alternatives, and potential outcomes. Therefore, use of application of mathematical theories, methods and models can be utilized to assess the substantial consideration circumstances and produce effective and efficient solutions. We applied logistics growth model for forecasting each livestock species: camel, horse, cattle, sheep and goat, in Mongolia for period of 2014-2030. The obtained result has a practical importance in planning for both sectors.

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