Original Paper

Evaluation and Analysis of Development Strategies for Pet Industry and Related Industries Based on Fuzzy Comprehensive Evaluation and Gray Prediction Modeling

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Abstract

With the deepening of economic development, the pet economy presents a multi-dimensional growth trend, however, the pet industry also exists in development dilemma. Therefore, this paper studies its current situation and factors affecting the development of the pet industry, predict the development trend, formulate effective development strategies to promote the sustainable development of the pet industry, and is committed to the deepening of the process of economic globalization.

*For Problem 1, firstly, data on the current development status of China's pet industry industry from 2019-2023 are collected and visualized to analyze its previous development. Secondly, the weights of the indicators of the development factors of China's pet industry are measured using **the expected value method**, and **the fuzzy comprehensive evaluation method** is used to establish a comprehensive evaluation index system for it, and the fuzzy comprehensive evaluation scores are obtained for each year from 2019 to 2023. Finally, using **the gray prediction model** to predict the development trend of China's pet industry in the next three years, it is found that the overall development of China's pet industry will show a steady upward trend, but the penetration rate of pet households is decreasing year by year.*

*For Problem 2, the effective data of global pet food demand from 2018-2023 are collected to explore the development of global pet industry through data visualization. Secondly, **the entropy weight-TOPSIS method** is utilized to obtain the relative proximity C value of global pet food demand for each year from 2018-2023. Constructing **the gray prediction model** and **multiple linear regression model** to predict the overall development of global pet food demand indicators-relative proximity C value and specific development data, we get that the overall pet food demand is on the rise, and the*

*predicted results of the global pet food demand in 2024-2026 are **US$ 152.32 billion, US$ 18.534 billion, and US$ 22.001 billion**, respectively.*

*For Problem 3, firstly, **a ridge regression model** is used to calculate China's pet food production and export volume from 2019-2023. Secondly, using **the gray forecasting model**, without considering the economic policy, based on the global pet food market demand trend and China's development, China's pet food production is forecasted to be **290,161 million pieces, 598,962 million pieces and 12,364,402 million pieces**, and demand is forecasted to be **1,509 million pieces, 1,877 million pieces and 2,335 million pieces**, respectively, for the period of 2024-2026.*

*For Problem 4, firstly, **the fuzzy comprehensive evaluation method** is utilized to solve the impact of the foreign economic policies of Europe and the United States on the pet food industry, and the impact scores of the foreign trade, foreign investment, monetary policy and tariff policy of Europe and the United States on China's pet food industry are obtained to be **46.45, 51.22, 16.67 and 37.78** respectively, and to formulate the strategy of sustainable development of China's pet food industry.*

Keywords

the fuzzy comprehensive evaluation method, the gray prediction model, a ridge regression model, the entropy weight-TOPSIS method

1. Introduction

1.1 Problem Background

Against the backdrop of people's changing consumption concepts, the pet industry has begun to emerge globally as an emerging field due to rapid economic development and rising per capita incomes. With the concept of "pet companionship" becoming more and more prevalent, pet food, pet clinics, pet supplies and pet care and other related industries are gradually moving towards a broad and fast-growing market pattern. The purpose of this article is to analyze the development of the pet industry and market needs, and based on the results of the analysis and the current economic situation, to formulate an appropriate strategic plan for the development of China's pet industry.

1.2 Our Work

In Problem 1: Collect data, based on the data in Attachment 1 and the collected data and the development trend of China's pet industry in the past five years according to the types of pets should be analyzed. At the same time, we analyze the factors that drive its development, and then construct a reasonable mathematical model to predict the direction of China's pet industry in the next three years.

In Problem 2: In recent years, the pet industry in Europe, America and other overseas regions has shown rapid development. Based on the data in Attachment 2 and other relevant data collected by the team, we analyze the development of the global pet industry according to different types of pets. A suitable mathematical model is then constructed to predict the global demand for pet food in the next three years.

In Problem 3: Analyze the development situation of China's pet food industry with reference to

China's pet food production and export data in Attachment 3, and forecast China's pet food production and export in the next three years (excluding the interference of economic policy changes), taking into account the demand trend of the global pet food market and China's own development status quo.

In Problem 4: China's pet food industry will inevitably be affected by the new foreign trade policies of the US and Europe (such as tariffs). In order to quantitatively analyze this impact, we need to construct an appropriate mathematical model by combining the attached data, self-collected data, and the results of the previous problem. Based on the conclusions of the calculations, a feasible strategy is planned for the lasting development of China's pet food industry.

2. Problem Analysis

2.1 Analysis of Problem 1

For question 1, Attachment 1 only gives the data of pet cats and dogs in China from 2019-2023. In order to study the development of China's pet industry in the past five years, it is necessary to take into account the data of household penetration rate and market size, etc. Therefore, this paper collects the data of household penetration rate of pet cats, market size of consumer market, consumption amount of a single cat, and the market size of food consumption and analyses the development of China's pet industry in the past five years according to the classification of pets. Therefore, this paper collects the data of pet cat household penetration rate, consumer market size, single consumption amount and food consumption market size, and analyses the development of China's pet industry in the past five years by pet classification, and considers adopting the 'combination of figures and shapes' to visually analyse the industry development. In view of the fuzzy and difficult to quantify the factors affecting the development of China's industry, it was decided to construct a fuzzy comprehensive evaluation model to convert qualitative indicators into quantitative data for analysis and evaluation. 2019-2023 data contains only five annual data values, the sample size is small, and it does not conform to the seasonal characteristics of the ARIMA model, so it is constructed as a grey prediction model to predict the development of the China's pet industry in the condition of a small amount of data and uncertainty. industry development under the conditions of small data volume and uncertainty.

2.2 Analysis of Problem 2

For question 2, due to the need to analyze the development of the global pet industry and the wide scope of the study, in order to improve the accuracy and reliability of the research results, the entropy weight-TOPSIS method is considered to be used to obtain the indicator weights of the various components of the global demand for pet food and the results of the overall development trend of the demand measurements, i.e., the relative proximity C-value, and the grey forecasting model is solved to forecast the next three years of the global pet food The grey forecast model is solved to predict the overall development of global pet food demand in the next three years. In addition, considering the strong linear relationship between the components of food demand, the global pet food sales volume is used to characterise the food demand, and a multiple linear regression model is constructed to predict

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the specific development trend of global pet food demand in the next three years.

2.3 Analysis of Problem 3

For question 3, the question requires that the export volume and production of Chinese pet food in the next three years be predicted based on the trend of global pet food market demand and China's development without taking into account the influence of other countries' foreign economic policies. In view of the strong correlation between the indicators, i.e., there is the problem of multicollinearity, the decision was made to establish a ridge regression equation to get the calculation equations for the production volume and the export volume. At the same time, it is considered to use the grey prediction model to predict the data of the independent variable in the ridge regression equation for the next three years, and substitute it back to the calculation equation to get the final result.

2.4 Analysis of Problem 4

For question 4, on the basis of question three, the impact of the new foreign economic policies of Europe and the United States on the development of China's pet food industry is introduced, and in order to transform the qualitative indexes into quantitative indexes for comprehensive evaluation, the fuzzy comprehensive evaluation method is finally considered to analyse the impact of different foreign economic policies on China. Based on this, the data obtained from the previous calculations are synthesised to formulate an effective sustainable development strategy for China's pet food industry.

3. Assumptions

1. It is assumed that the global pet food industry sales volume characterises the global pet food industry demand.

2. It is assumed that global pet food sales, global pet food market revenue growth rate, global pet food average price and global pet food market size together constitute the global pet food industry demand.

3. It is assumed that the new foreign economic policies of Europe and the United States mainly include foreign investment, trade, monetary policy and tariff policy.

4. Assume that the data collected in this paper are true and reliable.

4. Nomenclature

Symbols	Definition	Symbols	Definition
D_1	GDP per capita of prefecture-level cities in China	D_{11}	Amount of local government special bonds
D_2	Consumption expenditure per urban resident	D_{12}	Size of the fiscal deficit
D_3	Per capita disposable income of urban residents	D_{13}	Number of new veterinary drugs approved by the Ministry of Rural Affairs of China in the year

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D_4	Land area for urban construction	D_{14}	Number of patents granted for inventions
D_5	Urbanization rate of prefecture-level cities	α	Development coefficients of the gray forecasting model
D_6	China Pet Funeral Industry Market Size	u	Gray Role Quantities for Gray Predictive Models
D_7	Number of enterprises in China's funeral industry (households)	C	A posteriori difference ratios for the gray prediction model
D_8	Scale of education	e_j	The information entropy of the first j Information Entropy of the Indicator
D_9	Number of elderly people aged 65 years and above as a proportion of the total population of the country	w_j	The weights of the firstj Weighting of indicators
D_{10}	marriage rate	Y	Demand for food in the pet industry

5. Modelling and Solving

5.1 Modelling and Solving Problem 1

5.1.1 Analysis of the Development of China's Pet Industry in the Past Five Years

Firstly, considering the many types of pets, in order to simplify the model solving process, the categories of pet cats and pet dogs, which accounted for a relatively large proportion of the overall number of pets, were selected to explore the current situation of the development of the pet industry in China according to the classification of pets. Secondly, with the rise of the concept of pet ownership in China, the domestic pet industry has entered a brand new stage of development, the pet economy has further developed, the penetration rate of pet families has deepened, and the scale of the pet market has further expanded (Zhang & Chen, 2022). Only using the number of pets can not completely characterize the overall development of the pet industry, so this paper to the number of pets in towns and cities, pet family penetration, consumer market size, single consumption amount and food consumption market size of a total of five indicators to characterize the analysis of the past five years of China's pet industry industry development by pet category. The above data were collected through China Statistical Yearbook, China Pet Industry Research Report, China Pet Industry White Paper and Interface News, etc., see Table 1-Table 2 and Figure 1-Figure 3.
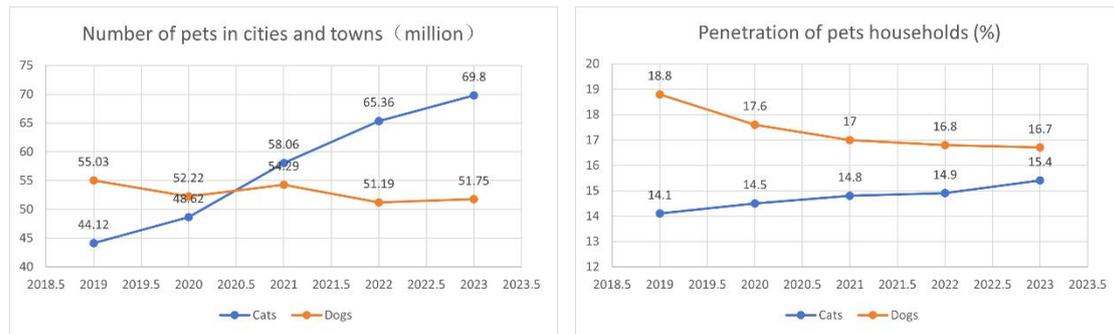
Table 1. Development of pet cats in China in the Past Five Years

Number of pet cats in towns and cities	Pet cat household penetration (%)	Consumer market size	Single consumption	Food consumption

	(millions)		(millions)	amount (yuan)	market size
2019	44.12	14.1	78000	1768	490
2020	48.62	14.5	88400	1818	510
2021	58.06	14.8	106000	1826	570
2022	65.36	14.9	123100	1883	660
2023	69.80	15.4	130500	1870	710

**Table 2. Development of pet Dogs in China in the Past Five Years**

	Number of pet dogs in towns and cities (millions)	Pet dog household penetration (%)	Consumer market size (millions)	Single consumption amount (yuan)	Food consumption market size
2019	55.03	18.8	124400	2261	750
2020	52.22	17.6	118100	2262	620
2021	54.29	17	143000	2634	720
2022	51.19	16.8	147500	2882	720
2023	51.75	16.7	148800	2875	748

Figure 1. Graph of Number of Pet Dogs and Cats in town (left) and Penetration of Pet Dog and Cat Households Graph (right)

Figure 2. Graph of the Amount of pet cat and Dog Consumption Market (left) and the Amount of Pet Cat and Dog Consumption Per unit (right)
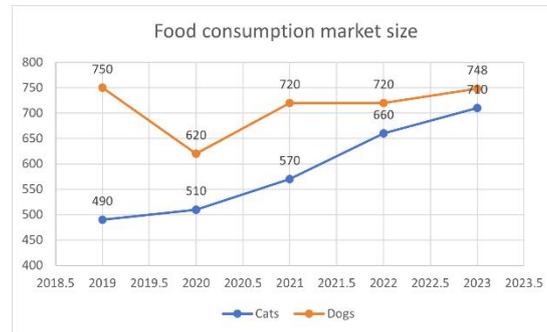
Figure 3. Consumption Market Size of Pet Food for Dogs and Cats

From Table 1-Table 2, Figure 1-Figure 3, it can be seen that the number of pet dogs and household penetration rate of China's pet industry in 2019-2023 as a whole shows a slow downward trend, which may be with the deep economic development in the past five years, the standard of living of the people and the quality of life continues to improve, but the growth rate slows down compared to the previous growth rate, or the number of pets in the era of epidemics plummeting, and other factors, which together lead to the number of pets and household penetration rate shows a Downward trend (Xie, 2024). In addition, from the pet consumption market size, pet single annual consumption amount and food consumption market size of the three indicators can be seen, China's pet industry in 2020 or by the economic development of the downward trend and the new crown epidemic during the epidemic control and other objective environmental factors superimposed on the people to raise pets to slow down the growth of the power of the impact of the decline and the short-lived fluctuation of decline (Qiu, 2024). However, the overall slow rise, especially the pet dog industry development data is particularly good, which suggests that the end of the epidemic era instead of bringing more potential pet emotional needs, etc. (ZHANG, WU, QIU, TIAN, & ZHANG, 2024).

To summarize, the development of China's pet industry in the past five years overall good rise, especially the rapid development of the pet dog industry, as evidenced by the expansion of the pet market scale, the growth of the amount of consumption of a single, as well as the growing scale of the consumer market for scientific food, but at the same time, there are also the number of pets and household penetration growth rate of the slowdown in the hidden dangers.

5.1.2 Modeling

The fuzzy comprehensive evaluation method is based on the affiliation theory of fuzzy mathematics, which accurately transforms the fuzzy and complex fixed-line evaluation factors into quantifiable indexes, and is capable of properly solving the fuzzy and difficult-to-quantify analytical and evaluation problems, while taking into account the interplay between multiple factors (WU, SHEN, & JIN, 2024). In addition, on the basis of the fuzzy comprehensive evaluation method, the comprehensive evaluation system is constructed to form the decomposition elements related to the decision-making factors, which can enhance the universality and rationality of the evaluation results (Xiong, n.d.). The development factors of China's pet industry, such as policy factors and economic factors, are characterized by

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qualitative indicators, which are suitable for the use of fuzzy comprehensive evaluation method scenario. Based on this, we consider applying the fuzzy comprehensive evaluation method to analyze the development factors of China's pet industry, and measure the weights of each indicator by using the expected value method, to comprehensively evaluate the development of its industry. On this basis, due to the small sample size of the data, the known information is more scarce, and the seasonality is weak, the gray prediction model is constructed to predict the development of China's pet industry in the next three years, and the process of establishing the specific model is as follows:

1) The establishment of a comprehensive evaluation index system for the development of China's pet industry:

Considering that the development of China's pet industry is affected by multiple factors, mainly including economic factors, social factors, policy factors and scientific and technological factors. Economically, the continuous improvement of the level of economic development is the cornerstone of the development and prosperity of the pet industry, with the improvement of people's living standards and the deepening of the urbanization process, the disposable income of the residents also continue to improve, more and more the pursuit of quality of life, boosting the continuous development of the pet economy. In terms of society and policy, changes in family structure, aging, and policy support make people's demand for pets continue to grow. In terms of science and technology, the continuous development of high and new technology brings new growth momentum for the prosperity of the pet economy and broadens the market space of the pet industry. Based on this, a comprehensive evaluation index system for the development of China's pet industry is constructed. The finally established comprehensive evaluation index system and related data are shown in Table 3.

Table 3. Comprehensive Evaluation Index System for the Development of China's pet Industry

Target level	Guideline layer 1	Guideline layer 2	Guideline layer 3
Development of Pet Industry in China$A$	Economic factorB_1	Level of economic developmentC_1	GDP per capita of prefecture-level cities in ChinaD_1
		People's standard of livingC_2	Consumption expenditure per urban residentD_2
			Per capita disposable income of urban residentsD_3
		Urbanization processC_3	Land area for urban constructionD_4
			Urbanization rate of prefecture-level citiesD_5
	Social factorB_2	People's love for petsC_4	China Pet Funeral Industry Market SizeD_6
			Number of enterprises in China's funeral industry (households)D_7
		SocioculturalC_5	Scale of educationD_8
		The aging of Chinese societyC_6	Number of elderly people aged 65 years and above as a proportion of the total population of the countryD_9

	Family structureC_7	Marriage rateD_{10}
Policy factorsB_3	Economic policyC_8	Amount of local government Special bondsD_{11}
		Size of the fiscal deficitD_{12}
	Pet Management PolicyC_9	Number of new veterinary drugs approved by the Ministry of Rural Affairs of China in the yearD_{13}
Technological and scientific factorsB_4	Technological innovationC_{10}	Number of patents granted for inventionsD_{14}

Through China Statistical Yearbook, China Pet Industry Research Report, China Pet Industry White Paper, and Interface News, we collect the cities in China in Table 3 for 2019-2023 D_1 - D_{14} The corresponding data were summed and organized by year, and the results are shown in Table 4.

Table 4. Data Table of Comprehensive Evaluation Index System for Pet Industry Development in China

	2019	2020	2021	2022	2023
D_1	61487.4	63329.32	69028.69828	75072.685	82637.5
D_2	21559	21210	24100	24538	26796
D_3	30733	32189	35128	36883	39218
D_4	41732	40015	42094	43743	44831
D_5	59.0722	61.2066	62.0458	63.2822	65.1803
D_6	862	1012	1252	1433	1820
D_7	998	1315	1685	2630	4459
D_8	948.6	1105	1242.4	1402.2	1553.7
D_9	12.6	13.5	14.2	14.9	15.4
D_{10}	6.6	8.1	7.5	4.8	4.1
D_{11}	2150	3750	3650	3650	3800
D_{12}	2760	3760	3570	3370	3880
D_{13}	95	70	75	78	89
D_{14}	453000	530000	696000	798000	921000

2) Calculation steps:

(1) Judgment of the development of the pet industry based on the fuzzy comprehensive evaluation method:

This paper utilizes the fuzzy comprehensive evaluation method to calculate the evaluation score of China's pet industry development, and its specific steps are shown in Figure 4.
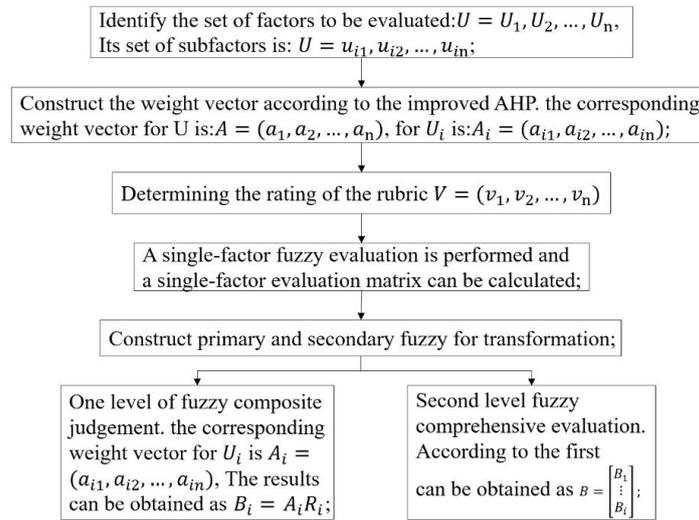
Identify the set of factors to be evaluated: $U = U_1, U_2, \ldots, U_n$, Its set of subfactors is: $U = u_{i1}, u_{i2}, \ldots, u_{in}$;

Construct the weight vector according to the improved AHP. the corresponding weight vector for U is: $A = (a_1, a_2, \ldots, a_n)$, for U_i is: $A_i = (a_{i1}, a_{i2}, \ldots, a_{in})$;

Determining the rating of the rubric $V = (v_1, v_2, \ldots, v_n)$

A single-factor fuzzy evaluation is performed and a single-factor evaluation matrix can be calculated;

Construct primary and secondary fuzzy for transformation;

One level of fuzzy composite judgement. the corresponding weight vector for U_i is $A_i = (a_{i1}, a_{i2}, \ldots, a_{in})$, The results can be obtained as $B_i = A_i R_i$;

Second level fuzzy comprehensive evaluation. According to the first can be obtained as $B = \begin{bmatrix} B_1 \\ \vdots \\ B_i \end{bmatrix}$;

Figure 4. Step-by-step Diagram of the Fuzzy Synthesis Evaluation Method

(2) Calculation of the forecast results of the development of the pet industry based on the gray forecasting model (SHANG, WANG, SHENG, & HU, 2024):

①Set the original sequence:

$$X^{(0)} = \{X^{(0)}(1), X^{(0)}(2), \ldots X^{(0)}(n)\} \tag{1}$$

②Accumulate the original sequence once to obtain a new sequence:

$$X^{(1)}(k) = \sum_{i=1}^{k} X^{(0)}(i), k = 1, 2, \ldots, n \tag{2}$$

③Establish the first-order linear minimization equations for the gray prediction model:

$$\frac{dX^{(1)}}{dt} + \alpha X^{(1)} = u \tag{3}$$

where denotes the development coefficient, and is the gray role quantity.

④Solve for the estimated parameters using the least squares method $\hat{\beta}$

$$\hat{\beta} = (B^T B)^{-1} B^T Y \tag{4}$$

⑤Restore the original data sequence

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1), k = 2, 3, \ldots, n, \hat{x}^{(0)}(1) = \hat{x}^{(1)}(1) \tag{5}$$

⑥Perform a posteriori difference tests

$$C = \sqrt{\frac{\frac{1}{n}\sum_{k=1}^{n}(x^{(0)}(k) - \bar{x})^2}{\frac{1}{n}\sum_{k=1}^{n}(\varepsilon(k) - \bar{\varepsilon})^2}} \tag{6}$$

5.1.3 Solving the Model

Firstly, based on the expected value method, the weights of the indicators of guideline layer 1, guideline layer 2 and guideline layer 3 are measured in turn, and the final weights of all indicators are obtained, and the results are shown in Table 5.

Table 5. Comprehensive Evaluation Index Weighting Table for China's Pet Industry Development

Target level	Standardized layer			Weights
	Guideline Layer 1	Guideline layer 2	Guideline layer 3	
China Pet Industry Development	Economic factor	Level of economic development	GDP per capita of prefecture-level cities in China	0.167
		People's standard of living	Consumption expenditure per urban resident	0.083
			Per capita disposable income of urban residents	0.083
		Urbanization process	Land area for urban construction	0.083
			Urbanization rate of prefecture-level cities	0.083
	Social factor	People's love for pets	China Pet Funeral Industry Market Size	0.038
			Number of enterprises in China's funeral industry (households)	0.038
		Sociocultural	Scale of education	0.075
		The aging of Chinese society	Number of elderly people aged 65 years and above as a proportion of the total population of the country	0.075
		Family structure	marriage rate	0.075
	Policy factors	Economic policy	Amount of local government special bonds	0.038
			Size of the fiscal deficit	0.038
		Pet Management Policy	Number of new veterinary drugs approved by the Ministry of Rural Affairs of China in the year	0.075
	Technological and scientific factors	Technological innovation	Number of patents granted for inventions	0.050

Secondly, the weight values of the indicators in Table 5 were substituted into Figure 4, and the fuzzy evaluation method was adopted for the indicators in the comprehensive evaluation index system for the development of China's pet industry to obtain the comprehensive evaluation scores for each year of 2019-2023, and the codes are shown in Appendix 1, and the results are shown in Table 6 below.

Table 6. Comprehensive Evaluation Score of China's pet Industry Development, 2019-2023

	2019	2020	2021	2022	2023
China Pet Industry Development Evaluation Score	1.00	12.50	25.00	37.50	50.00

Again, the data in Tables 1-2 and 6 were substituted into Eqs. (1) (2) (3) (6) to calculate the parameters required for the gray prediction model, and the results are shown in Table 7, and the codes are shown in Appendices A-E.

Table 7. Parameter Results for the First Order Linear Minimization Equation

	Development factor α	Gray quantity of action (math.)u	A posteriori difference ratio value (of an integer)
Comprehensive Evaluation Score of China's Pet Industry Development	-0.1361	37970.5147	0.0098
Number of pets in China's cities and towns (millions)	-0.058	94.439	0.043
Penetration rate of pet households in China (%)	0	31.955	0.143
Consumer market size in China (millions of dollars)	-0.067	113909.452	0.208
China Single Consumption Amount	-0.05	3873.557	0.097
China Food Consumption Market Size	-0.081	1013.506	0.041

As can be seen from Table 7, the a posteriori difference ratios of the comprehensive evaluation score of China's pet industry development, the number of pets in Chinese cities and towns (millions of pets), the penetration rate of pet households in China (%), China's consumer market size (millions of yuan), China's single consumption amount and China's food consumption market size indicators are all less than 0.35, indicating that the accuracy of the gray model is more reasonable. C values are all less than 0.35, indicating that the gray model has high precision and the results obtained from the solution have a large degree of rationality.

Substituting the data in Table 4 and Table 7 into Equation (5), a gray prediction model was used to predict the data on the development status of China's pet industry (including the number of pets in Chinese cities and towns, the penetration rate of pet households in China, the size of China's consumer

market, the amount of consumption of a single pet in China, the size of China's food consumption market, and the annual comprehensive score of the industry's development) for the next three years from 2024-2026, and the results are shown in Table 8, the The visualization charts are shown in Table 5, and the codes are shown in Attachmentes A-E:

Table 8. China Pet Industry Development Gray Forecast Result 2019-2023

|	Comprehensive Evaluation Score of China's Pet Industry Development	Number of pets in China's cities and towns (millions)	Penetration rate of pet households in China (%)	Consumer market size in China (millions of dollars)	China Single Consumption Amount	China Food Consumption Market Size
2019	1.00	99.15	32.90	125180.00	4029.00	1240.00
2020	12.50	100.84	32.10	118984.00	4080.00	1130.00
2021	25.00	112.35	31.80	144060.00	4460.00	1290.00
2022	37.50	116.55	31.70	148731.00	4765.00	1380.00
2023	50.00	121.55	32.10	150105.00	4745.00	1458.00
2024	62.50	125.02	31.50	173637.26	4871.45	1469.79
2025	75.00	139.27	31.21	210109.43	5324.86	1677.58
2026	87.50	155.15	30.92	254242.52	5820.46	1914.75

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Figure 5. Comprehensive Evaluation Score (left) and Number of Pets in Towns and Cities (right) Forecast for China's Pet Industry Development, 2019-2023

Figure 6. Forecast of Household Penetration (Left) and Consumer Market Size (Right) in China's Pet Industry, 2019-2023
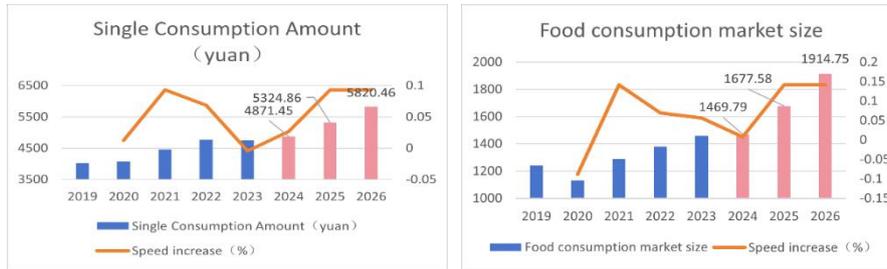
Figure 7. Forecast of China's Pet Industry Consumption Amount per Unit (Left) and Food Consumption Market Size (Right), 2019-2023

From the above Table 8 and Figure 5 - Figure 7 can be seen, the next three years the development of China's pet industry as a whole, the development trend is good, showing a steady upward trend, but the growth rate is declining or even tend to stagnate. Specific situation analysis, the number of pets in China's cities and towns, China's consumer market size, China's single consumption amount and China's food consumption market size in the next three years are showing a steady upward trend, and China's consumer market size is the fastest growing. While China's pet household penetration rate, although briefly rebounded in 2023, the next three years show a decreasing trend of change year by year.

5.2 Modelling and Solving Problem 2

5.2.1 Data Pre-processing

Considering that Attachment 2 only gives data on the number of pet dogs and cats in overseas countries such as Europe and America, including Germany, France and the United States, the amount of data is relatively small, which is not conducive to the establishment of the subsequent model and the reliability of the model solving results, this paper selects data such as the global sales of pet food, the global revenue growth rate of the pet food market, the average price of the global pet food, the global market size of the pet food, and the global sales of pet food to assist in modeling, in order to improve the reasonableness of the model solving results. Pet food sales and other data to assist in modeling, in order to improve the reasonableness of the model solving results. In addition, in the process of data collection, there is a general situation of missing data in some of the earlier years, so only the valid data for a total of six years from 2018 to 2023 are collected, see Table 9.

Table 9. Table of Data Preprocessing Results for Question Two

Particular year	Global pet food sales (in billions of dollars)	Global Pet Food Market Revenue Growth Change (%)	Global average price of pet food (USD)	Global pet food market size (billion kilograms)	Global pet food sales (billions of dollars)
2018	91.1	2.15	2.15	41.43	91.10

| 2019 | 93.9 | 6.15 | 2.26 | 43.61 | 93.90 |
2020	102.6	11.72	2.4	46.62	102.60
2021	114.37	11.47	2.55	47.39	114.37
2022	123.6	15.69	2.7	49.53	123.60
2023	133.9	12.15	2.79	51.44	133.90

5.2.2 Global Pet Industry Development Analysis

In view of the large number of countries around the world and the large differences in the development of their respective pet industries, it is not practical to comprehensively consider the development data of the pet industry in all countries. Based on this, this paper selects three typical and representative overseas countries, namely France, Germany and the United States, as the main research objects to analyze the development of the global pet industry, and the specific analysis results are as follows:

(1) Analysis of the overall development of the global pet industry by pets

Summing the data for the three countries in Attachment 2 by year, the results are shown in Table 10, and visualizing this result, the overall development of the global pet industry by pet can be obtained, see Figure 8.

Table 10. Table of Data Preprocessing Results for Question Two

|	2019	2020	2021	2022	2023
Dogs	10720	10345	10750	10790	10050
Cats	12190	9560	12600	10390	10610
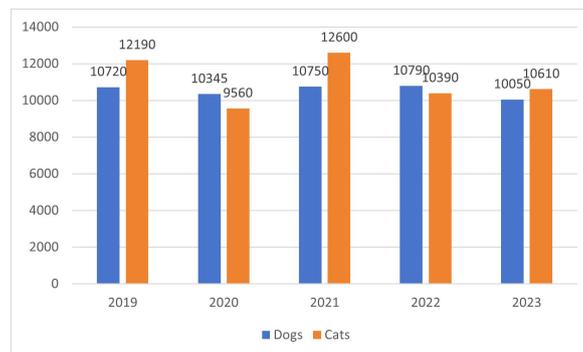
Figure 8. Overall Development of Global pet Industry by Pets

From Table 10 and Figure 8, it can be concluded that, from the global overall perspective, the development of the pet dog industry in the past five years has tended to flatten out, and the growth rate of industry development has tended to stagnate, while the development status of the pet cat industry has been more volatile.

(2) Global Pet Industry Segment Market Development by Pet Analysis

Visualizing the data in Attachment 2 by country gives the overall development of the global pet industry by pet in France, Germany and the US, as shown in Figure 9.
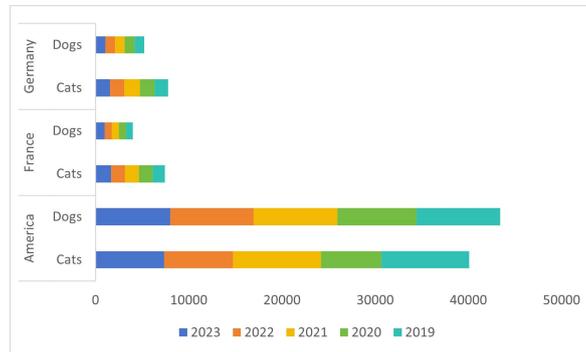
Figure 9. Global Pet Industry Segment Development by Pet in France, Germany and the United States

As can be seen from the above chart, the United States pet dog economic development is particularly prosperous, while the pet cat economy is in a period of recession, France, Germany and other countries in the overall development of the pet industry is more balanced, but compared with the United States in the existence of a large backward trend.

5.2.3 Modeling

This question of this paper requires that on the basis of the first question, the development of the pet industry in Europe and other countries in other regions is considered comprehensively and the global demand for pet food in the next three years is predicted, so this paper chooses to utilize the global pet food sales, global pet food market revenue growth rate, global average price of pet food, and the global pet food market size as the evaluation indicators , and adopts the entropy weighting-TOPSIS method to derive the weights and comprehensive evaluation results of each evaluation indicator. and then constructs a multiple linear regression model to characterize the global pet food demand in the next three years. The entropy weight-TOPSIS method is used to derive the weights and comprehensive evaluation results of each evaluation index. then a multiple linear regression model is constructed to characterize the global demand for pet food with the global pet food sales volume, and to make a prediction for the next three years. The specific modeling process is as follows:

1) Data standardization:

As the indicators selected in this paper have different scales, and except for the global average price of pet food, which is a negative indicator, the rest of the indicators are positive indicators, which will have a certain degree of impact on the comprehensive evaluation results. Therefore, firstly, the negative indicators are positively processed, and secondly, all the related indicators are standardized accordingly to eliminate the influence of the quantitative outline and enhance the rationality and reliability of the comprehensive evaluation results, and the standardized processing formula is as follows:

$$r_{ij}=\frac{X_{ij}-X_{min}}{X_{man}-X_{min}} \tag{7}$$

2) Measurement of the weights of indicators related to the demand for food in the pet industry based on the entropy weight method

(1) Construct a decision matrix:

$$A=(a_{ij})_{m\times n} \tag{8}$$

Where m is the number of samples , n is the number of indicators, n=5 in this question, corresponding to global pet food sales, global pet food market revenue growth rate, global pet food average price, global pet food market size, and global pet food sales data.

(2) Calculate the entropy value of the entropy value of the indicator e_j :

The entropy valuee_j It can reflect the number of$j$ information entropy of the indicator, which indicates the amount of information the indicator can provide.

$$e_j=-k\sum_{i=1}^{m}p_{ij}\ln p_{ij} \tag{9}$$

(3) Calculate the information utility value of the information utility value of the indicator :

$$d_j=1-e_j \tag{10}$$

(4) Calculate the weight of the$j$ weight of the indicator w_j

$$w_j=\frac{1-e_j}{\sum_{j=1}^{n}(1-e_j)} \tag{11}$$

3) Evaluation and analysis of food demand in the pet industry based on the TOPSIS method

(1) Calculate the positive ideal solution and the negative ideal solution:

$$A_j^+=max(a_{ij}),A_j^-=min(a_{ij}) \tag{12}$$

(2) Calculate the distance of each indicator from the positive and negative ideal solutions:

$$d_i^+=\sqrt{\sum_{j=1}^{n}w_j(a_{ij}-a_j^+)^2},d_i^-=\sqrt{\sum_{j=1}^{n}w_j(a_{ij}-a_j^-)^2} \tag{13}$$

(3) Calculate the closeness:

$$C_i=\frac{d_i^-}{d_i^++d_i^-} \tag{14}$$

4) Forecasting the demand for food in the pet industry in the next three years based on multiple linear regression models

$$Y=\mu_0+\mu_1C_1+\mu_2C_2+\cdots+\mu_nC_n+\varepsilon \tag{15}$$

Of which is the demand for food in the pet industry, and C_i is the posting progress of the corresponding indicator, and μ_i is the regression coefficient, and ε is the error term.

5.2.4 Solving the model

1) Solving the entropy weight-TOPSIS method

The data in Table 9 were brought into Eqs. (9)-(11), and the weights, positive and negative ideal

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solution distances, and relative proximity of each metric were calculated sequentially, and the results are shown in Table 11, and the codes are shown in Appendix 2 and Attachment M.

Table 11. Summary of the Results of the Entropy Method of Calculating Weights

Term (in a mathematical formula)	The information entropy value e	Information utility value d	Weighting factor w
Global Pet Food Sales (in billions of dollars)	0.9945	0.0055	6.67%
Global Pet Food Market Revenue Growth Change (%)	0.9319	0.0681	82.03%
Global average pet food price (USD)	0.9976	0.0024	2.87%
Global pet food market size (billion kilograms)	0.9985	0.0015	1.77%
Global pet food sales (billions of dollars)	0.9945	0.0055	6.67%

The data in Table 9 were substituted into equations (12)-(14) to evaluate and analyze the demand for food in the pet industry using the TOPSIS method, and the results are shown in Table 12, and the codes are shown in Appendix2 and Attachment M.

Table 12. Calculated results of TOPSIS evaluation

Term (in a mathematical formula)	Positive ideal solution distance D+	Negative ideal solution distance D-	Relative proximity C	Sorting results
2018	11.818	0.000	0.000	6
2019	8.688	3.292	0.275	5
2020	4.396	7.925	0.643	4
2021	3.922	7.954	0.670	3
2022	0.972	11.522	0.922	1
2023	2.904	9.144	0.759	2

Among them, both positive and negative ideal distance indicate the distance between the evaluation object and the optimal program, while relative proximity indicates the proximity between the evaluation object and the optimal program, and the larger the value indicates that its score is about ideal. Therefore, as shown in Table 12, the result of food demand in the pet industry from 2018-2023 is ideal, which verifies the applicability of the TOPSIS evaluation method to this problem.

(2) Global Pet Food Demand Forecast for the Next Three Years

First, the data from Tables 9 and 11 were substituted into Eqs. (1)-(6) to predict the overall development of food demand in 2024-2026 using a grey forecasting model, the results of which are

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shown in Table 13, and visualized graphically in Figure 10.

**Table 13. Table of Results of Gray Projections of the Overall Development of the Demand for Foodstuffs**

	2018	2019	2020	2021	2022	2023	2024	2025	2026
Relative proximityC	0	0.275	0.643	0.67	0.922	0.759	1.04	1.19	1.35
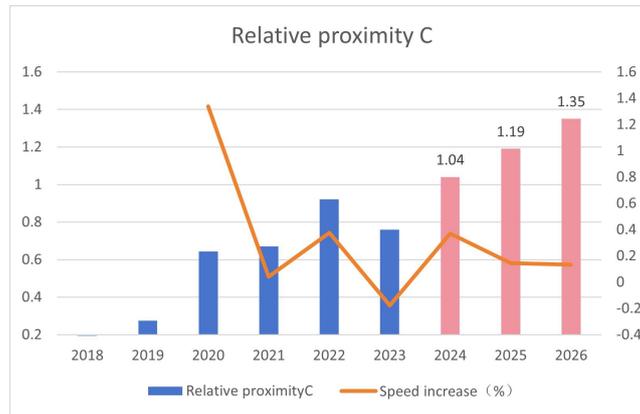
Figure 10. Global Pet Industry Segmentation Development by Pet

As can be seen from Table 13 and Figure 10, the global demand for pet food in the next three years shows a steady upward trend, and the volatility is significantly reduced compared with previous years, but the growth rate is more volatile.

Substituting the data in Table 9 into Equation (15), a multiple linear regression of pet food demand was performed and the specific development of global pet food demand over the next three years was predicted, with the results shown in Figure 11 and the code in Appendix 4.
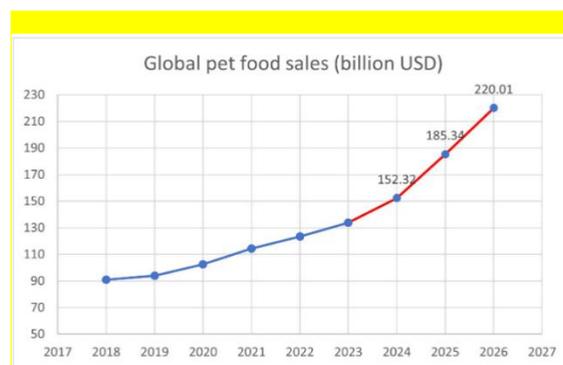
Figure 11. Forecast Results of Global Pet Food Demand for the Next Three Years

As can be seen in Figure 11, the projected global demand for pet food for the next three years from 2024-2026 results in US$ 152.32 bn, US$ 18.534 bn, and US$ 220.01 bn, respectively.

5.3 Modelling and Solving Problem 3

5.3.1 Modeling

Question 3 asked for a forecast of China's pet food production and export volume for the next three years without taking into account economic policy conditions, utilizing the data on China's pet food production and export volume in Attachment 3 and taking into account the trend of changes in market demand in the global pet food industry and the development of China's domestic pet industry.

First of all, in order to enhance the documentary and accuracy of the forecast results, all the data collected in the first two questions are considered as the components and influence indicators of the production and export volume of the pet food industry. Given that the above indicators, such as the global average price of pet food, global pet food market size, global pet food sales and the number of pets in Chinese cities and towns, have a strong correlation with each other, that is, there is a problem of multiple covariance, and the use of multiple linear regression model to solve the parameters will lead to a large error in relation to the actual results, so we choose to construct the ridge regression equation to get the production and export volume of the pet food industry. The model parameters of each constituent element, the specific modeling formula is as follows:

$$\hat{\beta}(k)=(X^TX+kI)^{-1}X^TY \tag{16}$$

Secondly, the gray prediction model is constructed again to predict the specific values of the components and influencing indicators of the production and export volume of the pet food industry in the next three years.

Finally, the predicted values are substituted into the ridge regression equation obtained from the model construction to obtain the output and export volume of Chinese pet food in the next three years under the influence of ignoring economic policies.

5.3.2 Solving the Model

1) Solve the ridge regression equation using SPSS

The data in Tables 1-2 and 9 were substituted into Equation (16) and the ridge regression equation was solved to obtain the unstandardized coefficients and yield expressions for each of the constituent indicators, the results of which are shown in Table 14, and the codes are given in Attachment N.

Table 14. Table of Results of Ridge Regression Analysis of Yield

K=0.091	Non-standardized coefficient	Standardized coefficient	t	P	R²
a constant (math.)	-7547.455	-		NaN	
Global Pet Food Sales (in billions of dollars)	15.508	0.271		NaN	
Global Pet Food Market Revenue Growth Change (%)	-89.507	-0.334		NaN	0.98
Global average pet food price (USD)	764.82	0.18		NaN	

Global pet food market size (billion kilograms)	89.647	0.291	NaN
China Food Consumption Market Size	0.047	0.007	NaN
China Single Consumption Amount	-0.24	-0.092	NaN
Penetration rate of pet households in China (%)	-37.328	-0.019	NaN
Global pet food sales (billions of dollars)	15.508	0.271	NaN
Consumer market size in China (millions of dollars)	0.001	0.021	NaN
Number of pets in China's cities and towns (millions)	20.395	0.218	NaN

Note. ***, **, * represent 1%, 5%, and 10% significance levels, respectively.

As can be seen in Table 14, the results of the analysis of ridge regression showed that the P-value of the F-test for significance was NaN, indicating the elimination of the disturbing factor of multiple covariance. and the R² was 0.98, indicating a good model fit. And the final yield calculation formula was obtained as:

Production = -7547.455 + 15.508 x global pet food sales - 89.507 x change in revenue growth in the global pet food market + 764.82 x average global pet food price + 89.647 x global pet food market size + 0.047 x Chinese food consumption market size - 0.24 x China's single consumer amount - 37.328 x China's pet household penetration rate + 15.508 x global pet food sales + 0.001 x China consumer market size + 20.395 x number of pets in Chinese cities and towns.

Substituting the data in Tables 1-2 and 9 into Equation (16) and solving the ridge regression equation, the unstandardized coefficients of the constituent indicators and the expression of the export volume were obtained, as shown in Table 15.

Table 15. Results of the Ridge Regression Analysis of Export Volume

K=0.091	Non-standardized coefficient	Standardized coefficient	t P	R²
a constant (math.)	-2525.549	-	NaN	
Global Pet Food Sales (in billions of dollars)	-0.557	-0.147	NaN	0.972
Global Pet Food Market Revenue Growth Change (%)	-1.422	-0.08	NaN	

Global average pet food price (USD)	-32.443	-0.116	NaN
Global pet food market size (billion kilograms)	-4.8	-0.236	NaN
China Food Consumption Market Size	-0.557	-0.147	NaN
China Single Consumption Amount	0.03	0.177	NaN
Penetration rate of pet households in China (%)	-0.521	-0.084	NaN
Global pet food sales (billions of dollars)	80.947	0.631	NaN
Consumer market size in China (millions of dollars)	0.22	0.461	NaN
Number of pets in China's cities and towns (millions)	0	0.111	NaN

Note. ***, **, * represent 1%, 5%, and 10% significance levels, respectively.

As can be seen from Table 15, the analytical results of ridge regression show that the p-value of F-test significance is still NaN, indicating that the interference factor of multiple covariance is eliminated once again. and the R^2 is 0.972, reflecting a good model fit. And the final export volume was obtained as:

Exports = -2525.549 - 0.557 x global pet food sales - 1.422 x change in revenue growth in the global pet food market - 32.443 x global average pet food price - 4.8 x global pet food market size - 0.557 x global pet food sales volume + 0.03 x China's per-unit consumption amount - 0.521 x number of pets in China's cities and towns + 80.947×Pet household penetration rate in China + 0.22×Chinese food consumption market size + 0.0×Chinese consumption market size (million dollars).

(2) Forecast of China's pet industry production and export volume in the next three years

Substituting the data in Tables 1-2 and 9 into Equations (1)-(6), a gray forecast model was used to forecast the demand-related indicators of each Chinese pet food industry for the years 2024-2026, and the results are shown in Table 16, and the codes are shown in Attachmentes A-J.

Table 16. Table of Results of Solving the Independent Variable Gray Prediction Model

	2024	2025	2026
Global Pet Food Sales (in billions of dollars)	125.02	139.27	155.15
Global Pet Food Market Revenue Growth Change (%)	11.22	10.99	10.75

Global average pet food price (USD)	2.71	2.88	3.06
Global pet food market size (billion kilograms)	48.17	48.97	49.78
Global pet food sales (billions of dollars)	127.33	141.92	158.19
Number of pets in China's cities and towns (millions)	125.02	139.27	155.15
Penetration rate of pet households in China (%)	31.50	31.21	30.92
Consumer market size in China (millions of dollars)	173637.26	210109.43	254242.52
China Single Consumption Amount	4871.45	5324.86	5820.46
China Food Consumption Market Size	1469.79	1677.58	1914.75

Secondly, based on the expressions of production and export volume obtained from the ridge regression analysis, the forecasts of production and export volume of China's pet industry in the next three years are calculated, and the forecast results are shown in the table below:

Table 17. Forecast Results of China's Pet Industry Production and Exports in the Next Three Years

	Production (billions of pieces)	Exports (billions of pieces)
2024	2901.61	15.09
2025	5989.62	18.77
2026	12364.02	23.35

5.4 Modelling and solving Problem 4

5.4.1 Modeling

This question of this paper requires quantitative analysis of the impact of foreign economic policies of European and American countries on China's pet food industry on the basis of the third question. Considering that the impact of foreign economic policies is also relatively vague and difficult to quantify, it is considered to utilize the fuzzy comprehensive evaluation method again to measure the impact of the economic policies of Europe and the United States on China's pet food industry.

5.4.2 Solving the model

Using the fuzzy comprehensive evaluation model, the relevant data of the foreign economic policies of Europe and the United States are substituted into the solving steps of Figure 4 to score the impact of the foreign economic policies of Europe and the United States on the pet food industry, and the results are shown in Table 18.

Table 18. Impact Score of Foreign Economic Policies of European and American Countries on Pet Food Industry

	foreign trade	external investment	monetary policy	tariff policy
Impact Score of Foreign Economic Policies on Pet Food Industry	46.45	51.22	16.67	37.78
Sorting results	2	1	4	3

From Table 18 and the results of previous calculations, it can be seen that the foreign investment policy of Europe and the United States has the greatest impact on China's pet industry, followed by foreign trade policy, tariff policy has less impact on it, and monetary policy has basically no impact on the development of China's pet industry.

Based on this, in order to promote the sustainable development of China's pet food industry, first of all, it is necessary to deal with the inhibiting effect of the new foreign economic policies of Europe and the United States on the development of the industry. In response to the foreign investment policy of Europe and the United States, it is necessary to adhere to the development strategy of scientific and technological innovation, enhance independent innovation and product research and development investment, and build a diversified network of partnerships, broaden financing channels, such as Chinese pet food distributors can strengthen the supply chain management and other ways to reduce the impact of the European and American economic embargo on them. In coping with the foreign trade policies of Europe and the United States, it is necessary to optimize the product structure, expand the market share, make full use of the high-tech background, implement the product differentiation strategy, and continuously promote product innovation. In response to the European and American tariff policy, it is necessary to strengthen cost control, enhance production efficiency and improve product competitiveness. Secondly, based on the trend of a steady rise in global demand for pet food and expanding growth rate in the next three years, China's pet food industry needs to strengthen brand building and reputation building through enhanced brand promotion and brand image shaping, while exploring the integration of online and offline channel development strategy.

6. Model Assessment

6.1 Strengths

1. In the process of problem analysis, the fuzzy comprehensive evaluation model fully considers the fuzzy characteristics of the objectives sought by the topic and various influencing factors, synthesizes multiple factors, and evaluates the development of China's pet industry in a systematic and effective way. The variables are set rigorously and reasonably, the constraints are comprehensive, and the data not given in the title are supplemented by data collection and reasonable assumptions.

2. Based on the relevant information given, it has been effectively merged and processed in a way that is actionable and facilitates relevant analysis.

3. In the process of model building, facing the lack of data, we not only chose the gray prediction model to accurately predict the development trend of the pet industry, but also clearly demonstrated the market dynamics through visualization, and used the ridge regression model to deal with the problem of multiple covariance, and stabilized the estimation of the parameters, effectively predicting the production and export volume.

4. When analyzing the results of the model, not only interpret the results, but also think and plan rationally and practically in relation to practical situations and real-life connections.

6.2 Weaknesses

1. Due to the comprehensive and complex model parameter settings, involving multiple aspects such as fuzzy comprehensive evaluation, grey prediction, ridge regression and multiple linear regression, the model is difficult to further optimize.

2. As the model assumes that the weight of the relationship between the factors obtained is known, according to real-life assumptions and thinking, affected by a variety of factors and some of the factors of the data is difficult to obtain, reducing the accuracy of the results.

6.3 Improvements

1. The model in question (2) is more limited in the selection of influencing factors. Consideration can be given to introducing more relevant factors, such as socio-economic indicators and changes in consumer preferences, in order to conduct a more comprehensive and integrated evaluation and to improve the model's forecasting accuracy and applicability.

2. More advanced statistical methods and machine learning techniques, such as cross-validation and sensitivity analysis, can be used in questions (3), (4) and (5) to rationally test the model results in order to improve the accuracy and reliability of the results.

6.4 Extension

The fuzzy comprehensive evaluation model and gray prediction model constructed in this paper can be extended to market evaluation and prediction in other industries, such as retailing and manufacturing, to provide decision support for more industries. The ridge regression and multiple linear regression models can be applied to policy effect evaluation and market trend analysis to help the government and enterprises develop more effective strategies.

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