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Estimating Bed Requirements for a Pediatric Department in a University Hospital in Egypt

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Abstract

Every day, a considerable number of children in need for health monitoring and control are turned away because of lack of beds in the Pediatric department in Zagazig University hospital in Egypt. This paper estimates the required number of beds needed for controlling this number of turned away children. The paper also investigates the effect of redistributing beds among different specialties on the service level. An Erlang Loss model is applied for estimating required capacity, then an optimization model is used for finding the optimum bed distribution that minimize number of turned away children.

Keywords

hospital bed planning, queuing theory, erlangloss models

1. Introduction

Many of the governmental Egyptian hospitals face the problem of high demand and limited resources. The reason is usually that there are no enough funds for extending current capacity when extension is required, but this is not always the only reason for this imbalance between demand and capacity. Sometimes lack of proper capacity planning leads to decisions in the wrong direction and here comes up the important role of Operational Research (OR) Sainfort (2001).

The Pediatric department at Zagazig University Hospital in Egypt suffers from these two problems. Like other governmental hospitals there are very limited funds that allow extension in capacity whenever needed and also there is a clear absence of scientific planning for capacity requirements and allocation. Hospitals capacity is usually expressed as number of operational beds. This paper estimates required capacity for each specialty in the Pediatric department and investigates the potential effect of redistributing beds among different locations on the service level expressed in number of turned away patients.

In the context of capacity planning, "design capacity" is the maximum amount of work that an organization is capable of completing in a given period, "effective capacity" is the maximum amount of work that an organization is capable of completing in a given period due to constraints such as quality problems, delays, material handling, etc. A discrepancy between the capacity and the demands results in inefficiency, either in under-utilized resources or unfulfilled demands. The goal of capacity planning is to minimize this discrepancy. Particularly, Hospital capacity planning is the process of determining the hospital capacity needed to meet changing demands for its Patients. Patients often experience serious delays due to highly variable patient demands and capacity constraints. Yet, hospitals are often unable to add capacity because of cost pressures, regulatory constraints, or a shortage of appropriate personnel. This makes it extremely important to use existing capacity most efficiently. Hospital bed planning, in particular, is an old problem and has been studied since the fiftieth of the twentieth century by Morgan (1950). For a thorough review on hospital capacity planning we refer to Smith-Daniel et al. (1988) and Green (2003/2004).

Target occupancy levels have been used during last decades as a performance measure to estimate required bed capacity following the assumption that it reflects the capacity levels that balance between cost, service level and resources utilization. But Green and Nguyen (2001) indicated that depending on Target Occupancy Level as the only performance measure is inadequate and leads to extra waiting times. Specially, Target occupancy Levels do not identify an accurate service level and when capacity levels are considered there must be a clear quantification of the desired service level. Lapierre et al. (1999) and Vissers (1994/1998) incorporate time series analysis of hourly census data in order to estimate bed demand and required capacity to fulfil expected demand. Based on Lapierre study, Murray (2005) uses a seasonality forecasting model instead of simulation to predict bed demand. Although the literature is so rich in the area of estimating hospital bed capacity, the incorporation of patient blocking as a performance metric is not widely used. Xie et al. (2007), Bretthauer et al. (2011) and Asaduzzaman et al. (2010) involve the probability of blocking patients in different contexts in order to estimate required bed capacity. Xie et al. (2007) developed a semi-open queuing model for computing expected number of blocked patients when all beds are occupied in a multi-stage geriatric department. Yet, they didn't consider the blocking effect between sequential stages. Bretthauer et al. (2011) studied the impact of implicit blocking between various stages in a hospital system in the USA on the probability of blocking new arrivals to the first stage. Asaduzzaman et al. (2010) involved the overflow probability along with refusal probability to determine required number of cots in a neonatal unit in the UK. Exact expressions were derived for calculating overflow and blocking probabilities. Their model results were validated and verified using observed data. However, overflow between units does not directly handle bed blocking. Typically, these studies make assumptions about arrival and service rates. Typically, these studies make assumptions about arrival and service distributions, arrival process was assumed to be Poisson and service times were exponential. However, sometimes the high variability in service times (patient length of stay) does not allow it to be fitted in a single exponential distribution and so the assumption of exponential service times may not be any longer valid and service times need to be modelled as a general distribution.

It has also to be noted that the literature about bed sizing in Pediatric departments is scarce; the only study in this context is the one by Kokangul (2008) for estimating required bed capacity in an intensive pediatric care unit.

Queuing theory was proved to be an accurate method for estimating required bed capacity especially in the presence of stochastic demand (McManus et al., 2004). For example, Adeleke et al. (2009) considered the waiting of patients in university health centers as a single channel queueing system with Poisson arrivals and exponential service time. Using an M/M/1 queueing system, they obtained the average number of patients and the average time spent by each patient as well as the probability of arrival of patients into the system. Laskowski et al. (2009) applied agent-based models and queueing models to investigate patient access and patient flow through Emergency Department. Li et al. (2009) employed queuing theory and multi-objective optimization for bed allocation in a hospital in China. Queuing model results are fed into a goal programming optimization algorithm to find a bed allocation that minimizes delay and increase patient flow. It was found that bed reallocation can result in overall profits increase and higher admission probability. A decision support system based on queueing theory to evaluate bed capacity in 24 clinical wards in a university medical center was developed by De Bruin et al. (2009). Data between 2004 and 2006 were analyzed. The system evaluated current bed capacity and investigated the significance of merging some departments. It was found that merging can have a great positive effect on the required number of beds for a given blocking percent. However, neither of the above mentioned models takes into account the dependent relation between patient blocking probabilities in different specialties. Especially in case if resources may be shared.

The main contribution of this paper is to determine the required capacity in a loss model with the assumption of exponentially distributed service times is relaxed. In this study we use Erlang Loss Model to represent patient flow. This model is insensitive to the service time distribution (De Bruin et al., 2009). The model assumes that patient length of stay follow a general distribution. First, we apply Erlang Loss Model to find the appropriate capacity level needed for a given service level. Then, an optimization model is applied to balance blocking probability among different Pediatric specialties with the existing capacity where no extension is available. Our results show that redistributing beds among specialties could result in high improvements in the service level. i.e., reducing overall blocking probability by 7%.

1.1 System Description and Data Collection

1.1.1 Patient Flow through Pediatric Department

The Pediatric department is a department at Zagazig University Hospital in Egypt. This is a Governmental hospital funded by Government and donors. The department contains 7 specialized wards: Kidney, Oncology, Nutrition, Chest, General, Gastrology and Cardiology. There are 160 beds in the department distributed over the 7 wards. The department suffers from high demand with limited resources which leads to high number of blocked patients due to beds unavailability. Patient flow through the department can be represented as a queue model which consists of entry point, service time and exit point (Bhattacharjee & Ray, 2014).

Patients enter the system either through emergency reception, or outpatient clinics. Emergency patients always have high priority in resources consumption and receive fast and good service. Once their health condition is stabilized, patients are discharged and may enter the system again through outpatient clinics if follow up is needed. Patients entering through outpatient clinics are first examined by a specialist who decides the next step for the patient. After consultation, patient either leaves the system (does not need to be admitted to the hospital), or directed to hospital reception (admission office) to be admitted to the hospital. At hospital reception, if there is available place (bed), the patient is admitted else this patient leaves the system as a blocked patient. In reality blocked patients are transferred unofficially to other hospitals that may have available capacity. A summary of the patient flow is displayed in Figure 1.

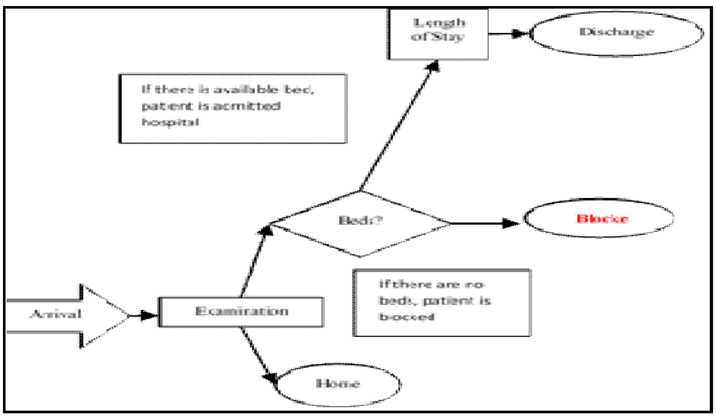

Figure1. Patient Flow through Pediatric Department

1.1.2 Data Collection and Flow

One of the main problems at Zagazig University Hospital is the absence of data records before admission to hospital (i.e., admission to outpatient clinic, how many referred to hospital, how many discharged from clinic, etc.). There is no accurate statistics about patients to be used for evaluating service level before admission to hospital. Most of data collected manually and through direct system observation. Patients register in clinic files before examination and receive an entrance ticket that accompanies them afterwards. Referred patients take their ticket to the hospital office to be admitted. If patient is admitted the ticket is replaced with an admittance file else the ticket is left at hospital reception (count of blocked patients). Table 1 represents collected data and source of each one. Data on arrival rates, referral rates and blocking rates are collected for 4 months starting from August 2014 till November 2014. Admissions and discharges data collected from hospital information system for admitted patients during the same 4 months.

Table 1. Data Sources

Data	Source
Number of Arrivals/Day	Clinics registration files
Number of Home/Day (do not need hospitalization)	Clinics registration files-Referred to hospital
Number of Referred to Hospital/Day	Number of tickets at hospital reception+Number of admitted patients
Number of Admitted/day	Hospital information system
Number of Blocked/day	Number of tickets at hospital reception

The remainder of this paper is organized as follows, in the next section, the models applied are detailed. Results are discussed in Section 3. Section 4 provides concluding remarks and future research directions.

2. Method

The current situation in Zagazig University Hospital, in general, and in the Paediatric department, in especial, necessitated developing models for examining and understanding resources requirements for the department.The models should provide hospital managers and decision makers with standards that they can use in capacity planning decisions.

2.1 Erlang Loss Model

In this section we represent the queuing model. Mind the concept of blocking patient introduced in section 1.1. We assume that if the referred patient found no available bed for admittance, then this patient is blocked.

Patients referred to hospital reception at random (unscheduled), so arrivals are assumed to follow a Poisson distribution with mean λ. Service time is represented by patient length of stay and it is assumed that each patient length of stay is independent and identically distributed with expected average μ_i for each specialized ward$i$. The service time distribution is assumed to be a general distribution. Beds are the servers. This model is known as an *M/G/c* model where, *M* represents Poisson arrivals, *G* represents general service times and c is number of servers. Probability of blocking patient in a specific ward can be computed using the formula:

$$\mathrm{P_c} = \frac{(\lambda\mu)^c/_{c!}}{\sum_{k=0}^{c}(\lambda\mu)^k/_{k!}} \tag{1}$$

Due to the expensive time complexity of the factorial function, a recursive function is used for computing the blocking probability. It is worth noting that this recursive function resulted in a run time reduction from days to minutes.

$$\mathrm{P_c} = \frac{\lambda\mu * P_{c-1}/_c}{1+\lambda\mu * P_{c-1}/_c} \tag{2}$$

We wish to find the minimum number of beds (c) that keeps this blocking probability under specific level. So we start with one bed (c=1) and blocking probability of 100% and then beds increased one by one until the desired blocking probability is reached. Different blocking levels have been applied and the results are discussed in section 4. The model results are validated against actual blocking data.

2.2 Optimisation Model

The second goal of this study is to investigate if there is a possibility of maximising utilisation of current available capacity in a way that reduces the blocking probability. With this aim in mind, we use an optimisation model that balances the distribution of beds among the 7 wards in the Paediatric department.

The model objective function is to minimise the Mean Absolute Deviation (MAD) of the blocking probability (P_{ci}) in each specialised ward from the average blocking probability in the whole department. Arrival rates are included in the objective function to incorporate arrivals effect on the blocking percentage. The beds will be redistributed to the 7 wards keeping total number of beds equals 160 (the current available beds). Decision variables, Objective function and constraints are as follows:

Decision Variables

$c_{i,}$ Where i=1,2,…,7 and c_i Number of beds at ward i,

Objective Function

$$Min \frac{1}{n}\sum_{i=1}^{n}\left|\lambda_i P_{c_i} - \bar{\lambda}\bar{P}\right|, Where \tag{3}$$

$$\mathrm{P}_{c_i} = \frac{\lambda\mu * P_{ci-1}/_{ci}}{1+\lambda\mu * P_{ci-1}/_{ci}} AND \bar{P} = \frac{1}{n}\sum_{i=1}^{n} P_{c_i}$$

Contraints

$$\sum_{i=1}^{n} c_i = 160 \tag{4}$$

It was noticed from the queuing model results that will be discussed in details in the next section that the blocking probability is not balanced among the 7 wards. We are here trying to find an optimal bed distribution that balances the blocking probability among all wards in the department. The new bed distribution will hopefully improve service level and decrease the blocking probability with the existing available resources. Model results are discussed in the next section.

3. Result

3.1 Queuing Model

We here describe the results of applying the Erlang Loss Model on the 7 specialized wards in the Paediatric department. Table 2 represents arrival rates, service times and bed capacities for the 7 wards. In Table 3 we present the required capacities needed for each ward to maintain a desired service level. For example, for keeping percentage of blocked patients in the kidney speciality under 10% we need to have at least 114 beds. We can see from this table, as we can expect, the more beds we have, the less patients we block. Figure 2 shows the relation between number of beds and blocking probability in the Chest ward.

Table 2. Main Service Features for Each Ward

	Arrival rate (λ)	Mean Service time (μ)	Current bed Capacity (c)
Kidney	6.23	19	33
Oncology	4.18	19.73	39
Cardiology	1.45	8.59	16
Chest	3.17	6.52	32
Gastrology	0.58	1.06	4
Nutrition	2.32	4.24	12
General	1.8	8.81	24

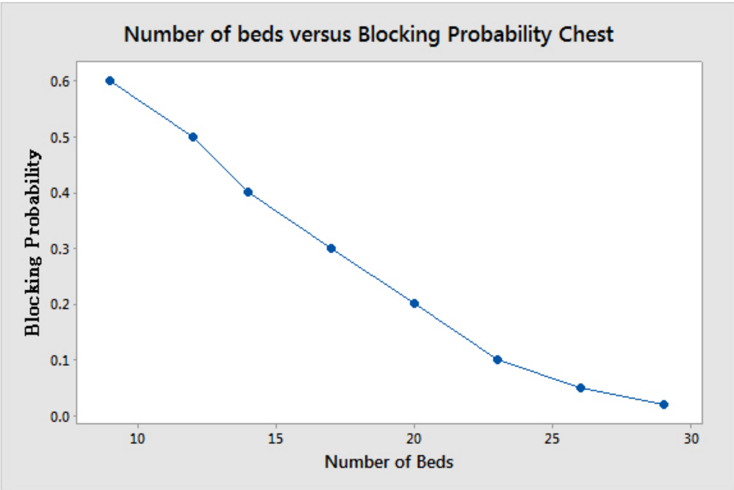

Figure 2. Relation between Number of Beds and Blocking Probability

Table 3. Minimum Number of Beds for Different Blocking Probabilities

	Number of required beds for							
	60% blocking	**50% blocking**	**40% blocking**	**30% blocking**	**20% blocking**	**10% blocking**	**5% blocking**	**2% blocking**
Kidney	49	61	73	86	99	114	123	132
Oncology	34	43	51	60	70	81	88	95
Cardiology	6	8	9	11	13	16	18	20
Chest	9	12	14	17	20	23	26	29
Gastrology	2	2	2	2	3	3	3	4
Nutrition	6	8	9	11	13	16	18	20
General	8	10	12	14	16	19	22	24

It was also noticed that not all of the Pediatric wards face the same problem. For example we see that the required capacity for blocking at most 5% of referred kidney patients is 123 beds with a need for extra 90 beds (more than the current 33 beds) while for keeping the same percentage in the Chest ward we need 26 beds with 6 beds less than we already have (32 beds currently). The relation between required and current capacity at each ward is illustrated in Figure 3. This result emphasized the previous notice of poorly planned capacity decisions.

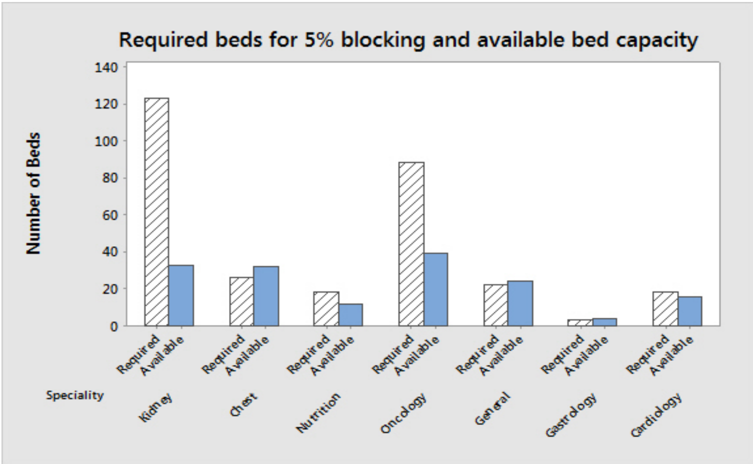

Figure 3. Required Beds Versus Available Beds

3.2 Bed Balancing

The results of applying the optimisation model are presented here. As the problem is relatively small and we want to find the exact optimal solution, a complete enumeration was done for all possible combinations. We implemented the model using MS Excel and Visual Basic for Applications (VBA). Table 4 represents the results of applying the optimisation model on the Paediatric department.

The model results indicate an overall improvement in the blocking probability for the whole department by 7%. It is also interesting to review how the new bed distribution improved the blocking probability in the three specialities Kidney, Nutrition and Cardiology by 3%, 9% and 4% respectively. We notice the bad effect on the Chest and General specialities (i.e., blocking increase by 6% and 3% respectively). Nevertheless, these blocking percentages are still acceptable.

Table 4. Blocking Probabilities for Optimal Bed Distribution

Objective Function Value (MAD)=0.68								
		Kidney	**Nutrition**	**Chest**	**Oncology**	**General**	**Gastro**	**Cardio**
Before	Beds	33	12	32	39	24	4	16
	Blocking	72%	12%	0%	52%	0%	0%	7%
After	Beds	37	15	25	39	22	4	18
	Blocking	69%	3%	6%	52%	3%	0%	3%
Blocking Improvement		3%	9%	-6%	0%	-3%	0%	4%
Overall improvement=7%								

4. Discussion

The methodology developed in this paper enables hospital managers and decision makers of making more informed decisions when planning for hospital bed capacity. The queuing model gives estimates of bed requirements for each specialized ward if there is available fund to be invested in extending the current hospital capacity. It provides a means of calculating the probability of blocking patients as a function of available beds. In order to apply this model we assumed that arrivals follow a Poisson process which reflects a steady state system. However, the model used in this paper is applicable to any length of stay distribution.Although this Erlang Loss Model may be criticized for its lack of other modelling techniques power like discrete event simulation for example, it still provides reliable results with very simple inputs such as average arrival rate and average length of stay.

The results of the queuing model revealed that some wards have high percentage of blocked patients while other wards have this percentage equal to 0. An optimisation model was developed for redistributing beds among the wards in a way that balances the service level. A limit was put on the number of beds assuming no extension is available. The optimisation results show that the new bed distribution leads the department to improved service levels with a decrease in the overall blocking probability by 7% with the same amount of resources (i.e., no financial needs). The results also show that while some specialities exhibits a considerable reduction in its blocking probability, other specialities have to accept a slightly increase in the number of blocked patients.

The optimisation model can be extended with more constraints, such as different priorities for each specialized ward, specific limits on the number of blocked patients at some of the specialities or adding a constraint on the number of idle beds at each speciality. In addition, the model can also be used for investigating the optimal distribution of new beds in case a hospital capacity extension is considered. The model can also be applied to different departments in the hospital to help make better informed decisions concerning bed capacity in the whole hospital which may result in an unexpected service improve.

With respect to time dependent effect, it has to be pointed out that this paper considers the steady state situation of the system. Future research may include the study of time dependent arrival rates on the required bed capacity at each unit time. The potential effect of controlling patient length of stay on the expected number of blocked patients may also be investigated. It must be noted that our approach is intended to give indications of the general system operations rather than a detailed description of the system behaviour. As such, our model can be considered a very useful tool used by hospital managers and decision makers in an attempt of making better informed decisions.

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