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Research on Case Reasoning Multi-attribute Group Decision-making Method Based on Hesitant Fuzzy Set

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Abstract

In this study, a hesitant fuzzy set-based case-based reasoning integration method is proposed for the multi-attribute group decision-making problem with unknown attribute weights and mixed forms of attribute values. First, from two perspectives, traditional distance measure and information theory, a multi-objective optimization model is constructed using the distance similarity measure and information entropy of each type of attributes to determine the attribute weights. Secondly, considering the hybrid and nonlinear characteristics of case data, based on the principle of symmetric interaction entropy and TOPSIS method, a global similarity measure based on symmetric interaction entropy is proposed and a case inference algorithm suitable for hesitant fuzzy environment is designed. Finally, by analyzing the arithmetic cases of the target case in the case base, the most similar historical cases to the target case are retrieved to determine the decision-making scheme, and the practicality and feasibility of the decision-making method are verified. The results show that considering hesitant fuzzy theory for case-based reasoning research will help improve the accuracy and reliability of decision-making and provide more effective support for multi-attribute group decision management.

Keywords

hesitant fuzzy sets, case-based reasoning, TOPSIS, multi-attributes group decision making

1. Introduction

Case-Based Reasoning (CBR) is an artificial intelligence approach that utilizes past cases of solving similar problems to solve new problems. It is based on the simple idea of retrieving cases of similar problems solved in the past and applying these solutions to the current problem with appropriate modifications to form a decision-making solution (Castro et al., 2009). CBR has been widely used in a number of domains, especially in areas that are rich in experience but lack a clear theoretical model, such as fault diagnosis (Pei et al., n.d.; Zhang et al., 2018), information sciences (Qin et al., 2018), business management decision-making (Li et al., 2021; Sartori, Mazzucchelli, & Gregorio, 2016), judicial decision-making (Wu et al., n.d.), paramedical diagnosis (Zhang et al., n.d.), emergency Emergency Management (Zhao, 2012; Zheng, Wang, & Zhang, 2017; Li et al., 2022; Yao et al., 2021; Zhu, Ren, & Pu, 2022) and so on. However, in general, scholars at home and abroad have conducted more comprehensive research on the concept definition, algorithm optimization research, system design, and related application scenarios of various aspects of case-based reasoning, and have affirmed the necessity of case expression and case retrieval in case-based reasoning, but the existing research on case expression and case retrieval in case-based reasoning generally suffers from the following inadequacies: firstly, in terms of case expression, most of the previous research is based on precise symbols, which are not the same as case retrieval. First, in case expression, most of the previous studies express cases based on the traditional single form of information characterization such as precise symbols, precise values, interval numbers, etc. There is a lack of more comprehensive and integrated information characterization tools to portray the complexity and uncertainty of case information, which can not effectively improve the accuracy of case expression, and thus reduce the accuracy of case retrieval, and also affect the efficiency of the case-based reasoning system. Second, in terms of case retrieval, the problem of case attribute weight allocation and the problem of similarity calculation and integration are the key research contents to optimize the accuracy and reliability of case retrieval, however, the research methods are both more concentrated and single. On the one hand, scholars are more inclined to adopt traditional weight allocation methods such as equalization method, expert assignment method, AHP method, etc (Gu et al., 2009). On the other hand, when calculating the similarity between the target case and the historical case, the traditional case inference methods are facing the real number information, and most of them apply the distance formula to construct the model (Chen, Marckilgour, & Hipel, 2008; Chen, Hipel, & Kilgour, 2007), which is difficult to solve the problem of decision-making with uncertain information. In similarity integration and final retrieval, the more widely used strategy is based on K-nearest neighbor (KNN) (Cover & Hart, 1967). It is mainly based on the attribute weights and their eigenvalues to calculate the similarity between the target case to be solved and the source cases in the case base, and then select one or some source cases with high similarity as the basis for case reuse (Lin & Chen, 2011).

In response to the first deficiency, and taking into account that the decision-making process of decision makers is not only affected by the uncertainty of objective things, but also limited by the decision makers' own lack of knowledge and the uncertainty of the environment and other real factors, in order to better express this kind of uncertain information, fuzzy sets (Zadeh, 1965), hesitant fuzzy sets (Torra, 2010), linguistic variables (Zadeh, 1975a; Zadeh, 1975b; Zadeh, 1975c) have been put forward, and some scholars have introduced fuzzy sets such as rough set theory[9] and triangular fuzzy numbers (Hu, Suan, & Sun, 2016), intuitive fuzzy numbers (Wang et al., 2015; Li, Zhu, & Liu, 2015) into the case-based reasoning methodology. Some scholars have introduced theories such as rough set theory and triangular fuzzy number, intuitionistic fuzzy number into the case-based reasoning method, which has expanded the research perspective of case-based reasoning. Compared with other theories, hesitant fuzzy set theory allows group decision makers to assign different evaluation values to the same set element. Hesitation fuzzy sets are a very useful tool when the decision maker hesitates about the degree of affiliation an element belongs to a set, and also integrates the group decision maker's decision making information. Considering the introduction of hesitation fuzzy sets into case-based reasoning can more reasonably solve the disadvantage that decision makers are unable to objectively describe the decision-making information of uncertainty and hesitation, and make the results of case-based reasoning and decision-making programs more accurate and reasonable (Zhao et al., 2020).

To address the second deficiency, in view of the important impact of weight allocation on similarity calculation and retrieval quality in case-based reasoning systems, and the importance of accurately and reasonably allocating weights to improve the performance of case-based reasoning models, some scholars have begun to use genetic algorithms (Ahn & Kim, 2009), SHO-SA algorithms (Yan & Ding, 2022), machine learning (Yeow, Mahmud, & Raj, 2014), scheme bias minimization (Zheng, Wang, & Zhang, 2017), and water-flooding allocation method (Yan, Qian, & Wang, 2014) to determine the weights respectively, and for the problem of weight allocation, there have been For the problem of weight allocation, the research mainly determines the weights from three perspectives: subjective, objective and the combination of the two methods (Fu, Xu, & Xue, 2018). Subjective methods include expert assignment method, AHP method, etc., there are problems such as uncertainty affected by the level of knowledge of experts and limitations of the scope of application, and the weights determined by different experts may have large differences, affecting the stability and reliability of the results, so the objective method is mostly used to allocate the weights, and there are mainly two kinds of machine learning and mathematical model solving. Machine learning requires a large amount of case data, and the computational cost is too high, so it is more appropriate to use mathematical models to determine the weights.

However, most of the existing studies on determining weights based on mathematical models analyze from traditional case data types, without considering the psychological behavior of decision-making subjects. In real decision-making, most decision-making subjects tend to hesitate due to the incompleteness of decision-making information and the uncertainty of the decision-making environment, thus making it difficult to reach a consensus. To address this problem, scholars have introduced the triangular fuzzy (Hu, Suan, & Sun, 2016), intuitionistic fuzzy, intuitionistic fuzzy(Wang et al., 2015; Li, Zhu, & Liu, 2015), gradient fuzzy(Dong & Wan, 2018) and other linguistic terminology sets into the case-based reasoning of the determination and optimization of the weights, which breaks through the limitations of the use of the traditional data types to portray fuzzy information, so that it has a stronger expressive power than the traditional data sets in dealing with uncertain information, and it also expresses the decision-making subject's psychological behavior and the psychological behavior of the decision-making body more intuitively and delicately close to reality. However, none of the above studies have considered how to reasonably express and make decisions on multiple attributes when the attribute values are in mixed form. Based on the above analysis, this paper will construct a multi-objective optimization model based on the distance similarity measure and information entropy of each type of attribute from the perspectives of traditional distance measure and information theory to determine the weights and solve the problem of weight allocation in the case of mixed attributes, so as to reflect the real level of attribute weights to a greater extent.

In addition, in terms of local similarity integration and ranking of various mixed feature attributes, scholars have defined the global similarity between two cases as the weighted sum of local similarity of various mixed attributes based on the weighted KNN strategy (Hu, Suan, & Sun, 2016; Wu et al., n.d.; Arroyo & Maté, 2009) and ranked them, and scholars have used the VIKOR method (Zhao et al., 2020) to determine the positive and negative ideal solutions for decision ranking, however, all of them have the limitation of not being able to accurately and efficiently differentiate between the merits and demerits of the decisions. In this regard, this paper combines the mixed and nonlinear characteristics of case data, and utilizes the principles of symmetric interaction entropy (Lu & Li, 2016) and TOPSIS method to define the global similarity based on symmetric interaction entropy, which is the improvement of the traditional TOPSIS method and the promotion of similarity integration, and it is known to be effective and feasible through the analysis of the calculation examples.

In summary, this paper conducts innovative research in the following aspects: (1) a multi-dimensional hybrid attribute feature system is constructed, including the number of definite symbols, the number of definite values, the number of interval values, and the number of hesitant fuzzy numbers, which is able to express the case more comprehensively and integrally from multiple dimensions; (2) an optimization model for the determination of the hybrid attribute weight is given, and at the same time, a multi-objective optimization model to determine the weights is constructed from the two perspectives of the traditional distance measure and the information theory, and the distance similarity measure and the information entropy of the attributes. attributes' distance similarity measure and information entropy to construct a multi-objective optimization model to determine the weights, with a view to reflecting the true and important level of attribute weights to a larger extent; (3) different from previous studies, this paper combines the hybrid and nonlinear characteristics of case data, and utilizes the principles of symmetric interaction entropy (Lu & Li, 2016) and the TOPSIS method, to define the global similarity based on symmetric interaction entropy, and on the basis of which, the symmetric interaction entropy based on symmetric interaction entropy mainly in the hesitant fuzzy environment of symmetric interaction entropy case inference algorithm.

2. Method of Presentation of Cases

In the case-based reasoning system, let the target case in the case base be N_0, the historical case set is $N = \{N_1, N_2, ..., N_m\}$ denoting the i th case in the case base, the case attribute set is $V = \{V_1, V_2, ..., V_n\}$, V_j denoting the j th attribute value of the case feature attribute, and the attribute value is characterized by a mixture of deterministic symbols, deterministic numeric values, interval numeric values, and hesitant fuzzy forms. The case feature attribute weight vector is, $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ denotes the weight of the case feature attribute V_j , $\sum_{j=1}^{n}\omega_j=1, 0\le\omega_j\le1$, as shown in Table 1:

Table 1. Formal Representation of Cases

causality		V_1	V_2	...	V_n
Target cases	N_0	...	...	...	V_{0n}
historical case	N_1	V_{11}	V_{12}	...	V_{1n}
	N_2	V_{21}	...	...	V_{2n}
	...	...	...	...	...
	N_m	V_{m1}	V_{m2}	...	V_{mn}
Attribute weights	ω	ω_1	ω_2	...	ω_n

3. Similarity Measures for Cases

As can be seen from Table 1, the case feature attributes contain four data types, and the fusion calculation of these data is a difficult problem, in fact, the fusion of heterogeneous information is involved in many problems. In order to accomplish the neighbor avoidance risk warning case reasoning problem in this paper in a targeted way, the following section focuses on the local similarity problem of the four types of data, namely, determining symbols, determining numerical values, interval numerical values, and hesitant fuzzy numbers.

Definition 1 Let the set of historical cases be $N=\{N_1,N_2,...,N_m\}$, the set of case attributes be $V=\{V_1,V_2,...,V_n\}$, the vector of attribute values for the target case N_0 be $V_0=\{V_{01},V_{02},...,V_{0n}\}$, the vector of attribute values for the historical case N_i be $V_i=\{V_{i1},V_{i2},...,V_{in}\}$, and null denote the missing attribute values. $sim(V_{0j},V_{ij})$ For the similarity function between the cases N_0 and $N_i\,(i=1,2,...,m)$ on the attribute $V_j\,(j=1,2,...,n)$, then $sim(V_{0j},V_{ij})$ satisfies the following three properties:

(1) $0\le sim(V_{0j},V_{ij})\le1$;

(2) $sim(V_{0j},V_{ij})=1$ when and only when $V_{0j}=V_{ij}$;

(3) $sim(V_{0j},V_{ij})=sim(V_{ij},V_{0j})$.

Based on the attribute value characteristics of the feature attributes in the case base of this paper, they are classified into hesitation fuzzy attributes as well as deterministic sign, deterministic value, and interval value attributes, and the similarity measure between the attributes under the consideration of hesitation fuzzy environment is proposed.

3.1 Similarity Calculation of Hesitant Fuzzy Attributes

Definition 1 Let X be a fixed set, then the hesitant set is a function (Torra, 2010; Torra & Narukawa, 2009) of each element of X mapped to a subset of $[0,1]$. Then the mathematical expression of hesitation fuzzy set can be expressed as $A=\{x,h_A(x)\,|\,x\in X\}$. Where $h_A(x)$ is the set of some values in $[0,1]$ representing some possible degrees of affiliation of the element x with respect to the set X. Call $h=h_A(x)$ a hesitant fuzzy element, $l_{h_A(x)}$ is the number of hesitant fuzzy number elements h and denote by Θ the set of all hesitant fuzzy elements (Xu & Xia, 2011).

Definition 2 Let A_1 and A_2 be two hesitant fuzzy on X, then the distance measure (Xu & Xia, 2011) between A_1 and A_2 can be defined as $d(A_1,A_2)$, which needs to satisfy the following properties:

(1) $0\le d(A_1,A_2)\le 1$;

(2) $d(A_1,A_2)=0$ when and only when $A_1=A_2$;

(3) $d(A_1,A_2)=d(A_2,A_1)$;

Definition 3 Let A_1 and A_2 be two hesitant fuzzy elements on X, then the similarity measure between A_1 and A_2 can be defined as $sim(A_1,A_2)$, which needs to satisfy the following properties:

(1) $0\le sim(A_1,A_2)\le 1$;

(2) $sim(A_1,A_2)=1$ when and only when $A_1=A_2$;

(3) $sim(A_1,A_2)=sim(A_2,A_1)$.

By analyzing Definition 2 and Definition 3, we can see that $sim(A_1,A_2)$ $sim(A_1,A_2)=1-d(A_1,A_2)$. When calculating the distance and similarity between hesitant ambiguities, the first thing to do is to make the number of elements of hesitant ambiguity numbers in the two sets equal. However, in most cases, $l_{h_{A_1}(x_i)}\ne l_{h_{A_2}(x_i)}$. Therefore, for hesitation fuzzy numbers with different number of elements, the following rule is applied to add to the hesitation fuzzy number with fewer elements to the same number (Xu & Zhang, 2013)

$$\overline{h}_A(x_i)=\xi h'_A(x_i)+(1-\xi)h''_A(x_i) \tag{1}$$

where, $h'_A(x_i)=min\{h_A^{\sigma(j)}(x_i)\,|\,\sigma(j)=1,2,...,l_{x_i}\}$

$h_A^{''}(x_i) = max\left\{h_A^{\sigma(j)}(x_i) \mid \sigma(j)=1,2,...,l_{x_i}\right\}$, $l_{x_i} = max\left\{l_{h_{A_1}(x_i)}, l_{h_{A_2}(x_i)}\right\}$, $\xi(0 \le \xi \le 1)$ are the optimization parameters and $\sigma(j)$ is the j th smallest element in the hesitation fuzzy element h. Then the generalized similarity of hesitation fuzzy numbers can be defined as:

$$sim(A_1, A_2) = 1 - \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}}\left|h_{A_1}^{\sigma(j)}(x_i) - h_{A_2}^{\sigma(j)}(x_i)\right|^{\lambda}\right)\right]^{\frac{1}{\lambda}} \tag{2}$$

where $\lambda > 0$. In particular, if $\lambda = 1$, the Hamming similarity of the hesitant fuzzy numbers $h_{A_1}(x_i)$ and $h_{A_2}(x_i)$ can be obtained:

$$sim(A_1, A_2) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}}\left|h_{A_1}^{\sigma(j)}(x_i) - h_{A_2}^{\sigma(j)}(x_i)\right|\right) \tag{3}$$

If $\lambda = 2$, the Euclidean similarity of hesitant fuzzy numbers $h_{A_1}(x_i)$ and $h_{A_2}(x_i)$ can be obtained:

$$sim(A_1, A_2) = 1 - \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}}\left|h_{A_1}^{\sigma(j)}(x_i) - h_{A_2}^{\sigma(j)}(x_i)\right|^{2}\right)\right]^{\frac{1}{2}} \tag{4}$$

Based on this, this paper gives the similarity definition of hesitation fuzzy attribute as follows: if V_{0j} is the target case the j th attribute for hesitation fuzzy attribute. V_{ij} is the j th attribute of the i th historical case, which is a hesitation fuzzy attribute, let the non-empty set $X = \{x_1, x_2, x_3, \ldots, x_m\}$, $h(x)$ be a hesitation fuzzy element defined on $x \in X$, and the set of hesitation fuzzy linguistic attributes is denoted as $V_{ij}(X) = \{x, h_A(x) | x \in X\}$. Then the similarity of the two cases for this hesitation fuzzy attribute can be expressed as:

$$sim_4(V_{0j}, V_{ij}) = \begin{cases} 1 - d(V_{0j}, V_{ij}) \; if \; V_{0j} \ne null \; and \; V_{ij} \ne null \\ null \quad if \quad V_{0j} = null \; or \; V_{ij} = null \end{cases} \tag{5}$$

Among them. $d(V_{0j}, V_{ij}) = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}}\left|h_{A_1}^{\sigma(j)}(x_i) - h_{A_2}^{\sigma(j)}(x_i)\right|^{\lambda}\right)\right]^{\frac{1}{\lambda}}$

3.2 Similarity Calculation for Determining Symbol Properties

Definition 4 If V_{oj} is the j th attribute of the target case, it is a deterministic symbol attribute. V_{ij} If j is the th attribute of the i th historical case, which is a deterministic symbolic attribute. The similarity (Hu, Chen, & Sun, 2016) of the deterministic attribute of the two cases can be expressed as follows:

$$sim_1\left(V_{0j},V_{ij}\right)=\begin{cases}1 \ \ if \ \ V_{0j}=V_{ij}\\ 0 \ \ if \ \ V_{0j}{}^{1}V_{ij}\\ null \quad if \ \ V_{0j}=null \ or \ V_{ij}=null\end{cases} \tag{6}$$

3.3 Determining Similarity Calculations for Numerical Attributes

Definition 5 If V_{oj} is the j th attribute of the target case and V_{ij} is the j th attribute of the i th historical case, which is a determined numerical attribute. Then the Euclidean distance is used to calculate the similarity of the two cases for that deterministic numerical attribute (Hu, Chen, & Sun, 2016) can be expressed as:

$$sim_2\left(V_{0j},V_{ij}\right)=\begin{cases}1-d\left(V_{0j},V_{ij}\right) if \ V_{0j}\neq null \ and \ V_{ij}\neq null\\ null \qquad if \quad V_{0j}=null \ or \ V_{ij}=null\end{cases} \tag{7}$$

included among these $d\left(V_{0j},V_{ij}\right)=\dfrac{\left|V_{ij}-V_{0j}\right|}{maxV_{ij}-minV_{ij}},i\in m.$

3.4 Similarity Calculation of Numerical Attributes between Zones

Definition 6 If V_j is the j th attribute of the target case, it is an interval-valued attribute, $V_{0j}=\left[V_{0j}{}^{L},V_{0j}{}^{U}\right]$. V_{ij} is the j th attribute of the i th historical case, which is an interval-valued attribute, and $V_{ij}=\left[V_{ij}{}^{L},V_{ij}{}^{U}\right]$. Then the similarity (Hu, Chen, & Sun, 2016) of the interval value attribute of the two cases can be expressed as:

$$sim_3\left(V_{0j},V_{ij}\right)=\begin{cases}exp\left[-d\left(V_{0j},V_{ij}\right)\right] if \ V_{0j}\neq null \ and \ V_{ij}\neq null\\ null \qquad if \quad V_{0j}=null \ or \ V_{ij}=null\end{cases} \tag{8}$$

Among them. $d\left(V_{0j},V_{ij}\right)=\dfrac{\sqrt{\left(V_{ij}{}^{L}-V_{0j}{}^{L}\right)^2+\left(V_{ij}{}^{U}-V_{0j}{}^{U}\right)^2}}{max\left\{\sqrt{\left(V_{ij}{}^{L}-V_{0j}{}^{L}\right)^2+\left(V_{ij}{}^{U}-V_{0j}{}^{U}\right)^2} \mid i\in m\right\}}$

In summary, the similarity between the target case and the historical case on each genus subspace is calculated separately using the above similarity measures to obtain the similarity matrix $S=\left(s_{ij}\right)_{m\times n}$.

4. Multi-attribute Weight Determination Optimization Model

Due to the complexity of the decision-making scenario and with full consideration of the characteristics of hesitant fuzzy decision-making information, the attribute weight determination model in hesitant fuzzy environment is constructed.

4.1 Model for Determining Multi-attribute Weights under the Distance Measure

If the attribute difference value of each case under the attribute V_j is larger or more dispersed, the attribute plays a greater role in case decision-making and sorting, and then the attribute is given a larger weight; on the contrary, if the attribute difference value of each case under the attribute V_j is smaller or more centralized, the attribute V_j plays a smaller role in case decision-making and sorting, and then the attribute is given a smaller weight. Based on this, the following model is constructed based on the distance measure in Eqs. (1)~(8):

$$M_1:\begin{cases} maxf_1(W)=\sum_{j=1}^{n}\frac{1}{m}\sqrt{\sum_{i=1}^{m}d^2\left(V_{0j},V_{ij}\right)}\omega_j \\ s.t. \quad \sum_{j=1}^{n}\omega_j^2=1, 0\le\omega_j\le 1 \end{cases} \tag{9}$$

4.2 Multi-attribute Weight Determination Model under the Information Entropy Measure

The entropy value can be used to assess the amount of information carried by an attribute, with a lower entropy value indicating a greater amount of information and a more significant influence of the attribute on the case. When determining the weights, greater weights are given to indicators with lower entropy values.

For the hesitant fuzzy attribute, its information entropy is defined as:

$$E_j=\frac{1}{m}\sum_{i=1}^{m}E\left(V_{ij}\right) \tag{10}$$

formula $E\left(V_{ij}\right)=\frac{1}{l\left(\sqrt{2}-1\right)}\sum_{\lambda=1}^{l}\left(\begin{array}{l} sin\frac{\pi\left(\gamma^{\lambda}+\gamma^{l-\lambda+1}\right)}{4}+ \\ sin\frac{\pi\left(2-\gamma^{\lambda}+\gamma^{l-\lambda+1}\right)}{4}-1\end{array}\right)$

(1) For determining the symbol properties, define its information entropy as:

$$E_j=-\frac{1}{lnm}\sum_{i=1}^{m}P_{ij}lnP_{ij} \tag{11}$$

Where, P_{ij} denotes the frequency of occurrence of the characteristic attribute value V_{ij} of the attribute j in the i case, and m denotes the number of species of the symbol sequence.

(2) For determining the numerical attributes, define their information entropy as:

$$E_j=-\frac{1}{lnm}\sum_{i=1}^{m}P_{ij}lnP_{ij} \tag{12}$$

which is $P_{ij}=\frac{V_{ij}}{\sum_{i=1}^{m}V_{ij}}$.

(3) For the interval value attribute, define its information entropy as:

$$E_j=\frac{E_j^L+E_j^U}{2} \tag{13}$$

Eq. $\begin{cases} E_j^L = -\dfrac{1}{lnm}\sum_{i=1}^{m} P_{ij}^L lnP_{ij}^L \\ E_j^U = -\dfrac{1}{lnm}\sum_{i=1}^{m} P_{ij}^U lnP_{ij}^U \end{cases}, \begin{cases} P_{ij}^L = \dfrac{V_{ij}^L}{\sum_{i=1}^{m} V_{ij}^L} \\ P_{ij}^U = \dfrac{V_{ij}^U}{\sum_{i=1}^{m} V_{ij}^U} \end{cases}$

The optimization model M_2 is constructed based on the information entropy of each case in terms of the values of the attributes V_j attributes E_j :

$$M_2 : \begin{cases} maxf_2\left(W\right) = \sum_{j=1}^{n}\left[1 - E_j\right]\omega_j \\ s.t. \quad \sum_{j=1}^{n}\omega_j^2 = 1, 0 \le \omega_j \le 1 \end{cases} \tag{14}$$

4.3 Optimization Model for Multi-Attribute Weight Determination under Integrated Measurement

Considering the two levels of departure maximization and information entropy, M_1 and M_2 are combined into a single-objective optimization model M :

$$M : \begin{cases} maxf\left(W\right) = \alpha\sum_{j=1}^{n}\frac{1}{m}\sqrt{\sum_{i=1}^{m} d^2\left(V_{0j}, V_{ij}\right)}\omega_j + \beta\sum_{j=1}^{n}\left[1 - E_j\right]\omega_j \\ s.t.\sum_{j=1}^{n}\omega_j^2 = 1, 0 \le \omega_j \le 1 \end{cases} \tag{15}$$

Where α, β is the preference coefficient and $0 \le \alpha, \beta \le 1, \alpha + \beta = 1$ is based on the decision maker's preference, a Lagrangian auxiliary function is constructed to solve the model:

$$L\left(W, \lambda\right) = f\left(W\right) + \frac{1}{2}\lambda\left(\sum_{j=1}^{n}\omega_j^2 - 1\right) \tag{16}$$

Find the partial derivative with respect to $\omega_j\left(j = 1, 2, \ldots, n\right), \lambda$ while making

$$\begin{cases} \dfrac{\partial L}{\partial \omega_j} = \alpha\sum_{j=1}^{n}\dfrac{1}{m}\sqrt{\sum_{i=1}^{m} d^2\left(V_{0j}, V_{ij}\right)} + \beta\sum_{j=1}^{n}\left[1 - E_j\right] + \lambda\omega_j = 0 \\ \dfrac{\partial L}{\partial \lambda} = \dfrac{1}{2}\left(\sum_{j=1}^{n}\omega_j^2 - 1\right) = 0, \left(j = 1, 2, \ldots, n\right) \end{cases} \tag{17}$$

Solve for the attribute weights $\omega_j^*\left(j = 1, 2, \ldots, n\right)$ as

$$\omega_j^* = \frac{\alpha\sum_{j=1}^{n}\frac{1}{m}\sqrt{\sum_{i=1}^{m} d^2\left(V_{0j}, V_{ij}\right)} + \beta\sum_{j=1}^{n}\left[1 - E_j\right]}{\sqrt{\sum_{j=1}^{n}\left(\alpha\sum_{j=1}^{n}\frac{1}{m}\sqrt{\sum_{i=1}^{m} d^2\left(V_{0j}, V_{ij}\right)} + \beta\sum_{j=1}^{n}\left[1 - E_j\right]\right)^2}} \tag{18}$$

Normalizing ω_j^* gives

$$\omega_j^* = \frac{\alpha \sum_{j=1}^{n} \frac{1}{m} \sqrt{\sum_{i=1}^{m} d^2\left(V_{0j}, V_{ij}\right)} + \beta \sum_{j=1}^{n} \left[1 - E_j\right]}{\sum_{j=1}^{n} \left(\alpha \sum_{j=1}^{n} \frac{1}{m} \sqrt{\sum_{i=1}^{m} d^2\left(V_{0j}, V_{ij}\right)} + \beta \sum_{j=1}^{n} \left[1 - E_j\right]\right)^2} \tag{19}$$

5. Case-based Reasoning Global Similarity integration Method Design

A similarity matrix $S = \left(s_{ij}\right)_{m\times n}$ is derived based on the computed distances between the target case and the source case in the case base on each genus subspace. The core of the case inference model is to integrate the similarity matrix to get the global similarity. The global similarity based on symmetric interaction entropy and TOPSIS is proposed by integrating the basic idea principles of symmetric interaction entropy and TOPSIS methods.

Definition 7 Symmetric interaction entropy For $s_k, s_l \in S, s_k = \left\{s_{k1}, s_{k2}, \ldots, s_{kn}\right\}, s_l = \left\{s_{l1}, s_{l2}, \ldots, s_{ln}\right\}$.

Where $0 \le s_{kj}, \ s_{lj} \le 1, j = 1, 2, \ldots, n$. Then the degree of difference between the two elements of the similarity matrix $S = \left(s_{ij}\right)_{m\times n}$ can be expressed as the symmetric interaction entropy (Wang et al., 2015) .

$$D\left(s_k, s_l\right) = \sum_{j=1}^{n}\left[s_{kj} ln \frac{2s_{kj}}{s_{kj}+s_{lj}} + \left(1-s_{kj}\right) ln \frac{2\left(1-s_{kj}\right)}{2-s_{kj}-s_{lj})} + s_{lj} ln \frac{2s_{lj}}{s_{kj}+s_{lj}} + \left(1-s_{lj}\right) ln \frac{2\left(1-s_{lj}\right)}{2-s_{kj}-s_{lj})}\right] \tag{20}$$

The symmetric interaction entropy can indicate the degree of difference between two elements, but not the distance. The smaller the value of symmetric interaction entropy, the smaller the degree of difference between two elements. In order to measure the distance between two elements in the similarity matrix $S = \left(s_{ij}\right)_{m\times n}$, the TOPSIS method is integrated in the symmetric interaction entropy. By using TOPSIS method, the positive ideal solution S^+ and negative ideal solution S^- in the similarity matrix $S = \left(s_{ij}\right)_{m\times n}$ can be determined.

$$S^+ = \left\{S_1^+, S_2^+, S_3^+, \ldots, S_i^+\right\} = \left\{\left(maxS_{ij} | j \in I\right), \left(minS_{ij} | j \in J\right)\right\} \tag{21}$$

$$S^- = \left\{S_1^-, S_2^-, S_3^-, \ldots, S_i^-\right\} = \left\{\left(minS_{ij} | j \in I\right), \left(maxS_{ij} | j \in J\right)\right\} \tag{22}$$

s_k For any element in the similarity matrix S, fusing the positive and negative ideal solutions S^+, S^- and the symmetric interaction entropy in the TOPSIS method, and combining the weights of each type of attribute, the symmetric interaction entropy of the element s_k can be expressed as:

$$Es_k^+ = \sum_{j=1}^{n}\omega_j\left[S_j^+ ln\frac{2S_j^+}{S_j^+ + S_{ij}} + \left(1-S_j^+\right)ln\frac{2-S_j^+}{2-S_j^+ - S_{ij}} + S_{ij}ln\frac{2S_{ij}}{S_j^+ + S_{ij}} + (1-S_{ij})ln\frac{2(1-S_{ij})}{2-S_j^+ - S_{ij}}\right] \quad (23)$$

$$Es_k^- = \sum_{j=1}^{n}\omega_j\left[S_j^- ln\frac{2S_j^-}{S_j^- + S_{ij}} + \left(1-S_j^-\right)ln\frac{2-S_j^-}{2-S_j^- - S_{ij}} + S_{ij}ln\frac{2S_{ij}}{S_j^- + S_{ij}} + (1-S_{ij})ln\frac{2(1-S_{ij})}{2-S_j^- - S_{ij}}\right] \quad (24)$$

Definition 8 Global Similarity The global similarity of the case retrieval of S_k is defined using the symmetric interaction entropy of the element S_k:

$$E_{S_k}^* = \frac{Es_k^-}{Es_k^+ + Es_k^-}\left(k = 1, 2, \ldots, m\right) \quad (25)$$

The historical cases $N_k\left(i = 1,2,...,m\right)$ are sorted according to the size of the global similarity $E_{S_k}^*\left(k = 1,2,...,m\right)$, and the historical cases that most closely match the target case are selected and relevant decisions are made.

6. Examples and Analysis

6.1 Decision-making Steps

Step 1: The decision maker measures and scores each case according to various types of attributes, where V_1,V_2 is a deterministic symbolic attribute, V_3,V_4 is a deterministic numerical attribute, V_5,V_6 is an interval numerical attribute, and V_7,V_8,V_9,V_{10} is a hesitant fuzzy attribute, forming a decision matrix as follows:

Table 2. Case Reasoning Decision Matrix

case (law)	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}
N_0	2	1	0.7	0.5	0.4,0.6	0.3,0.6	0.4,0.7	0.5,0.7,0.8	0.2,0.5	0.4,0.5,0.7
N_1	1	1	0.5	0.6	0.4,0.5	0.5,0.9	0.1,0.3	0.2,0.3,0.4	0.1,0.2	0.5,0.6,0.8
N_2	4	2	0.4	0.5	0.2,0.3	0.1,0.3	0.5,0.8	0.1,0.3	0.8,0.9	0.1,0.3,0.5
N_3	1	1	0.8	0.2	0.4,0.8	0.5,0.7	0.2,0.6	0.2,0.5	0.5,0.6,0.7	0.1,0.3
N_4	2	3	0.6	0.8	0.4,0.6	0.6,0.9	0.5,0.9	0.2,0.5,0.6	0.1,0.4	0.5,0.6
N_5	3	1	0.8	0.6	0.7,0.9	0.8,0.9	0.1,0.2	0.4,0.6	0.1,0.2	0.4,0.7

Step 2: Construct a normalized decision matrix for the sample data:

Table 3. Standardized Case Reasoning Decision Matrix

case (law)	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}
N_0	2	1	0.	0.	0.4,0.	0.3,0.	0.4, 0.4,	0.5,0.7,0.	0.2,0.2,0.	0.4,0.5,0.

			7	5	6	6	0.7	8	5	7
N_1	1	1	0.	0.	0.4,0.	0.5,0.	0.1,0.1,0.	0.2,0.3,0.	0.1,0.1,0.	0.2,0.3,0.
			5	6	5	9	3	4	2	4
N_2	4	2	0.	0.	0.2,0.	0.1,0.	0.5,0.5,0.	0.1,0.1,0.	0.8, 0.8,	0.1,0.1,0.
			4	5	3	3	8	3	0.9	3
N_3	1	1	0.	0.	0.4,0.	0.5,0.	0.2, 0.2,	0.2,0.2,0.	0.5,0.6,0.	0.2,0.2,0.
			8	2	8	7	0.6	5	7	5
N_4	2	3	0.	0.	0.4,0.	0.6,0.	0.5, 0.5,	0.2,0.3,0.	0.1,0.1,0.	0.2,0.3,0.
			6	8	6	9	0.9	4	2	5
N_5	3	1	0.	0.	0.7,0.	0.8,0.	0.1,0.1,0.	0.2,0.3,0.	0.1,0.1,0.	0.4,0.4,0.
			8	6	9	9	2	4	2	5

Step 3: Calculate the similarity matrix between the target case and each historical case according to the various types of attribute feature similarity formulas (1) to (7), as shown in the following table:

Table 4. Case Reasoning Similarity Matrix

	N_1	N_2	N_3	N_4	N_5
$\mathrm{sim}(V_1,\ V_{i1})$	0	0	0	1	0
$\mathrm{sim}(V_2,\ V_{i3})$	1	0	1	0	1
$\mathrm{sim}(V_3,\ V_{i3})$	0.5	0.75	0.25	0.25	0.25
$\mathrm{sim}(V_4,\ V_{i4})$	0.1667	0.0000	0.5000	0.5000	0.1667
$\mathrm{sim}(V_5,\ V_{54})$	0.7900	0.4275	0.6241	1.0000	0.3679
$\mathrm{sim}(V_6,\ V_{i6})$	0.5388	0.5388	0.6815	0.4831	0.3679
$\mathrm{sim}(V_7,\ V_{74})$	0.8234	0.9439	0.9048	0.9216	0.8037
$\mathrm{sim}(V_8,\ V_{i8})$	0.8078	0.7464	0.8037	0.8078	0.8078
$\mathrm{sim}(V_9,\ V_{i9})$	0.8953	0.7315	0.8357	0.8953	0.8953
$\mathrm{sim}(V_{10},\ V_{i10})$	0.8716	0.8078	0.8716	0.8909	0.9282

Step 4: According to the data in Table 3, take the preference coefficient to calculate the indicator weights, and the weights of each type of attribute can be obtained from Equation (15)~(19):

$$W_j = \left(0.013, 0.0449, 0.007, 0.0156, 0.0139, 0.0195, 0.2053, 0.2260, 0.245, 0.2099\right)^T$$

Step 5: Apply TOPSIS method to rank the cases by global similarity, which can be obtained from Eqs. (20) to (23).

(1) Calculate the positive and negative ideal solutions for each type of attribute S^+ With the S^-

$$S^+ = \{1,1,0.75,0.5,1,0.6815,0.9439,0.8078,0.8953,0.9282\}$$

$S^{-}=\{0,0,0.25,0.25,0.3679,0.3679,0.5145,0.4796,0.4592,0.6293\}$

(2) Calculate the cross-symmetric interaction entropy $E_{S_k}^{+}$, $E_{S_k}^{-}$,and global similarity $E_{S_k}^{*}$ of each historical case based on the positive and negative ideal solutions and the weights of each type of attributes:

Table 5. Symmetric Interaction Entropy and Global Similarity Matrix for Case-based Reasoning

	$E_{S_k}^{+}$	$E_{S_k}^{-}$	$E_{S_k}^{*}$
N_1	0.0655	0.0817	0.5550
N_2	0.1613	0.011	0.0804
N_3	0.0072	0.1646	0.9580
N_4	0.0839	0.0667	0.4428
N_5	0.0620	0.0930	0.6000

From the above table, The combined mean value of R_i yields a ranking of the historical cases as $N_3 \succ N_5 \succ N_1 \succ N_4 \succ N_2$. Among the five cases taken from the historical cases, the case with the highest closeness to the target case is the case N_3, which has a closeness of 0.9580 and is an acceptable rational solution.

6.2 Sensitivity Analysis

In practical decision making, experts can have different risk preferences and thus will take different optimization parameters ξ, with the change of ξ, the case ranking results will also be affected. Therefore, this paper analyzes the value of ξ in steps of 0.1, and obtains 11 different groups of global similarity sorting results as shown in Table 6 $E_{S_k}^{i}$ (global similarity of the i historical case), and further examines the stability of the model by analyzing the sensitivity of the 11 groups of data.

Table 6. Sorting Results of Cases under not Same as ξ

ξ	$E_{S_k}^{1}$	$E_{S_k}^{2}$	$E_{S_k}^{3}$	$E_{S_k}^{4}$	$E_{S_k}^{5}$	arrange in order
0	0.5807	0.1022	0.9640	0.4494	0.5882	$N_3 \succ N_5 \succ N_1 \succ N_4 \succ N_2$
0.1	0.5654	0.0960	0.9654	0.4441	0.5830	$N_3 \succ N_5 \succ N_1 \succ N_4 \succ N_2$
0.2	0.5488	0.0899	0.9669	0.4365	0.5754	$N_3 \succ N_5 \succ N_1 \succ N_4 \succ N_2$
0.3	0.5277	0.0831	0.9676	0.4242	0.5639	$N_3 \succ N_5 \succ N_1 \succ N_4 \succ N_2$
0.4	0.5363	0.0810	0.9648	0.4308	0.5767	$N_3 \succ N_5 \succ N_1 \succ N_4 \succ N_2$
0.5	0.5550	0.0804	0.9580	0.4428	0.6000	$N_3 \succ N_5 \succ N_1 \succ N_4 \succ N_2$

0.6	0.5663	0.0790	0.9452	0.4486	0.6232	$N_3 \succ N_5 \succ N_1 \succ N_4 \succ N_2$
0.7	0.5794	0.0784	0.9368	0.4549	0.6391	$N_3 \succ N_5 \succ N_1 \succ N_4 \succ N_2$
0.8	0.6092	0.0809	0.9512	0.4718	0.6378	$N_3 \succ N_5 \succ N_1 \succ N_4 \succ N_2$
0.9	0.6288	0.0816	0.9574	0.4830	0.6360	$N_3 \succ N_5 \succ N_1 \succ N_4 \succ N_2$
1	0.6444	0.0818	0.9590	0.4914	0.6355	$N_3 \succ N_1 \succ N_5 \succ N_4 \succ N_2$

As can be seen from Table 5, under different ξ, the sorting results of different historical cases basically do not change. Taking $\xi = 0.9$ as the boundary, only two kinds of sorting results are produced in the end, which are mainly reflected in the difference between the sorting results of N_1 and N_5. Moreover, under 11 perturbations of ξ, the solution is sorted by $N_3 \succ N_5 \succ N_1 \succ N_4 \succ N_2$ 10 times (91.99%), so it can be considered that the model is insensitive to the perturbation of ξ. Further analysis shows that N_3 is the most stable solution result, and the perturbation of ξ value has no effect on the ranking of N_3 and N_5. In the actual decision-making scheme comparison, the decision maker often gives an optimal scheme and an alternative scheme, and the traditional decision-making method can only sort the schemes, and the decision maker can only judge the optimal scheme, but cannot give an acceptable set of other schemes. In summary, based on the sensitivity analysis, it is known that the method has good stability.

6.3 Comparative Analysis

(1) Mixed-attribute decision-making scenario analysis

Previous studies have addressed multi-attribute decision-making problems containing symbolic numbers, exact numbers, interval numbers, intuitionistic fuzzy numbers, trapezoidal fuzzy numbers, trigonometric fuzzy numbers, etc., but less on decision-making situations with hesitant fuzzy language. In this paper, considering the existence of fuzzy uncertainty and the advantages of hesitant fuzzy characterization information in real problems, hesitant fuzzy language is used to portray the decision-making information of relevant attributes, and the mixed multi-attribute decision-making problem in the decision-making environment of hesitant fuzzy language is investigated.

(2) Analysis of global similarity integration methods for hybrid attributes

In terms of similarity integration of various mixed feature attributes, most of the traditional researches are based on weighted sum or weighted average to integrate the similarity of various feature attributes, but there are limitations that may not be able to accurately and efficiently differentiate between the merits and demerits of decisions. In this paper, the global similarity based on symmetric interaction entropy is defined by utilizing the principle of symmetric interaction entropy and TOPSIS method, which is an improvement of the traditional TOPSIS method and the promotion of similarity integration, and the method is effective and feasible through the analysis of examples.

(3) Analysis of hybrid attribute weight calculation methods

In the problem of assigning weights to mixed attributes is, most studies mainly determine weights from three perspectives: subjective, objective and the combination of the two methods. The subjective method has uncertainty and limitations in the scope of application due to the subjective assignment of weights by the decision maker, which reduces the quality of decision making. While the combination method has greater randomness and other shortcomings, so this paper simultaneously from the traditional distance measurement and information theory from two perspectives, based on the various types of attributes of the distance similarity measure and information entropy to build a multi-objective optimization model to determine the weights comprehensively, in order to reflect the attribute weights to a greater extent of the true and important level, improve the objectivity and scientific decision-making results.

7. Conclusion

Aiming at the multi-attribute group decision-making problem in which the attribute values are in mixed form and the weights of the attributes are unknown, this paper effectively integrates the hesitation fuzzy, case-based reasoning and TOPSIS methods, and proposes an integrated method based on hesitation fuzzy set case-based reasoning, which is conducive to the improvement of the accuracy and validity of the case expression and the reliability of the case retrieval, which is specifically shown in the following points:

(1) Comprehensiveness and synthesis. For the uncertain decision-making problem with mixed attributes, considering the fuzzy and unstructured characteristics of the decision-making information, it breaks through the limitation of traditional fuzzy set to describe the information and introduces the theory of hesitant fuzzy set, which is able to portray and describe the uncertain information in a more delicate way, making the information closer to the actual decision-making situation.

(2) Validity and rationality. An optimization model for determining the weights of mixed attributes is given, and at the same time, a multi-objective optimization model is constructed to determine the weights based on the distance similarity measure and information entropy of each type of attributes from two perspectives of the traditional distance measure and information theory, which reflects the true and important level of the attribute weights to a larger extent.

(3) Practicality and feasibility. Combining the mixed and nonlinear characteristics of the case data, the global similarity based on symmetric interaction entropy is defined by utilizing the principle of symmetric interaction entropy and TOPSIS method, and on this basis, the symmetric interaction entropy case inference algorithm is designed mainly in hesitant and fuzzy environments, which provides a convenient and quick way of thinking for the multi-attribute group decision-making problem where the attribute values are in mixed form and the weights of the attributes are unknown.

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References

Ahn, H., & Kim, K.-j. (2009). Global optimization of case-based reasoning for breast cytology diagnosis. Expert Systems with Applications, *36*(1), 724-734. https://doi.org/10.1016/j.eswa.2007.10.023

Arroyo, J., & Maté, C. (2009). Forecasting histogram time series with k-nearest neighbors methods. *International Journal of Forecasting*, *25*(1), 192-207. https://doi.org/10.1016/j.ijforecast.2008.07.003

Castro, J. L. et al. (2009). Loss and gain functions for CBR retrieval. *Information Sciences*, *179*(11), 1738-1750. https://doi.org/10.1016/j.ins.2009.01.017

Chen, Y., Hipel, K. W., & Kilgour, D. M. (2007). Multiple-Criteria Sorting Using Case-Based Distance Models With an Application in Water Resources Management. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, *37*(5), 680-691. https://doi.org/10.1109/TSMCA.2007.902629

Chen, Y., Marckilgour, D., & Hipel, K. (2008). A case-based distance method for screening in multiple-criteria decision aid☆. *Omega*, *36*(3), 373-383. https://doi.org/10.1016/j.omega.2006.04.016

Cover, T., & Hart, P. (1967). Nearest neighbor pattern classification. *IEEE Transactions on Information Theory*, *13*(1), 21-27. https://doi.org/10.1109/TIT.1967.1053964

Dong, J. Y., & Wan, S. P. (2018). A PROMETHEE-FLP Method for Heterogeneous Multi-Attributes Group Decision Making. *IEEE Access*, *6*, 46656-46667. https://doi.org/10.1109/ACCESS.2018.2865773

Fan, Z.-P. et al. (2014). Hybrid similarity measure for case retrieval in CBR and its application to emergency response towards gas explosion. *Expert Systems with Applications*, *41*(5), 2526-2534. https://doi.org/10.1016/j.eswa.2013.09.051

Fu, C., Xu, D.-L., & Xue, M. (2018). Determining attribute weights for multiple attribute decision analysis with discriminating power in belief distributions. *Knowledge-Based Systems*, *143*, 127-141. https://doi.org/10.1016/j.knosys.2017.12.009

Gu, D. X. et al. (2009). Research and application of case retrieval and weight optimization methods. *Journal of Systems Engineering*, *24*(6), 764-768.

Hu, J., Chen, Z., & Sun, J. H. (2016). Research on the integration method of reasoning for non-complete information cases based on sequential relations. *Computer Applications and Software*, *33*(12), 220-223+317.

Hu, J., Suan, H. F., Sun, J. H. (2016). Research on matching supply and demand of shallow tacit knowledge based on fuzzy case-based reasoning. *Intelligence Theory and Practice*, *39*(4), 84-88.

Li, C. P. et al. (2016). The accident early warning system for iron and steel enterprises based on combination weighting and Grey Prediction Model GM (1, 1). *Safety Science*, *89*, 19-27. https://doi.org/10.1016/j.ssci.2016.05.015

Li, L. et al. (2022). A review of research on demand forecasting of emergency relief supplies under the perspective of natural disasters. *Technology and Innovation Management*, *43*(5), 581-588.

Li, P., Zhu, J. J., & Liu, S. F. (2015). An intuitive fuzzy decision-making method based on case-based reasoning. *China Management Science*, *23*(7), 113-118.

Li, S. Q. et al. (2021). A hybrid method of blockchain and case-based reasoning for remanufacturing process planning. *Journal of Intelligent Manufacturing*, *32*(5), 1389-1399. https://doi.org/10.1007/s10845-020-01618-6

Lin, S.-W., & Chen, S.-C. (2011). Parameter tuning, feature selection and weight assignment of features for case-based reasoning by artificial immune system. *Applied Soft Computing*, *11*(8), 5042-5052. https://doi.org/10.1016/j.asoc.2011.05.054

Lu, G. X., & Li, B. Q. (2016). A multi-attribute decision ranking method based on symmetric interaction entropy. *Journal of Mathematics*, *36*(6), 1253-1260.

Mei, F. J., & Li, Y. M. (2019). Parameterized hesitant fuzzy entropy and its application. *Computer Engineering and Science*, *41*(12), 2202-2210.

Pei, D. F. et al. (n.d.). A hybrid model for line fault diagnosis based on improved differential evolution. *Control Engineering*, 1-9.

Qin, Y. C. et al. (2018). Towards an ontology-supported case-based reasoning approach for computer-aided tolerance specification. *Knowledge-Based Systems*, *141*, 129-147. https://doi.org/10.1016/j.knosys.2017.11.013

Sartori, F., Mazzucchelli, A., & Gregorio, A. D. (2016). Bankruptcy forecasting using case-based reasoning: The CRePERIE approach. *Expert Systems with Applications*, *64*, 400-411. https://doi.org/10.1016/j.eswa.2016.07.033

Torra, V. (2010). Hesitant fuzzy sets. *International Journal of Intelligent Systems*, 2010: n/a-n/a. https://doi.org/10.1002/int.20418

Torra, V., & Narukawa, Y. (2009). On hesitant fuzzy sets and decision[C]//2009 IEEE International Conference on Fuzzy Systems. Jeju Island, South Korea. *IEEE*, 1378-1382. https://doi.org/10.1109/FUZZY.2009.5276884

Wang, L. Y. et al. (2015). An emergency material demand forecasting model based on intuitionistic fuzzy case-based reasoning. *Journal of China University of Mining and Technology*, *44*(4), 775-780.

Wu, S. S. et al. (n.d.). Extraction and amount estimation of executor's property concealment cases based on heterogeneous information similarity. *China Management Science*, 1-12.

Xu, Z. S., & Xia, M. M. (2011). Distance and similarity measures for hesitant fuzzy sets. *Information Sciences*, *181*(11), 2128-2138. https://doi.org/10.1016/j.ins.2011.01.028

Xu, Z. S., & Xia, M. M. (2011). Induced generalized intuitionistic fuzzy operators. *Knowledge-Based Systems*, *24*(2), 197-209. https://doi.org/10.1016/j.knosys.2010.04.010

Xu, Z. S., & Zhang, X. L. (2013). Hesitant fuzzy multi-attribute decision making based on TOPSIS with incomplete weight information. *Knowledge-Based Systems*, *52*, 53-64. https://doi.org/10.1016/j.knosys.2013.05.011

Yan, A. J., & Ding, K. (2022). Optimization of feature weights for case-based reasoning prediction model based on SHO-SA algorithm. *Journal of Beijing Institute of Technology*, *48*(4), 355-366.

Yan, A. J., Qian, L. M., & Wang, P. (2014). A comparative study of assignment models for case-based reasoning attribute weights. *Journal of Automation*, *40*(9), 1896-1902.

Yao, X. et al. (2021). Intelligent generation of similar cases for landslide disaster emergency based on case-based reasoning. *Systems Engineering Theory and Practice*, *41*(6), 1570-1584.

Yeow, W. L., Mahmud, R., & Raj, R. G. (2014). An application of case-based reasoning with machine learning for forensic autopsy. *Expert Systems with Applications*, *41*(7), 3497-3505. https://doi.org/10.1016/j.eswa.2013.10.054

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

Zadeh, L. A. (1975a). The concept of a linguistic variable and its application to approximate reasoning-I. *Information Sciences*, *8*(3), 199-249. https://doi.org/10.1016/0020-0255(75)90036-5

Zadeh, L. A. (1975b). The concept of a linguistic variable and its application to approximate reasoning-II. *Information Sciences*, *8*(4), 301-357. https://doi.org/10.1016/0020-0255(75)90046-8

Zadeh, L. A. (1975c). The concept of a linguistic variable and its application to approximate reasoning-III. *Information Sciences*, *9*(1), 43-80. https://doi.org/10.1016/0020-0255(75)90017-1

Zhang, Q. et al. (2018). Rule-based and case-based reasoning expert system for fault diagnosis of turbine generator sets. *Electric Power Science and Engineering*, *34*(6), 52-59.

Zhang, W. B. et al. (n.d.). A personalized assisted decision-making method for chronic diseases considering adaptive reaching process. *Systems Engineering*, 1-14.

Zhao, H. et al. (2020). Research on VIKOR hesitant fuzzy multi-attribute decision-making method based on prospect theory. *Practice and Understanding of Mathematics*, *50*(4), 124-136.

Zhao, X. L. (2012). Demand forecasting of emergency supplies for catastrophic earthquakes based on paradigmatic reasoning. *China Journal of Safety Science*, *22*(8), 3-9.

Zheng, J., Wang, Y. M., & Zhang, K. (2017). A case retrieval and attribute weight determination method based on emergency decision-making perspective. *Journal of Shandong University* (Science Edition), *52*(1), 56-64.

Zhu, L. N., Ren, K., & Pu, J. Y. (2022). Case-based reasoning for emergency decision-making modeling of ship pipeline breakage. *Ship Electronic Engineering*, *42*(9), 141-145+150.