Original Paper

Analysis of Theoretical Concepts for Interpretation the Result of the Experimental Studies of Free Groundwater Level Oscillation in Wells

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Abstract

A number of publications present the results of an experimental study of free oscillations of groundwater piezometric level in wells with their eigenfrequencies. The oscillations were initiated by a pulsed impact on the aquifer through the well. Also in a number of publications a theoretical interpretation was proposed for the established phenomenon. However, the existing theoretical ideas about free oscillations of the groundwater level seem to be incorrect. In the present work, a critical analysis of these available theoretical concepts is performed. The analysis served as an impetus to the development of a consistent theory of relaxation filtration of groundwater.

Keywords

aquifer, groundwater level, free level oscillations, theory, critical analysis

1. Introduction

Bredehoeft, Cooper, and Papadopoulos in 1966 studied inertial effects in aquifers on the basis of analog modeling of GroundWater (GW) filtration [6]. The paper shows that one of the likely results of the manifestation of such effects may be the occurrence of waves in aquifers that determine the free oscillations of the piezometric level of GW in wells with eigenfrequencies.

In the middle of the 70s I. Krauss in his works [20, 21] (published in German) presented the results of field experimental studies of some problems of GW dynamics were (Figure 1). Thus, in the experiments, free damped oscillations of the piezometric GW level in disturbing wells after pulsed excitation of the aquifer were recorded. The piezometric GW level in the test well was pressed out by compressed air for several tens of centimeters. After the level stabilization at a certain level, the air pressure in the well were

dropped abruptly, as a result of such a pulsed excitation of the aquifer in the well, free damped level fluctuations with eigenfrequencies were initiated.

In the same papers [20, 21], the first theoretical interpretation of the observed wave effect was proposed. Subsequently, in a number of publications (see, for example, [5, 17, 18, 22]), the results of a theoretical interpretation of the process of free pressure oscillations p (or level H) in a fluid in an aquifer after its pulsed excitation were also considered, including those based on the model proposed in the works of I. Krauss [20, 21]. Later, at the end of the 80s, the works of S.F. Grigorenko [5, 17], devoted to the same problem. Its logical conclusion was the Ph.D. thesis [18] presented by Grigorenko in 1992. It should be noted that in his theoretical constructions he used a physical and mathematical model of GW movement, in principle, not different from that proposed by I. Krauss.

The need of such a theory development is dictated by a number of reasons. On the one hand, such a development promises the creation of a fundamentally new experimental base in hydrogeology; interest in the problem of wave processes in aquifers has already been realized by the emergence of methods for experimental filtration testing (EFT) of water-bearing sediments and the interpretation of the results of these experiments [5, 17, 18, 20-22]. On the other hand, many aspects of these methods are poorly theoretically established and for a number of positions they are simply incorrect.

Such situation required the creation of a theory that adequately represents the results of experimental studies of free oscillations of the piezometric level of the GW in disturbing wells. The theory received the name of the relaxation theory of GW filtration. The theory considering the phenomenon of free oscillations of a piezometric level in a well after pulsed excitation of an aquifer was completed in [7-15, 30, 31]. The authors used a fundamentally different approach, in comparison with the one used I. Krauss and S.F. Grigorenko, to justificate a physical and mathematical filtering model. In these works the possibility of techniques for the EFT of aquifers construction is analyzed based on the theory proposed.
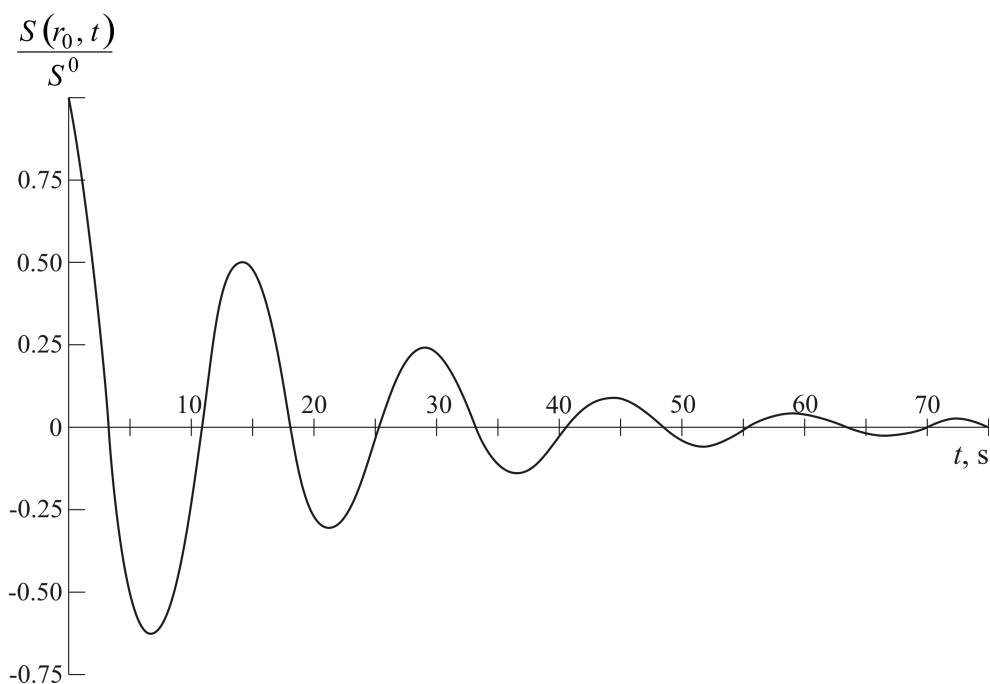
Figure 1. The Indicator Curve for Tracking of the Piezometric Level Oscillation Decrease in the Well (according to I. Krauss [20, 21])

Here $S(r_0, t)$ is the lowering of the level in the perturbing well; S^0 —initial (pressurized by compressed air) lowering of the level at the time of the disturbance start.

Accordingly, in the present work a critical analysis of the basic principles of the physical and mathematical model of the filtration of GW used in the papers by I. Krauss and S.F. Grigorenko has been carried out which prompted the development of the theory of relaxation GW filtration.

It should be noted that in the process of such analysis it is not enough to declare the inconsistence of the available theoretical ideas being analyzed. The fact is that, according to the theory of evidence, such a statement is a negative statement. And it requires not only a detailed analysis of the available theoretical concepts, which allows answering the question why they are not correct, but also answering the next obvious question: which should be the correct theory? In other words, the negative statement made should be supported by the developed alternative theory that correctly represents the process in question, the phenomenon.

2. Method

2.1 Formal Problems of Description of the Pressure Wave Propagation in Aquifer

By now, as it was already noted above, there are a significant number of publications that have studied wave processes in aquifers [2, 6, 20-22, 26, 32]. In these works, the data of studies of forced GW level oscillations in wells are mainly presented, although in [5, 6, 17, 18, 22, 21] the results of field observations were considered, during which the natural oscillations of the level were recorded. In [32],

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the presence of the oscillations having eigenfrequencies was assumed a priori, and the frequencies of forced oscillations were substantiated, which provide resonance with their own. In the majority of published works, mechanical analogs were used in the formal description of experiments (self-damped oscillations of the load attached to the free end of the spring). Such mechanical analogs are the basis for the formalization of forced level oscillations.

In some cases, the description of the propagation of waves with eigenfrequencies in water-bearing formations is carried out on the basis of the already accepted physical and mathematical concepts of GW motion (the main one being the linear filtration law and the filtration equation of parabolic type). However, such a description is incorrect and can not always be used in the analysis of wave processes.

Let us explain what was said by example [7, 13]. In [20] an attempt was made to find a solution to the problem of natural GW level oscillations in a disturbing well and in an aquifer near it. It was supposed that these oscillations are initiated by an instantaneous decrease or increase in pressure in the well (Figure. 2). The process of wave propagation is described by the well-known parabolic equation of a one-dimensional axisymmetric nonstationary filtration, written in the piezometric level declining as:
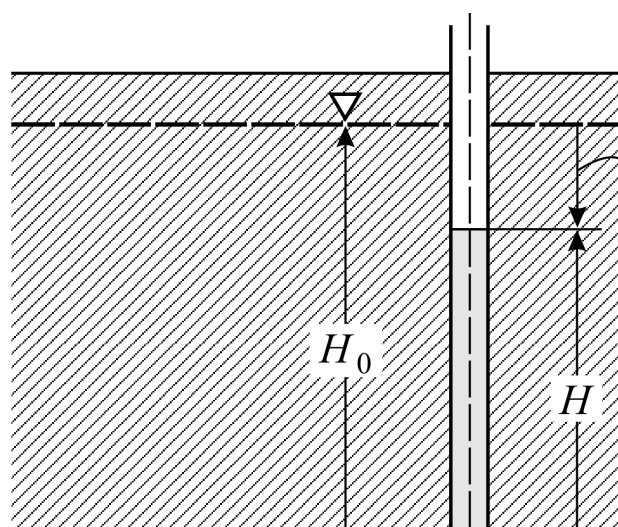

Figure 2. Schematic Diagram of the Well—Aquifer

$$\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r} - \frac{1}{\chi}\frac{\partial S}{\partial t} = 0,$$ (1)

where $S(r,t)$ is the decrease (increase) in the level of GW at the point of the reservoir with coordinate r at the moment of time t from the beginning of the experiment; $\chi = \dfrac{T}{\mu^*}$ —piezoconductivity of the reservoir (T—water conductivity, and μ^*—elastic capacity of water-bearing sediments); $T = Km$ (K is the filtration coefficient of water-bearing sediments, m is the thickness of the aquifer).

Equation (1) was solved under the following boundary conditions [20]:

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$$S(\infty, t) = 0, \quad Tr_0 \left. \frac{\partial S}{\partial r} \right|_{r \to r_0} = -\frac{Q(t)}{2\pi} = \frac{F_F}{2\pi} \frac{\partial S(r_F, t)}{\partial t}. \tag{2}$$

Here $Q(t)$ is the rate of the axial flow of water in the wellbore (inflow of GW into the well or outflow from it); $F_F = \pi r_F^2$ —the area of the internal cross-section of the perturbing well; r_F —radius of the internal cross section of the filter column; r_0 —well radius (outer radius of the filter column).

The solution of the problem with conditions (2) is presented in [20] as following:

$$S(r, t) = f(r) \cdot e^{\gamma t}, \tag{3}$$

$$\gamma = \omega_w \left(-\beta + i\sqrt{1 - \beta^2} \right), \quad \beta < 1, \tag{4}$$

$$\gamma = \omega_w \left(-\beta + \sqrt{\beta^2 - 1} \right), \quad \beta > 1, \tag{5}$$

where $f(r)$ is a function of the r coordinate; γ—parameter; ω_w—eigenfrequency; β—damping parameter of natural oscillations.

Substituting (3) into (1) leads to the ordinary differential equation:

$$\frac{d^2 f}{dr^2} + \frac{1}{r} \frac{df}{dr} - \kappa^2 f = 0, \quad \kappa^2 = \frac{\gamma}{\chi}, \tag{6}$$

the solution of which [1, 28] is:

$$f(r) = C_1 I_0(\kappa r) + C_2 K_0(\kappa r).$$

Here C_1 and C_2 are the constants determined from the boundary conditions; $I_0(\kappa r)$ and $K_0(\kappa r)$—modified Bessel functions of zero order, respectively, of the first and second kinds [1].

Without considering here the final form of the solution obtained in [20], let us analyze the approach for finding it.

From equation (6) it is obvious that the value of γ should be positive. However, in this case it turns out from (3) that the function $S(r, t)$ increases indefinitely, and this is not possible, since additional energy does not enter into the well—aquifer system.

In equation (5) value γ, as it could be easily seen, is negative for any $\beta > 1$. Therefore, when substituting (3) into (1) instead of equation (6), we get the following equation [7, 13]:

$$\frac{d^2 f}{dr^2} + \frac{1}{r} \frac{df}{dr} + \kappa^2 f = 0. \tag{7}$$

The solution of the latter for natural oscillations, provided that the function $S(r, t)$ must remain finite for $r_0 \to 0$, is represented as [23]:

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$$f(r) = C_1 J_0(\kappa r), \tag{8}$$

where $J_0(\kappa r)$ is the Bessel function of the first kind of zero order [1].

If we substitute the value γ from (5) into (3) and use the initial condition $S(r_0, t) = S(r_F, t) = S_0$ and $J_0(\kappa r_0) \to 1$ with $r_0 \to 0$ to find the constant C_1 in (8), then the final solution will be:

$$S(r, t) = S_0 \cdot e^{-\omega_w \left(\beta - \sqrt{\beta^2 - 1}\right) t} J_0(\kappa r).$$

It follows that with an instantaneous decrease (increase) of the level in the well (at $t = 0$), the distribution of the decrease (increase) of the head in the reservoir will also instantly take the form:

$$\frac{S(r, 0)}{S_0} = J_0(\kappa r)$$

(see Figure 3). With time the change in the piezometric level at each point monotonously (without oscillation) decreases according to the law $A\, e^{-\omega_w \left(\beta - \sqrt{\beta^2 - 1}\right) t}$ [7, 13].

Obviously, the solution shown has no physical meaning, since it is difficult to explain the occurrence of standing waves (if they can be called waves at all) in a reservoir unlimited in the radial dimension. In other words, it does not satisfy the requirement of monotonicity of the characteristics of GW motion (in particular, the pressure distribution) within the filtration area for equations of parabolic type in the absence of a periodic component of forced oscillations.

If we specify γ in the form of a complex number [7, 13]:
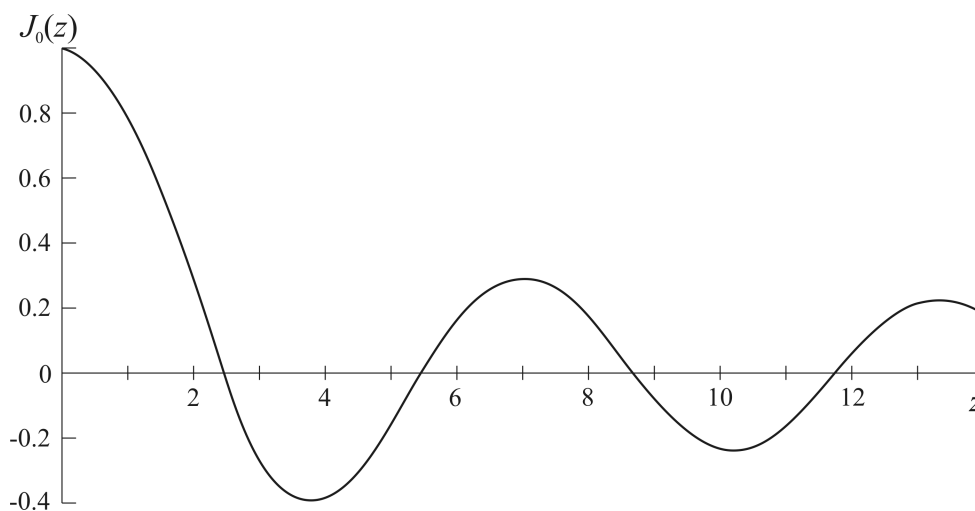
$$\gamma = -\left(\varsigma + i\omega_w\right)$$

Figure 3. Graph of the Bessel Function of the First Kind of Zero Order

(here ς is the oscillation damping parameter), similar to that specified in [20] from equation (4), and use the second condition from (2) to find the constant C_1 in (8), it is easy to show that this condition actually determines damped forced oscillations of the water level in a disturbing well. The detailed justification for such a problem for plane one-dimensional fluid motion is considered in [23]. In our case, the solution is represented as a cylindrical wave:

$$S(r,t) = S_0 \cdot e^{-\varsigma t} \cos \omega_w t \, J_0(\kappa r).$$

So far, based on the assumption of presence of natural oscillations of the GW level, we have considered solutions corresponding to standing cylindrical waves. However, the latter can occur only in finite-volume media [23], and this contradicts the formulation of the problem adopted in [20] (the reservoir is assumed to be unlimited in the radial plane). For forced oscillations of the GW level in an unlimited aquifer, the solution of equation (7) should be sought in the form corresponding to a traveling diverging wave [23]. Therefore we get the following equation:

$$S(r,t) = S_0 \cdot e^{-(\varsigma + i\omega_w)t} \, H_0^{(1)}(\kappa r),$$

where $H_0^{(1)}(\kappa r)$ is the Hankel function [1].

Thus, neither the solutions obtained in [20] nor the equations presented above do not describe the natural oscillations of the GW level. Forced oscillations in unlimited aquifers can be determined by waves with arbitrary frequencies. Accordingly, the latter do not have any unambiguous connection with the geometric and filtration parameters of the aquifer, as assumed in [20, 21] for natural oscillations. In addition, the solution obtained in [20] has nothing to do with the formulation of the problem adopted in the paper (it is important to note that the experimental studies performed by I. Krauss, the results of which are presented in [21], correspond to the formulation of the problem but not its solution) about the instantaneous head decrease (increase) in a perturbation well with the linear law of GW flow leading to a filtration equation of a parabolic type. The correct solution of the problem in this formulation should be fundamentally different from that given in [20], mainly by the absence of a periodic component of the head change [7, 13].

It is not difficult to show that the latter equation in a simplified form in Laplace images (by Laplace-Carson) is:

$$S^0(r, t_p) = \frac{S_0 \, K_0(\kappa r)}{K_0(\kappa r_0) + \dfrac{2\mu^*}{\kappa r_0} K_1(\kappa r_0)}, \quad \kappa^2 = \frac{1}{\chi t_p}. \tag{9}$$

Here $S^0(r, t_p)$ is the image of the function $S(r,t)$; t_p—conversion parameter; $K_1(\kappa r_0)$ is a modified second-order Bessel function of the second kind [1]. The remaining notation is the same.

Solutions of a similar problem in a more general formulation (taking into account the imperfections of the perturbing well by the pattern and degree of opening of the aquifer, by the well and changes in the well filter radius), obtained using the method of integral transforms (Laplace and Laplace-Carson) are given in [16, 29] (in [29], moreover, a review of such solutions is given).

The results of the numerical transform from images of the form (9) to the original, presented in [16, 29] in the form of reference curves make it possible to perform a qualitative analysis of the laws of the GW level restoration in a well after its instantaneous decrease or increase. Such an analysis clearly indicates, in particular (and this is the main thing), the absence of natural level oscillations. Thus, the description of such GW head oscillations in aquifers based on parabolic equations, as proposed in [20, 21], is incorrect.

So, the formal problems of the pressure waves propagation in an aquifer were actually investigated above, but so far the consistency of the original physical and mathematical model of fluid motion has not been considered. Therefore, it remains unclear whether the noted discrepancy is a consequence of partial interpretation errors, or a consequence of the inadequacy of the physical and mathematical model used to describe the phenomenon observed in the experiments. We will try to analyze this physical and mathematical model here.

2.2 Description of the Piezometric Oscillations in a Perturbing Well in a Hydrodynamic Form

In accordance with the ideas of I. Krauss in the already mentioned papers, the process of damped oscillations of a load suspended from a spring fixed by one end acts as a mechanical analogue to the process of level oscillations in the well—aquifer system. This process is described by the following equation:

$$\frac{d^2z}{dt^2} + 2\beta\omega_w \frac{dz}{dt} + \omega_w^2 z = 0,\qquad(10)$$

where z is the coordinate of the displacement of the load relative to the equilibrium position (along the vertical axis z). The remaining notation is the same. Further in [20] the presented model is recorded for the liquid column in the wellbore.

I. Krauss gives the solution of the problem of free oscillations of the load [20]; the same solution for damped oscillations is given in the book by L.D. Landau and E.M. Lifshits [25]. We will not dwell in detail on its presentation; we will consider only the physical assumptions used in the justification of equation (10) and well detailed in the same monograph [25]. In parallel, we will trace the derivation of the equation of motion obtained by I. Krauss on the basis of these assumptions.

In work [25] it is indicated that when the body moves in the medium, the latter provides resistance, tending to slow down the movement, which, strictly speaking, results in damped oscillations. The energy of a moving body in this case goes into heat or dissipates. The process of movement under these conditions is no longer purely mechanical; its consideration requires consideration of the movement of the medium itself and the internal thermal state, both of the medium and of the body. However, as noted in [25], there is a category of phenomena for which motion in a medium can be described using

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mechanical equations of motion by introducing some additional terms into them. These include, in particular, oscillations with frequencies that are small compared with those characteristic of internal dissipative processes in the medium. When this condition is fulfilled, it is permissible to assume that a friction force acts on the body, depending only on its speed.

If this speed is small enough, then the friction force can be expanded in its powers. The zero term of the expansion is equal to zero, since the friction force does not act on a fixed body, and the first non-vanishing term is proportional to the velocity [25]. Accordingly, we will assume that the fluid moving laminarly in the wellbore experiences a resistance proportional to the first degree of movement velocity. Considering the liquid being purely viscous, and neglecting the change in its mass in the well, we can present the force of resistance (friction) as:

$$f_f = -\alpha \frac{dz}{dt}, \tag{11}$$

where α is a positive coefficient. The minus sign indicates that the force acts in the direction opposite to the direction of the velocity vector of the body (water column in the wellbore).

At the bottom of the water column in the wellbore there is a pressure force from the aquifer:

$$f_p = -\pi r_F^2 \left[p_0 - p\left(r_F, t\right) \right],$$

where r_F is, as before, the radius of the internal cross section of the filter column, $p\left(r_F, t\right)$ is the current value of pressure in the fluid in the opened part of the aquifer (in the well itself), and p_0 is the pressure in the fluid in the undisturbed formation.

According to the Dalamber principle, the sum of the forces acting on the system of material points is equal to the inertia force [19]. The latter, in neglecting the change in the mass of fluid in the well, is:

$$f_{in} = m \frac{\partial^2 z}{\partial t^2}.$$

From here we get for $r = r_0$

$$-m \frac{1}{t_0} \frac{\partial z}{\partial t} - m \frac{\partial^2 z}{\partial t^2} = \pi r_F^2 \left[p_0 - p\left(r_F, t\right) \right],$$

or, after simple transformations—

$$\frac{\partial^2 z}{\partial t^2} + \frac{1}{t_0} \frac{\partial z}{\partial t} = g \left(1 - \frac{H_0}{H} \right), \tag{12}$$

where m is the mass of fluid in the wellbore; $p = \rho g H$ and $m = \rho \pi r_F^2 H$; H—piezometric pressure in the well; H_0—piezometric head in an unperturbed well; t_0—characteristic time.

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The last equation completely coincides with the equation of free damped oscillations of a water column in a well bore, used in [20, 21].

It is necessary to pay attention to the fact that the term on the right-hand side of equation (11) is included in the algebraic sum on the left-hand side of equation (12) in the form $-\dfrac{m}{t_0}\dfrac{\partial z}{\partial t}$, in other words, in (12) it is assumed that

$$\alpha = \frac{m}{t_0}.\tag{13}$$

Such a transformation follows from the relation introduced in [25]:

$$\frac{\alpha}{m} = 2\beta,$$

determining the coefficient α through the parameters of the process of its own damped oscillations—the mass of the load fixed at the free end of the spring m, and the damping coefficient of the oscillations β. Taking into account that $\beta = \dfrac{1}{2t_0}$ [8, 13], one can easily follow from the last expression to relation (13).

When analyzing the obtained equations of free damped oscillations, it is necessary to pay attention to two circumstances that determine the physical features of the presented model, which were completely ignored in [20]. First, the model was based on a system with concentrated parameters. Accordingly, the mass of the moving body in (12) is assumed to be concentrated in a limited volume of space, so that, in accordance with the presentation of equations (10) and (12), the elastic force of the spring is applied to the whole body mass. Secondly, equation (12), as noted above, is true for an isolated dissipative system, and the system is limited only by the wellbore. In other words, in the model of damped level oscillations in the well, according to the condition, there is no provision for the exchange of energy and matter (water) between the well and the aquifer. The energy source of the level oscillations themselves can only be the energy of elastic compression of the fluid in the wellbore.

In order to overcome this difficulty, in [5, 17, 18, 20, 21], the effect on the reservoir in the model is proposed to take into account by applying the boundary condition of the form (2), written in [20] in the following form:

$$-Tr\left.\frac{\partial H(r,t)}{\partial r}\right|_{r\to r_0} = \frac{Q(t)}{2\pi} = r_F^2\,\frac{\partial z}{\partial t}.\tag{14}$$

Here $H(r,t)$ is the piezometric head at the point of the aquifer with the coordinate r at the moment of time t from the start of the disturbance; the remaining notation is the same.

We repeat, an isolated dissipative system cannot exchange mass with other systems, the exchange of energy for an isolated system is also prohibited. Thus, a condition of the form (14) connects two systems,

initially (when deriving the equation of motion), are not related. In this case, the damped oscillations are considered separately in an isolated oscillatory system—a well, and condition (14), albeit illegally, should provide a description in the model by I. Krauss and S.F. Grigorenko transfer the power of the oscillations of this system to the aquifer. In other words, according to the approach to describe the propagation of oscillations in an aquifer, two independent problems of fluid movement are taken: one describes the oscillations of a water column in a wellbore with eigenfrequencies (in an isolated system that does not exchange mass and energy with other systems, and the damping of oscillations in this system is determined only by the friction of the fluid on the inner walls of the well), the other describes the forced oscillations of the GW level in the aquifer (excluding the well itself)—an open system with the linear law of filtration and in the absence of the inertial component of resistance to the motion of a fluid. Both of these systems are artificially connected by condition (14). In this case, indeed, the propagation of waves in an aquifer can be described by a parabolic filtration equation, however, all the given constructions for the formal description of GW level oscillations with eigenfrequencies in the works by I. Krauss and S.F. Grigorenko are meaningless.

An example of namely such a description of the propagation and registration of waves in aquifers can be found in the paper by A.Zh. Muftakhova [26]. In this paper are considered both, the reaction of a well in the riverine zone to oscillations in the level of GW caused by fluctuations in the water level in the river and the possibility to determine the filtration parameters of the aquifer based on the data tracking. At the same time, such oscillations are represented in [26] by one or several superimposed harmonics, and the plane flow of the GW is uniquely directed from the river to the well. Obviously, in this case, there are no doubts that the natural or forced oscillations of the level of GW are registered in the well.

In general, it is puzzling that in the works by I. Krauss to build a physical and mathematical model of oscillations of a water column in a well with its eigenfrequencies, an analogy with the oscillations of a load suspended at the free end of a spring rigidly fixed by the other end, i.e. a kinematic model is used. Such a model does not provide an estimate of the eigenfrequencies based on the parameters of the system itself (for example, the water compression modulus, its density).

Yet this possibility is obvious. After all, if we neglect a change in the mass of fluid in the wellbore, as it is used in the work by I. Krauss [20], then we inevitably come to the problem of longitudinal natural oscillations of the rod (length H_0) with one end fixed, described in detail in [24] (a hydrodynamic model). And this solution already provides the possibility of calculating the eigenfrequencies of such oscillations, based on the parameters of the material (liquid) from which the rod is made, and its geometrical characteristics (length). Due to the isolation of such an oscillatory system, these frequencies are in no way dependent on the filtration parameters and spatial dimensions of the aquifer.

It is necessary to emphasize the following. Earlier, in [13], with reference to the work [3, 4], the justification of the isothermal filtration process of a liquid in a porous medium was provided. At the same time, if there is a movement in a deformable body, then the temperature of such a body, generally

speaking, is not constant, but varies with time from point to point along the body. This circumstance greatly complicates the exact equations of motion.

However, the situation is simplified due to the very slow transfer of heat from one part of the body to another (by means of simple thermal conductivity). If it can be assumed that heat exchange practically does not occur during periods of time on the order of the oscillatory motion period in the body, then each part of the body is considered to be thermally isolated, i.e the motion is adiabatic [24]. Accordingly, adiabatic values are taken as the calculated values of the compression modulus.

Longitudinal oscillations in the rod (along the vertical axis z, coinciding with the longitudinal axis of the rod) are described by the equation of the form [24]:

$$\frac{\partial^2 H}{\partial z^2} - \frac{\rho}{\left(E_\rho\right)_{AD}} \frac{\partial^2 H}{\partial t^2} = 0,\qquad(15)$$

where $\left(E_\rho\right)_{AD}$—adiabatic values of the compression modulus; ρ is the density of the rod material (water in the well bore) as mentioned before. The remaining notation is the same.

In equation (15) there is no member that determines the resistance to the movement of fluid in the wellbore due to its wall friction . Accordingly, the oscillations are turn out to be continuous. Obviously, this is a definite abstraction, however, equation (15) provides a completely reliable estimate of the eigenfrequencies of water level fluctuations in a well.

At the fixed end of the rod (at the top level of the pressure reservoir), with $z = 0$, should be $H = 0$, and at the free end (static piezometric water level in the well), with $z = H_0, -\dfrac{\partial H}{\partial z} = 0$ [24].

The solution of equation (15) is found in the form [24]:

$$H = A\cos\left(\omega t + \alpha\right)\sin kz, \quad k = \omega\sqrt{\frac{\rho}{\left(E_\rho\right)_{AD}}}.$$

All notation here is the same.

From the condition when $z = H_0$ one can obtain $\cos\left(kH_0\right) = 0$, where for the eigenfrequencies is

$$\omega = \sqrt{\frac{\left(E_\rho\right)_{AD}}{\rho}}\ \frac{\pi}{2H_0}\left(2N + 1\right),\qquad(16)$$

where N is integer.

The relationship of the adiabatic and isothermal modulus of water compression is expressed by the equation [24]:

$$\frac{1}{\left(E_\rho\right)_{AD}} = \frac{1}{E_\rho} - \frac{T\alpha^2}{c_p}, \quad \left(\frac{\partial V}{\partial T}\right)_p = \alpha, \tag{17}$$

where E_ρ is, as before, the water compression modulus (isothermal); T—absolute water temperature;

c_p is the specific heat of water at constant pressure; α is the relative change in the volume of water V

when heated.

Taking into account the fact that the water temperature in the wellbore during the initiation of level oscillations according to the method by I. Krauss and S.F. Grigorenko actually changes only by fractions of a degree, with great accuracy the relative change in the volume of water when heated α can be

neglected, i.e. in the first equation in (17) we can assume $\left(E_\rho\right)_{AD} \cong E_\rho$. In addition, as follows from

equation (16), such a premise (including $N = 0$) provides the absolute minimum estimate of the eigenfrequency level oscillations.

Let us perform here such an assessment. Taking the water compression modulus $E_\rho = 2.03 \cdot 10^9$ Pa [3,

4], water density $\rho = 1000$ kg/m^3, piezometric head $H_0 = 100$ m and N, as already noted, equal to 0, from equation (16) we get that the probable minimum eigenfrequency of oscillations of the piezometric level in the wellbore is $\omega \cong 22.4$ s^{-1}. Obviously, as it follows from (16), with decreasing H_0 and increasing N, the eigenfrequencies should increase.

From the above test, it follows that the characteristic minimum value of the eigenfrequency of the level in the well (we repeat, considered as an isolated system) significantly (approximately by 2-3 orders of magnitude) exceeds the frequencies obtained at the EFT of real aquifers and is equal in order of magnitudes $\omega \cong \cdot n \cdot (10^{-2}\text{-}10^{-1})$ s^{-1}. This is primarily determined by the low value of water compressibility. Then it becomes unclear what, in fact, determines the water level oscillations in a well with eigenfrequencies of the order of $\omega \cong \cdot n \cdot (10^{-2}\text{-}10^{-1})$ s^{-1}, recorded in experiments [20, 21]?

So, the kinematic model of the oscillation of the load, suspended at the free end of the spring, used as an analogue for description of the free oscillations of the piezometric level in the well, does not allow to perform the test calculations to assess its applicability. It is simply not intended to calculate the eigenfrequencies using the "internal" parameters of the oscillating system—the mass of the suspended load, the geometric parameters (for example, the length) of the spring and spring rate; the model does not contain physical prerequisites for oscillations. By the way, for this reason, the entire physical interpretation of the natural oscillations of the level in the wells in the works by S.F. Grigorenko [5, 17, 18] is reduced to the arguments about some "transient processes" in the system of a well—aquifer.

In addition, the absence of strict initial physical notions about the process allows to operate arbitrary and without any justification (including the introduction of new ones) with boundary conditions of the system. Thus, the water exchange condition is introduced into the kinematic model.

The need to ensure consistent interpretation of the oscillation process of the piezometric level in the well involuntarily causes a direct association with the oscillating load in the well, fixed on the free end of the spring (see Figure 4), prompted by the concepts of oscillatory processes used in [20, 21]. Only in this case a source of energy necessary for initiation of oscillations appears, and the natural oscillations of the water column in the well will not be determined by the parameters of this column (its length H_0 and water compressibility), but by the characteristics of the spring and the inertia of the load suspended to it. Attenuation of oscillations will be determined by the friction of the load on the water.

However, then the initial premise that both oscillations of the water column in the wellbore and pressure (or piezometric pressure) oscillations in the aquifer (outside the perturbating well itself) will become forced oscillations. And then it becomes indifferent what type of equation (parabolic or some other type) will describe the process of propagation of pressure waves in the aquifer.
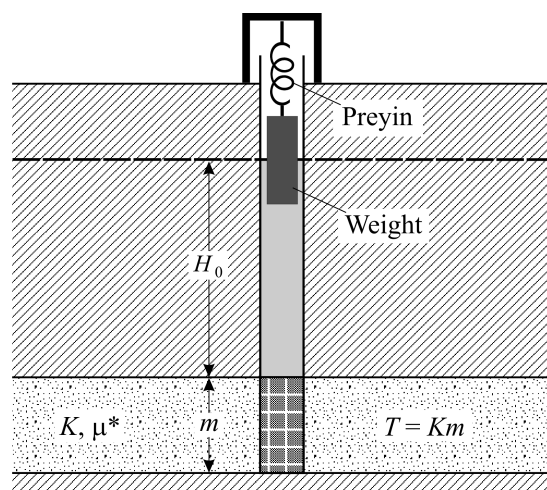

Figure 4. Scheme of Initiation of Oscillations of a Piezometric Level in a Well According to I. Krauss

Thus, a system in which oscillations with eigenfrequencies can be observed should either be isolated but unified (i.e., oscillations should be considered as a single process at once in the entire system), or an open system, initially connected to another system, only in this case, the exchange of mass (water) and energy between the aquifer and the well is possible. In this case, the source of oscillations should be the aquifer, and the probe well should be considered as a reacting system, although the primary impact on the reservoir goes through the well.

In order to open the system, you must enter the flow component [27]. As such a streaming component, for opening a mechanical system, we can consider the equation of GW filtration. In this case, the flow

component is considered as applied to the system (aquifer) with distributed parameters, therefore it should be of a gradient type.

Let us repeat the derivation of an equation that provides a description of free decaying oscillations of a piezometric level in a well with eigenfrequencies, taking into account the flow component.

So, the fluid in the well in accordance with the third law of Newton is acted upon by the fluid in the aquifer. Taking into account the relation of change in pressure in the aquifer in the process of its perturbation, and neglecting the change in the mass of fluid in the well, we will present this force as following:

$$f_p = m \frac{\chi}{t_0} \Delta H(r_0, t).$$

Here t_0 is, as before, the characteristic time; $H(r_0, t)$ is the pressure at the point of the aquifer with the coordinate r_0 (on the borehole wall) at the moment of time t from the beginning of the disturbance; Δ is the Laplace operator. This is exactly the flow component that ensures the opening the well—aquifer system.

The moving laminarly in the wellbore, fluid experiences resistance proportional to the first degree of movement speed. Assuming the fluid is purely viscous, and again neglecting the change in its mass in the well, we can present the force of resistance (friction), as in (11), in the following form:

$$f_f = -\frac{m}{t_0} \frac{\partial H(t)}{\partial t}.$$

As before, according to the Dalamber principle, we consider that the sum of the forces applied to the system of material points is equal to the force of inertia. The latter neglecting the change in the mass of fluid in the well can be written as:

$$f_{in} = m \frac{\partial^2 H(t)}{\partial t^2}.$$

Hence, using the principle of continuity, in other words, assuming that $H(r_0, t) = H(t)$, where $H(t)$ is, as before, the piezometric head in the well, we obtain for $r = r_0$

$$-m \frac{1}{t_0} \frac{\partial H}{\partial t} + m \frac{\chi}{t_0} \Delta H = m \frac{\partial^2 H}{\partial t^2}. \tag{18}$$

Since all the forces in equation (18) applied to the same mass of fluid, then this equation can be simplified and written as following:

$$t_0 \frac{\partial^2 H}{\partial t^2} + \frac{\partial H}{\partial t} - \chi \Delta H = 0 \quad \text{or} \quad t_0 \frac{\partial^2 p}{\partial t^2} + \frac{\partial p}{\partial t} - \chi \Delta p = 0,$$

where, as before, $p = \rho g H$.

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Thus, the correct formulation of the problem leads to a completely different type of filtration equation, compared to that used in the works by I. Krauss and S.F. Grigorenko,—to the hyperbolic (wave) equation. At the same time, the introduction of the flow component fundamentally changes the characteristics of the system—the natural oscillation frequencies in such a system are determined by the geometrical and filtration parameters of the disturbed part of the aquifer.

However, it is not yet clear from the last equation how the level oscillations in an aquifer with eigenfrequencies can be described, for now nothing has been told about the geometrical parameters of the disturbed region in the aquifer, but it has been repeatedly emphasized with reference to L.D. Landau and E.M. Lifshits [23], that the eigenfrequencies strongly depend on the geometric characteristics of this region, its limitations in the plane. In other words, the question comes to the fore, if oscillations of the piezometric level with eigenfrequencies are recorded during the EFT of an aquifer, then the condition of limitation in plane of the perturbation region should be a prerequisite for this. What physical prerequisites determine such a condition? Which in this case becomes the physical meaning of the characteristic time t_0? Answers to these questions in accordance with the principle of constructivism actually require the development of a new filtration model (theory) that provides an adequate description of the process of propagation of pressure waves (or pressure) in an aquifer.

3. Result

A critical analysis of the available theoretical ideas about the process of pressure waves propagation in the aquifer is presented. These waves determine free oscillations of the GW piezometric level with eigenfrequencies in the disturbing well, through which the impulse impact on the aquifer was applied.

Firstly, the description of such a process based on a parabolic-type filtration equation that does not take into account the inertial component of the resistance to the GW motion, without the need to specify the boundary condition determining the oscillations of the piezometric GW level in the well due to some extraneous influence, in principle is possible. And then these oscillations are not free, but forced, and the frequencies of such oscillations are determined by the given frequencies of such an external source of oscillations.

Secondly, in the existing theoretical concepts, a perturbation well is actually considered as an isolated dissipative system. The estimation of the eigenfrequencies of the longitudinal free oscillations of the water column in the well significantly (by 2-3 orders of magnitude) exceeds the frequencies obtained by the EFT of real aquifer horizons. In other words, such a system cannot adequately represent the phenomenon of free oscillations of the piezometric level observed in the experiments in a probe well with eigenfrequencies.

Thirdly, the correct formulation of the problem leads to a completely different type of filtration equation for the GW, as compared to that used in the existing theoretical concepts, to a hyperbolic (wave) equation. At the same time, the introduction of a flow component fundamentally changes the characteristics of a disturbing (experimental) well—aquifer system—it should become isolated, but uniform (ie, oscillations

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should be considered as a single process within the whole system). Only in such a system the eigenfrequencies are determined by the geometrical and filtration parameters of the disturbed part of the aquifer. In this case, the source of oscillations should be the aquifer, and the probe well should be considered as a reacting system, although the primary impact on the reservoir has been made through this well.

Namely these provisions that were used as the basis for the theory of relaxation filtration, developed by the author, and will be presented in subsequent publications in English.

4. Discussion

I believe that in this article I was able to convincingly show the inconsistency of the currently used theoretical concepts of free oscillations in the GW level in disturbing wells to describe the interpretation of these oscillations. There is an obvious need to develop a consistent physical and mathematical theory that provides such a description and interpretation. I have proposed a version of such a theory, which will be presented in subsequent publications in English.

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