

## Original Paper

# Research on Monitoring Plan for Xiaolangdi Reservoir Based on Nonlinear Programming

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### **Abstract**

*The water and sediment flux of the Yellow River fluctuates under the influence of various natural and human factors, significantly impacting ecological management and water resources in the Yellow River basin. This study investigates the patterns of sediment-water flux variation in the Yellow River and evaluates the practical effectiveness of the "water diversion and sediment regulation" project. Utilizing multiple linear regression models, seasonal ARIMA models, and grey prediction models, it comprehensively employs Excel, Matlab, and SPSS to analyze the relationship between sediment concentration and time, water level, and flow rate. The research summarizes the patterns of sediment-water flux variation and assesses the actual outcomes of the "water diversion and sediment regulation" initiative to enhance Yellow River basin management and address practical challenges.*

*Addressing the first issue, Excel was first used to clean the water-sediment monitoring data from a specific hydrological station on the Yellow River between 2016 and 2021 (Appendix 1), removing invalid data lacking sediment concentration values. Subsequently, Matlab was employed to plot scatter diagrams of sediment concentration versus time, water level, and flow rate. These plots indicated a preliminary linear relationship between sediment concentration and water level/flow rate, suggesting the applicability of a multiple linear regression model. SPSS was employed to solve the multiple linear*

regression model, yielding regression coefficients of 0.568 for water level and 0.002 for flow rate, confirming the linear relationship between independent and dependent variables. A multiple linear regression model was then re-established to estimate sediment concentration in the Yellow River over the past six years. Excel was used to calculate annual total water flow and annual total sediment discharge.

For Problem 2, the water-sediment flux research methodology was first defined, dividing water-sediment flux into water flux and sediment flux. Water flux was measured by flow rate indicators, while sediment flux was measured by sediment discharge indicators. Excel was used to filter and consolidate the required research data. Second, employing a “combination of numbers and shapes,” annual line charts for water and sediment fluxes were plotted, along with 72-month variation patterns (January to December) for each year from 2016 to 2021. This enabled visual analysis of the hydrological station’s water-sediment flux characteristics, including abrupt changes, seasonality, and periodicity.

For Problem 3: First, aggregate the water and sediment flux data from 2016 to 2021 at this hydrological station. Construct a seasonal ARIMA model and solve it using SPSS to verify that the original sequence passes the stationarity test. Analyze the autocorrelation and partial autocorrelation plots to obtain the model results. Use Excel to plot line charts showing the future two-year trends in water and sediment flux at this hydrological station. Second, based on the patterns of water and sediment flux variation, a nonlinear programming model was constructed. Using Matlab to solve the nonlinear programming model, an optimal sampling and monitoring plan for the next two years was formulated. This approach aims to understand the dynamic changes in water and sediment flux while minimizing sampling and monitoring costs and resource utilization.

For Question 4, first use Excel to plot a line chart based on the water and sediment data from May to October 2016–2021 in Appendix 1 and the riverbed elevation data in Appendix 3. This will enable a comparative analysis of the actual effectiveness of the “water and sediment regulation” measures at Xiaolangdi Reservoir. Second, exclude the June data from Appendix 2 to calculate the elevation changes (either rise or fall) of the riverbed over the two-year period. Construct a grey prediction model and solve it using SPSS to forecast the future changes in riverbed elevation over the next decade.

### **Keywords**

*Multiple linear regression, Seasonal ARIMA model, Nonlinear programming model, Grey prediction model*

## **1. Problem Restatement**

### *1.1 Problem Background*

With deepening industrialization and accelerated development, ecological governance of the Yellow River faces unprecedented challenges. The 14th Five-Year Plan for National Economic and Social Development of the People’s Republic of China and the Long-Range Objectives Through the Year 2035,

released in 2021, explicitly calls for deepening regional major strategies and advancing ecological conservation and high-quality development in the Yellow River Basin (Han, 2023). Thus, investigating the patterns of water-sediment flux in the Yellow River basin is not only a practical necessity for resolving the current ecological governance challenges, implementing ecological development concepts, reinforcing green value principles, and transforming the Yellow River into a river that benefits human well-being, but also an essential requirement for promoting sustainable economic and social development (Mao & Liu, 2024).

### *1.2 Problem Restatement*

This paper primarily analyzes and mines monitoring data on the Yellow River's water and sediment, cross-sections, and other parameters to investigate the patterns of water-sediment flux variation and its impacts on environmental governance, climate change, and people's livelihoods along the river basin. This research aims to provide theoretical guidance for water resource allocation, human-land relationship coordination, and flood and drought disaster prevention in the Yellow River Basin.

Data Set Introduction: Appendix 1 contains hydrological monitoring data (water level, discharge, and sediment concentration) from a specific Yellow River hydrological station between 2016 and 2021. Appendix 2 provides cross-sectional measurement data for the station over the past six years, while Appendix 3 offers monitoring data from selected observation points at this station. Based on these datasets, mathematical models will be developed to address the following research questions:

**Problem 1:** Investigate and analyze the relationship between sediment load, water level, and discharge in the Yellow River basin. Based on this relationship, estimate the annual total water volume and annual total sediment discharge for each of the six years from 2016 to 2021.

**Problem 2:** Based on the data compiled in Problem 1, investigate the abrupt changes, seasonal patterns, and periodic characteristics of sediment-water flux at this hydrological station over the past six years, and summarize the recent trends in sediment-water flux of the Yellow River.

**Problem 3:** Based on the patterns of sediment-water flux variation, predict the future trends of sediment-water flux at this hydrological station for the next two years. Subsequently, develop an optimal sampling and monitoring plan to minimize monitoring costs and resource utilization.

**Problem 4:** Based on the hydrological and sediment flux data and riverbed elevation changes at this hydrological station over the past six years, investigate the actual effectiveness of the Xiaolangdi Reservoir's "water and sediment regulation" project conducted annually from June to July. Subsequently, predict the impact on riverbed elevation in the Yellow River basin ten years from now if the "water and sediment regulation" project is not implemented.

## **2. Problem Analysis**

### *2.1 Analysis of Problem 1*

Regarding Question 1, which requires investigating the relationship between sediment load and water level/flow rate using known monitoring data from 2016 to 2021, the first step is to compile daily water

level and flow rate measurements at 8:00 AM for each day over these six years that include sediment load data. Given the presence of two independent variables (water level and flow rate) and the availability of three indicators over nearly six years, preliminary analysis suggests this data should be suitable for multiple linear regression analysis. Therefore, a multiple linear regression model can be attempted. Substituting the preprocessed data yields the linear relationship between sediment load and water level/flow rate. Additionally, Question 1 requires estimating annual total flow and annual sediment discharge based on this linear relationship. First, provide the calculation formula for sediment discharge to facilitate subsequent computations and descriptions. Second, the sum of water flows at each time point can be considered as the estimated annual total water flow. Finally, based on the linear relationship among sediment content, water level, and water flow, missing sediment content data for the six time points other than 8:00 daily can be supplemented. Combined with the sediment discharge calculation formula, the sum of sediment discharge at each time point can be used as the estimated annual total sediment discharge.

### *2.2 Analysis of Problem 2*

Regarding Problem 2, which requires investigating the abruptness, seasonality, and periodicity of water-sediment flux, the first step is to define the concept of water-sediment flux to determine the research methodology. Excel is then used to filter, merge, and aggregate the overall data into the required dataset for analysis. Additionally, to analyze these characteristics more intuitively and clearly, the aggregated data will be plotted as line graphs to identify abrupt changes, seasonal patterns, and periodic cycles in sediment-water flux.

### *2.3 Analysis of Problem 3*

Regarding Question 3, it requires predicting the future two-year trends in water-sediment flux based on the patterns identified in Question 2, and developing an optimal sampling monitoring plan. First, considering the pronounced seasonality of the data provided in this paper, it is preliminarily decided to employ a seasonal ARIMA model to forecast the future two-year trends in water-sediment flux. Second, a nonlinear programming model will be constructed. Taking the first year's sampling monitoring plan as an example for prediction, the second year's plan will be inferred by analogy.

### *2.4 Analysis of Problem 4*

For Question 4, we first consider using Excel to plot a line chart comparing data before and after June to analyze the actual effectiveness of the "water and sediment regulation" measures. Second, we consider constructing a grey prediction model to forecast the impact of not implementing "water and sediment regulation" on the future ten-year changes in riverbed elevation based on elevation rise and fall data over the two-year period.

## **3. Model Assumptions**

- 1) Assume the data provided in the question stem is accurate and reliable.
- 2) Assume that the monitoring data from this hydrological station on the Yellow River downstream of

Xiaolangdi Reservoir reflects the overall conditions of the Yellow River basin, possessing typicality and representativeness.

3) Assume the sediment discharge calculation formula is the product of sediment concentration and water flow rate.

4) Assume that the research method for water-sediment flux combines studies of water flux and sediment flux.

#### 4. Symbol Explanation

The primary symbols used in this paper and their meanings are shown in Table 1:

**Table 1. Symbol Explanation**

No.	Symbol Name	Symbol Meaning
1	$\hat{y}$	Sediment content of Yellow River water
2	$X_1$	Water Level
3	$X_2$	Flow rate
4	$i$	Month
5	$j$	This hydrological station monitoring point
6	$x_{ij}$	Monitoring point for the current month
7	$C_{ij}$	Monitoring cost
8	$f_j$	Monitoring Point Capacity
9	$b_i$	Monitoring Requirements

#### 5. Model Development and Solution

##### 5.1 Model Establishment and Solution for Problem One

Given that the sediment content of the Yellow River at the hydrological station correlates with time, water level, and water flow rate, this study establishes a multiple linear regression model to further investigate the relationship between sediment content and these variables. Here, time, water level, and water flow rate serve as independent variables, while sediment content is the dependent variable. Since the relationship between sediment content and time, water level, and water flow rate in the Yellow River remains relatively stable over a specific period without significant fluctuations, this study uses relevant data from in 2016 to investigate this relationship.

##### 5.1.1 Data Preprocessing

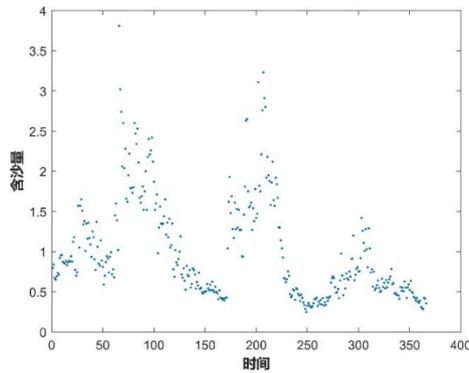
The data in Appendix 1 provided herein is characterized by its large volume, encompassing various parameters recorded at seven specific times daily—0:00, 4:00, 8:00, etc.—from 2016 to 2021. Since the provided sediment load data only pertains to the 8:00 time slot each day, the water level, flow rate, and sediment load data for this time slot over the past six years have been aggregated on a daily basis.

The data is presented in the appendix.

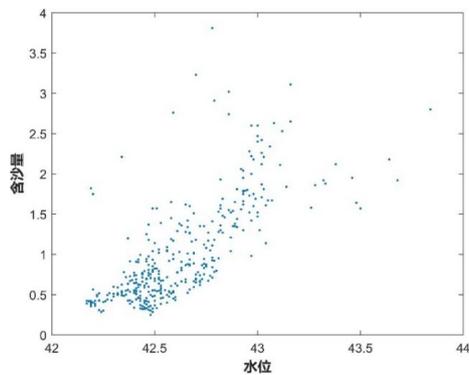
### 5.1.2 Linear Relationship Assessment

Initially, scatter plots were generated using Matlab to visualize the relationship between sediment content and time, water level, and flow rate, enabling a preliminary assessment of linear relationships between independent and dependent variables.

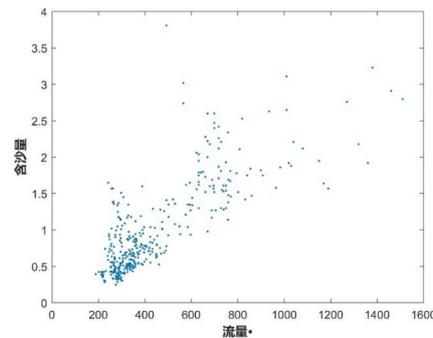
The scatter plots reveal a strong linear correlation between sediment content and both water level and flow rate, while no linear correlation exists with time.



**Figure 1. Scatter Plot of Time and Sediment Load**



**Figure 2. Scatter Plot of Water Level Versus Sediment Content**



**Figure 3. Scatter Plot of Flow Rate Versus Sediment Content**

### 5.1.3 Model Development

The expression for the multiple linear regression model is as follows:

$$\hat{y} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \dots + \beta_pX_p + \varepsilon \tag{1}$$

In equation (1),  $\hat{y}$  represents the dependent variable, which in this paper refers to the sediment load of the Yellow River.  $\beta_0$  is the constant term.  $\beta_i$  ( $i=1,2,\dots,p$ ) denotes the average change in the dependent variable Y when a specific independent variable X changes by one unit, while the other independent variables  $X_i$  remain constant.  $\varepsilon$  is the residual, representing the difference between the actual value of the dependent variable and its estimated value, and is a random variable.

5.1.4 Model Solution and Analysis

We employed SPSS to solve the multiple linear regression model. The results are as follows:

**Table 2. ANOVAa Table (Intermediate Process)**

Model	Sum of Squares	Degrees of Freedom	Mean Square	F	Significance
1 Regression	95.128	3	31.709	230.356	.000 <sup>b</sup>
Residual	49.831	362	.138		
Total	144.959	365			

As shown in the table above,  $F=230.356$ ,  $p<0.05$ , indicating that the linear regression model is statistically significant overall, meaning that at least one X variable influences Y.

**Table 3. Model Summary**

Model	R	R <sup>2</sup>	Adjusted R-squared	Standard error of the estimate	R-squared change	F change	Degrees of freedom 1
1	.810 <sup>a</sup>	.656	.653	.371017	.656	230.356	3

As shown in the table above, the adjusted R-squared value is 0.653, indicating that the established regression model explains 65.3% of the variation in sand content, demonstrating good model fit.

**Table 4. Linear Regression Analysis Results**

Model	Unstandardized Coefficient (B)	Unstandardized Coefficient (Standard Error)	Standardized Coefficient (Beta)	t	Significance	Collinearity Statistics (Tolerance)
(Constant)	-23.880	5.078		-4.703	.000	
Date	-5.167E-9	.000	-.059	-1.809	.071	.900

Water level (m)	.568	.120	.249	4.726	.000	.342
Flow rate (m <sup>3</sup> /s)	.002	.000	.576	11.042	.000	.349

As shown in the linear regression analysis results table, the p-values for the t-tests corresponding to water level and flow rate are both less than 0.05, indicating significant characteristics. The p-value for time is greater than 0.05, so water level and flow rate have a significant effect on sediment content, while time has no effect on sediment content. The tolerances are all greater than 0.2, indicating no multicollinearity among the independent variables.

Performing linear regression analysis with water level and flow rate as independent variables and sediment content as the dependent variable yields a significant regression model:  $F=230.356$ ,  $p<0.05$ .

The model equation is:

$$\hat{y} = -23.880 + 0.568 * X_1 + 0.002 * X_2 \tag{2}$$

Thus, sediment load exhibits a linear correlation with water level and discharge. Specifically, water level (Beta=0.249,  $p<0.01$ ) and discharge (Beta=0.576,  $p<0.01$ ) exert significant positive effects on sediment load, while time (Beta=-0.059,  $p=0.071$ ) has no impact.

Following this approach, we employed the same methodology to establish another multiple linear regression model using sediment load, water level, and discharge data from the Yellow River over the past six years. This model estimated the sediment load for the same period, ultimately calculating the annual total discharge and annual total sediment discharge. Our results are presented in the table below:

**Table 5. Summary of Results**

Year	Annual Total Water Flow (m <sup>3</sup> )	Annual Sediment Discharge (kg)
2016	1,089,444	127,335.028
2017	1079887	1612606.454
2018	4,406,970	22,715,325.67
2019	4,601,865	26,377,458.66
2020	4,951,508	32,291,741.77
2021	5,386,611	39,355,610.45

## 5.2 Model Development and Solution for Problem 2

### 5.2.1 Data Preprocessing

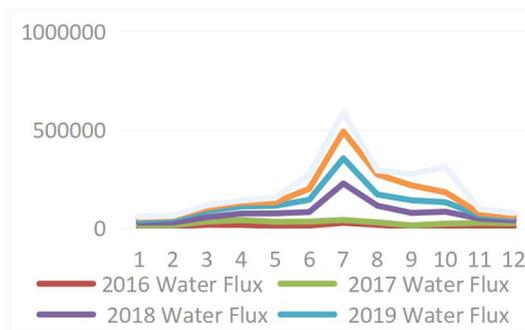
To facilitate the study of characteristics such as abrupt changes, seasonality, and periodicity in water and sediment flux, the data in Appendix 1 were aggregated and analyzed on a monthly basis. Using Excel to process the data in Appendix 1, monthly flow and sediment discharge data for 2016–2021

were obtained and subjected to integrated analysis.

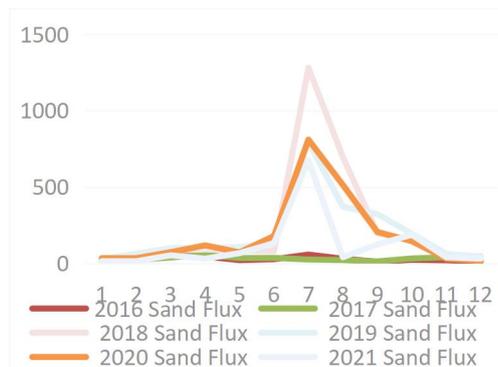
### 5.2.2 Model Solution and Analysis

Given the multiple definitions of water-sediment flux, this study adopts Lu (2022), which separates water-sediment flux into water flux (i.e., flow velocity) and sediment flux (i.e., sediment discharge). Due to the Yellow River’s natural characteristics, its flow velocity is influenced by factors such as climate, precipitation, topography, and basin area. For instance, higher summer precipitation leads to faster flow velocities. Consequently, the Yellow River’s water flux exhibits seasonal variations. The sediment flux of the Yellow River is influenced by natural factors such as the climate, landforms, and vegetation within its basin. To better monitor and understand the dynamic changes in water-sediment flux, it is necessary to analyze the abrupt changes, seasonal variations, and periodicity of water-sediment flux at this hydrological station, thereby summarizing the patterns of its variation.

For Question 2: Analyze the abrupt changes, seasonal patterns, and periodicity of water and sediment flux at this hydrological station. First, analyze the water and sediment flux data. Use Excel to filter and consolidate the data, summarizing the water and sediment flux for each month (January to December) from 2016 to 2021. Next, plot the data as line charts to visually analyze their characteristics. The results are shown below:



**Figure 4. Line Chart of Water Flux for January–December, 2016–2021**

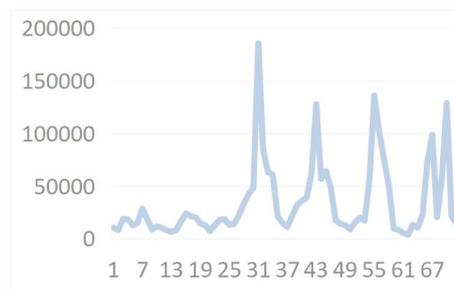


**Figure 5. Sand Flux Line Chart for January–December 2016–2021**

To analyze its abrupt change patterns, data indicates that abrupt changes in water-sediment flux refer to significant short-term variations in this flux. Such changes may result from sudden natural factors like torrential rain, floods, and ice melt, or human activities such as reservoir releases and river channel

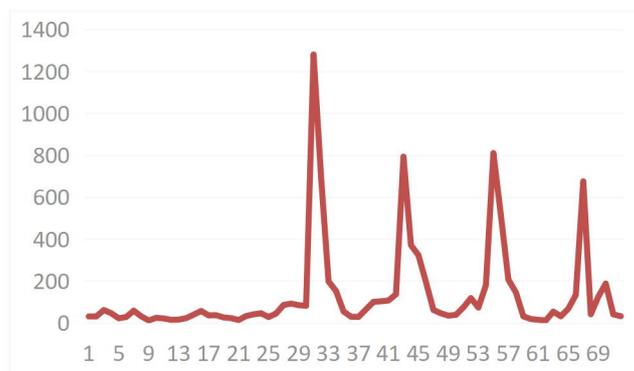
regulation (Characteristics of Water-Sediment Flux at Hydrological Stations: Abruptness, Seasonality, and Periodicity). As shown in the figure, water flux exhibited relatively stable variation from January to December 2016–2018. However, a sudden surge in water flux occurred in October 2021, indicating abruptness. Regarding sediment flux, August 2021 witnessed a rapid and substantial decline, also demonstrating abruptness. Additionally, it can be observed that the sediment flux values in 2018 were significantly higher than in other years, also exhibiting abrupt changes.

Regarding seasonal patterns, as shown in the figure above, water and sediment flux trends at this hydrological station on the Yellow River generally followed similar trajectories from 2016 to 2021: higher in summer and lower in winter. They rise gradually from January to June, surge rapidly from June to July, and reach the first peak in water and sediment flux in July. Subsequently, they decline from July to August, rise slowly again from August to October, and reach the second peak in water and sediment flux in October before gradually decreasing thereafter. This demonstrates that both water and sediment fluxes exhibit strong seasonality. For instance, summer is a period of relatively high precipitation in the Yellow River basin, resulting in higher peak values for water and sediment fluxes during this season.



**Figure 6. Variation Pattern of Water Flux from 2016 to 2021**

To analyze their periodic patterns, this study examines the interannual variations in water and sediment fluxes. First, data from January to December across 72 months (2016–2021) were compiled and summarized. Subsequently, data analysis was conducted, with results presented in the figure below:



**Figure 7. Variation Pattern of Sediment Flux from 2016 to 2021**

As shown in the figure, over six cycles with a 12-year period, both water flux and sediment flux exhibit distinct interannual cyclical variations. Water flux and sediment flux significantly increase during summer each year while decreasing relatively in winter, reflecting the influence of seasonal precipitation and snowmelt on the basin’s hydrological processes. It is evident that both water flux and sediment flux exhibit not only seasonal variations within a year but also regular interannual fluctuations, meaning that the annual changes are largely consistent.

5.3 Model Development and Solution for Question Three

Building upon Question 2, this study has investigated the variation patterns of water and sediment fluxes at this hydrological station and calculated their values for the past six years. Therefore, for Question 3, a seasonal ARIMA model is constructed to forecast the future two-year trend of water flux at this station. Based on this, an optimal sampling and monitoring plan for the next two years is formulated for this hydrological station.

5.3.1 Development of the Seasonal ARIMA Model

The seasonal ARIMA model is a method for handling time series data exhibiting seasonal variations. It serves as an extension of the ARIMA model, specifically designed to address data with pronounced seasonal patterns. The model is expressed as:

$$\left(1 - \sum_{i=1}^p \phi_i L^s\right) A_p(L^s) (1-L)^d \Delta_s^D X_t = \left(1 + \sum_{i=1}^q \theta_i L^s\right) B_Q(L^s) \epsilon_t \tag{3}$$

where P denotes the maximum lag order of seasonal autoregression, Q represents the maximum lag order of the periodic moving average operator, and D indicates the order of seasonal differencing. When P=D=Q=0, the SARIMA model is equivalent to the ARIMA model.

5.3.2 Solving the Seasonal ARIMA Model

After organizing the nearly six years of water flux and sediment flux data from the hydrological station summarized in Problem 2, this study employs SPSS to construct a seasonal ARIMA model to forecast the future two-year trends in water and sediment flux at this station.

**Table 6. ADF Test Results for Water Flux**

Variable	Series	t	P	AIC	Critical Value		
					1%	5%	10%
Water Flux	Original sequence	-1.624	0.471	1392.843	-3.546	-2.912	-2.594
	First-order difference	-3.707	0.004***	1371.982	-3.546	-2.912	-2.594
	First-order differenced	-					
	First-order seasonally differenced	-7.724	0.000***	1123.028	-3.551	-2.914	-2.595
	Second-order difference	-6.973	0.000***	1385.164	-3.546	-2.912	-2.594
	2nd-order differenced	-2.917	0.043**	1112.479	-3.578	-2.925	-2.601

1st-order seasonally differenced

Note. \*\*\*, \*\*, and \* denote significance levels of 1%, 5%, and 10%, respectively.

**Table 7. Sand Flux ADF Test Table**

Variable	Series	t	P	AIC	Critical Value		
					1%	5%	10%
Sand Flux	Original sequence	-1.696	0.433	804.012	-3.546	-2.912	-2.594
	First-order difference	-3.646	0.005***	793.442	-3.546	-2.912	-2.594
	First-order differenced	-					
	First-order seasonally differenced	-7.897	0.000***	652.474	-3.551	-2.914	-2.595
	Second-order difference	-7.099	0.000***	806.169	-3.546	-2.912	-2.594
	Second-order differenced	-					
	First-order seasonally differenced	-3.096	0.027**	651.767	-3.578	-2.925	-2.601

Note. \*\*\*, \*\*, \* denote significance levels of 1%, 5%, and 10%, respectively.

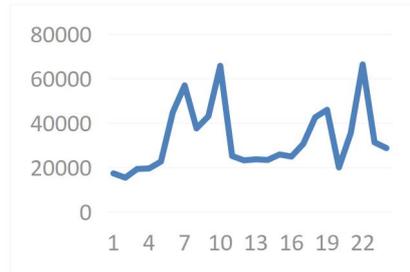
As shown in the table above, the original sequence passed the stationarity test after first-order differencing. Subsequently, the autocorrelation function (ACF) and partial autocorrelation function (PACF) were calculated for the stationary time series. By analyzing the autocorrelation plot and partial autocorrelation plot, the model results were obtained.

**Table 8 Model Evaluation Table - SARIMA Model (1, 0, 0)x(2, 0, 0, 12)**

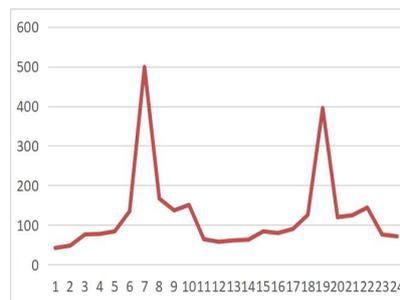
Term	Symbol	Value
Sample Size	N	72
Q statistic	Q6 (p-value)	0.995
	Q12 (p-value)	1
	Q18 (p-value)	1
	Q24 (p-value)	1
	Q30 (p-value)	1
Information criterion	AIC	948.656
	BIC	960.039
Goodness of Fit	R <sup>2</sup>	0.489

Based on the combined chart analysis, the model results are SARIMAx(1, 0, 0) x (2, 0, 0, 12). Since water and sediment fluxes are susceptible to factors such as weather and geographical location, making

precise predictions difficult, the model ARIMA(0,1,1) exhibits a relatively weak fit with an  $R^2$  value of 0.489. However, this does not compromise the prediction of future trends in water and sediment flux over the next two years. This study employed the SARIMA(1,0,0)x(2,0,0,12) model to forecast water and sediment fluxes at this hydrological station for the next two years. A line chart was generated using Excel, as shown below:



**Figure 8. Trend Chart of Water Flux for the Next Two Years**



**Figure 9. Future Two-Year Trend Chart of Sediment Flux**

As shown in the figures above, the water flux at the hydrological station will continue to exhibit seasonal variation over the next two years, with flow peaks occurring in July and October each year, followed by a decline. The sand flux trend, likely influenced by water flux, also shows seasonal variation, peaking in July and October each year before declining.

### 5.3.3 Development of a Nonlinear Programming Model

Based on the patterns of water and sediment flux variation, this study constructs a nonlinear programming model to formulate an optimal sampling and monitoring plan for the next two years. This plan aims to both capture the dynamic changes in water and sediment flux in a timely manner and minimize monitoring costs and resource consumption.

As shown in Appendix 3, this hydrological station has 25 monitoring points. To better formulate the monitoring plan, these points are numbered from 1 to 25. Due to their varying locations, each monitoring point has distinct monitoring costs and capabilities. In this study,  $x_{ij}$  denotes monitoring points,  $C_{ij}$  represents the monitoring cost required for each point in different months,  $f_j$  indicates the monitoring capacity of each point, and  $b_i$  signifies the monthly monitoring demand. Here,  $i$  ( $i=1,2,3, \dots, 12$ ) represents the 12 months of the year, and  $j$  ( $j=1,2,3, \dots, 25$ ) denotes the number corresponding to each of the 25 different monitoring points. The model is constructed as follows:

$$x = [x_{1,1} \cdots x_{1,25}, \cdots x_{12,1}, \cdots x_{12,25}]^T \tag{4}$$

$$C_{all} = [c_{1,1} \cdots c_{1,25}, \cdots c_{12,1}, \cdots c_{12,25}]^T \tag{5}$$

$$f = [f_1, f_2, f_3, \cdots f_{25}]^T \tag{6}$$

$$\min C_{all}^T \cdot x \tag{7}$$

$$s. t. \begin{cases} \sum_j f_{ij} x_{ij} \geq b_i \\ 0 \leq x_{ij} \leq 1 \end{cases} \quad x, y \in Z \tag{8}$$

### 5.3.4 Solving the Nonlinear Programming Model

This model was solved using MATLAB, with results shown below.

**Table 9. Monitoring Scheme Table**

No. Month	1	2	3	4	5	6	7	8	9	10	11	12
1	√	√	√	√	√	√	√	√	√		√	
2							√			√		√
3							√			√		
4							√					
5						√				√		
6			√				√					
7		√		√				√		√		
8					√	√			√		√	
9	√	√		√				√		√		√
10					√		√		√			
11	√	√	√	√	√	√		√		√		√
12						√	√	√			√	
13						√	√		√			
14				√	√						√	
15	√	√	√					√		√		√
16						√	√		√		√	
17							√				√	
18							√			√		
19							√	√		√		
20			√		√				√			
21		√						√		√	√	√
22						√	√			√		
23				√				√				
24							√				√	

As shown in the table above, a certain number of monitoring points are arranged for monitoring in different months. Since July and October each year are peak flow periods, a larger number of monitoring points are arranged for these months in this study, while the number of monitoring points in other months is relatively balanced. Since water and sediment fluxes exhibit regular interannual variations, with sudden changes being low-probability events, this paper establishes a one-year monitoring plan. The monitoring plan for the second year remains identical to the table above.

5.4 Model Development and Solution for Problem 4

5.4.1 Study on Water Diversion and Sediment Transport Effects

(1) Studying Water Diversion and Sediment Transport Effects Through Changes in Water-Sediment Flux

The Xiaolangdi Reservoir conducts annual water and sediment regulation from June to July, significantly impacting riverbed sedimentation and scouring, which in turn affect channel stability and navigability. By regulating water and sediment transport, sedimentation can be reduced or prevented, thereby maintaining navigability and ecological functions of the river channel. To investigate the impact of water and sediment regulation on water and sediment fluxes, we utilized data from May to October 2016-2021 (Appendix 1) to calculate monthly average flow and sediment discharge. A line chart correlating water flux and sediment flux was plotted for analysis, as shown below:

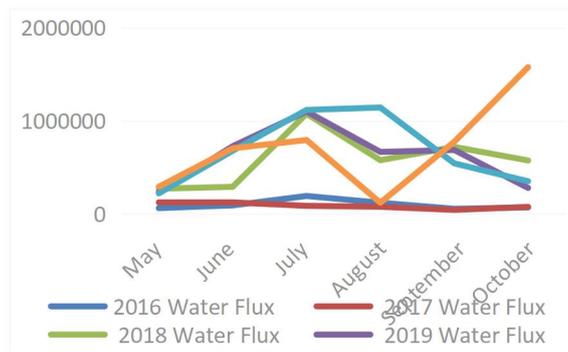


Figure 10. Water Flux Line Chart for May-October, 2016-2021

Available data indicate that without water and sediment regulation, the Yellow River’s water flux would continue to rise during the flood season in July and August, significantly increasing the risk of flooding. Observing the figure above reveals that water flux increases rapidly each year from June to July, peaking in July. After implementing water and sediment regulation, water flux shows a downward trend from July to August. This reduces the likelihood of flood disasters during the flood season while ensuring sufficient flow remains to scour sediment and maintain channel patency. Particularly after August-September, the effects of water and sediment regulation become increasingly evident. With improved channel conditions, water and sediment flow velocities increase, leading to higher flow rates

in most years. Without such regulation, the July peak would be higher, causing the channel to rapidly narrow and shallow, hindering subsequent drainage and sediment discharge.

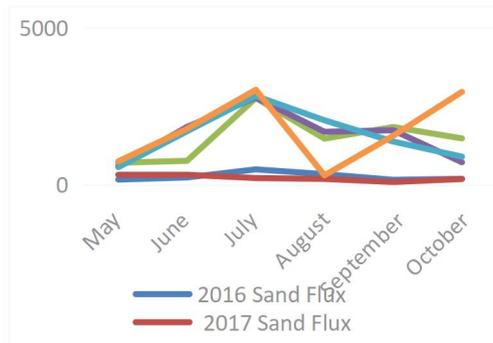


Figure 11. Sand Flux Line Chart, May–October 2016–2021

Similarly analyzing sand transport changes, although sand transport rose continuously from May to July, it gradually decreased after the June–July water and sediment regulation, achieving the effect of preventing sediment accumulation. This demonstrates that water and sediment regulation plays a crucial role in regulating water and sediment transport, exploring an effective approach for Xialangdi and other reservoirs to discharge sediment, reduce siltation, and extend their service life.

(2) Studying Water and Sediment Regulation Effects Through Riverbed Elevation Changes

To investigate the actual effects of Xiaolangdi Reservoir’s annual June–July “water and sediment regulation,” this study utilized riverbed elevation data from Appendix 3 to create a line chart in Excel. Comparative analysis revealed the practical outcomes of this regulation.

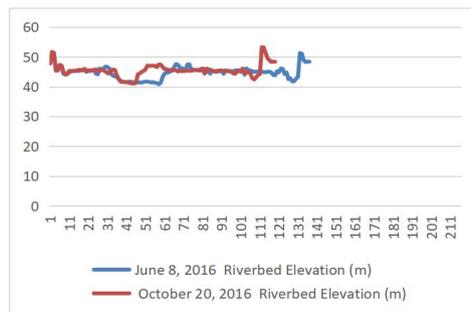


Figure 12. Comparison of Riverbed Elevation in June and October 2016

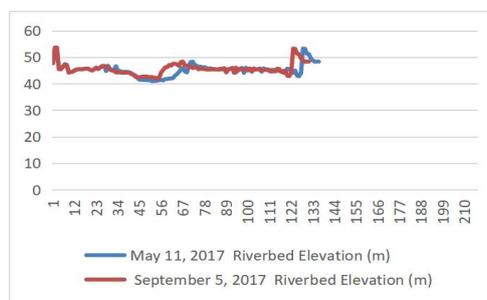
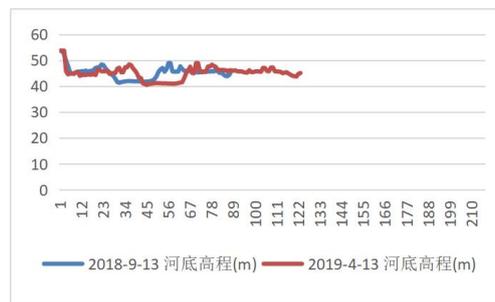
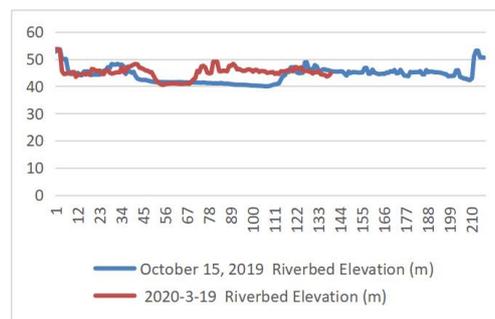


Figure 13. Comparison of Riverbed Elevation in May and September 2017

As concluded in Question 2, July marks the annual peak for both water and sediment transport. Figures 1 and 2 illustrate changes in riverbed elevation before and after the July flow peak. The figures show that riverbed elevation did not significantly increase after the July flow peak, remaining relatively stable. Common sense dictates that water and sediment flux positively correlates with riverbed elevation. Typically, increased water and sediment flux leads to a corresponding rise in riverbed elevation. However, the riverbed elevation line charts above indicate no substantial change after July. This demonstrates that the Xiaolangdi Reservoir’s water and sediment regulation has been highly effective, enhancing the stability of the riverbed elevation.



**Figure 14. Comparison of Riverbed Elevation in September 2018 and April 2019**



**Figure 15. Comparison of Riverbed Elevation between October 2019 and March 2020**

From the conclusions of Question 2, we also observe that October each year represents the second peak period for both water and sediment transport. During this time, the Xiaolangdi Reservoir did not conduct water and sediment regulation operations. Figures 3 and 4 show that after October, the riverbed elevation experienced a slight increase and exhibited relatively fluctuating levels. Water and sediment fluxes in July are significantly higher than those in October. However, riverbed elevation does not rise substantially after July. In contrast, changes in riverbed elevation after October are relatively unstable. This further demonstrates the effectiveness of the annual water and sediment regulation operations conducted at Xiaolangdi Reservoir from June to July.

#### 5.4.2 Projected Impact of Discontinuing “Water and Sediment Regulation” on Future Riverbed Elevation

To further investigate how the riverbed elevation at the hydrological station would change over the next decade without water diversion and sediment regulation, this study excluded June data from Appendix

II and calculated the elevation rise or fall between two consecutive years. The results are presented in Table 10 below. Subsequently, a grey prediction model GM(1,1) was constructed using the data in the table as input to forecast future changes in riverbed elevation over the next decade.

**Table 10. Riverbed Elevation Changes from 2016 to 2021**

Time Span	Riverbed Elevation Change (m)
2016/10/20–2017/5/11	-0.2214
September 5, 2017 - September 13, 2018	-0.14118
October 15, 2019 - March 19, 2020	0.969518
March 19, 2020 - March 14, 2021	-0.6547

(1) Model Development

First, the original data underwent a rank ratio test to assess its suitability for model construction. Results indicated the original data failed the rank ratio test, necessitating a shift transformation. Following this transformation, all rank ratio test values fell within the standard interval [0.67,1.492] , confirming the data’s suitability for constructing a GM(1,1) model. Detailed results are presented in Table 11.

**Table 11 Log-rank Test Results**

Index Item	Raw Value	Log-rank Value	Translated Value	Sequence	Translated Value	Ratio
1	-0.221	-	2.779		-	
2	-0.141	1.568	2.859		0.972	
3	0.97	-0.146	3.97		0.72	
4	-0.065	-14.809	2.935		1.353	

(2) Model Solution

This study employed SPSS to solve the GM(1,1) model, predicting changes in riverbed elevation over the next decade. The prediction results are presented in the table below.

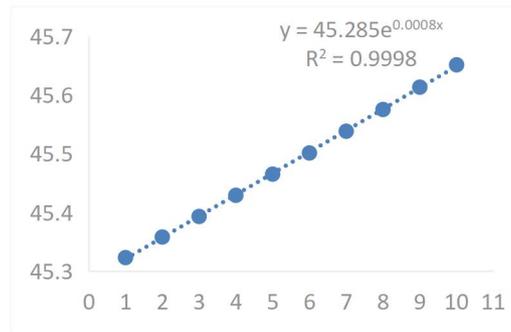
**Table 12. GM(1,1) Model Prediction Values**

Prediction Order	Predicted Value
1	45.323
2	45.358
3	45.393
4	45.429

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5	45.465
6	45.501
7	45.538
8	45.575
9	45.613
10	45.651

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**Figure 16. Riverbed Elevation Changes Over the Next Decade**

As shown in the above charts, without water diversion and sediment flushing operations, the riverbed elevation at this hydrological station will increase annually over the next decade, potentially impacting people’s livelihoods and daily activities. Therefore, it is essential for the Xiaolangdi Reservoir to conduct water diversion and sediment flushing operations every year from June to July.

**6. Model Evaluation and Optimization**

*6.1 Advantages and Limitations of the Model*

6.1.1 Advantages of the Model

- 1) During data preprocessing, different methods were employed to handle various types of data. Based on the attachment information provided in the question stem, Excel software was used to filter, merge, and calculate the data, demonstrating strong operability and practicality while simplifying the calculation process. Alternatively, Matlab was utilized to process data sets with large volumes and numerous missing values.
- 2) During model development and solution, reliable data support is provided alongside data visualization using tools like Matlab. Visual representations go beyond simple statistical charts, featuring diverse graphical configurations.
- 3) Intelligent optimization algorithms are predominantly employed for model solution, enhancing both reliability and applicability.
- 4) Analysis of model results goes beyond interpreting raw data. It employs a “combination of numbers and shapes” approach, integrating other real-world factors such as seasonal natural attributes. This comprehensive consideration yields highly accurate analytical outcomes.

### 6.1.2 Limitations of the Model

1) Regarding water-sediment flux research methods, this paper relies solely on a single literature source. The identified patterns of water-sediment flux variation still leave room for further exploration and refinement, as the concept of “water-sediment flux” itself lacks clear definition. More research methodologies should be considered in conjunction with modeling to ultimately refine an optimal approach for studying water-sediment flux.

2) This paper employs a single metric to measure both water and sediment fluxes.

3) Although this does not affect predictions of water and sediment flux trends over the next two years, the seasonal ARIMA model exhibits weak fitting accuracy. This is because both water and sediment fluxes are susceptible to unpredictable natural factors, making precise forecasting difficult.

### 6.2 Model Optimization

1) The seasonal ARIMA model can be extended and optimized by incorporating the probability of uncertain events during forecasting to enhance prediction accuracy.

2) The GM(1,1) model constructed in this paper does not account for the impact of uncertain events on riverbed elevation when forecasting future trends over the next decade. Therefore, the GM(1,1) model also has room for optimization to enable predictions based on a comprehensive consideration of all influencing factors.

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