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Original Paper

Foundations to Algebraic Mastery

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Abstract

Realizing that Algebra 1 is a gatekeeper to not only higher mathematics but STEM careers in general, it is imperative that our students master the content matter. Our Nation's report card shows we are not progressing in this area. To assist in algebraic mastery, this paper describes and provides concrete examples of four research-based pedagogical elements that can aid in student success: (a) basic skill development, (b) computational ease, (c) step-by-step scaffolding, and (d) the extensive use of the Explain-Practice-Assess (EPA) Strategy. Basic skill development assures that all students begin with the requisite background, providing equal opportunity for success, which can promote student engagement. By eliminating unnecessary computational complexity, students are more likely to participate and persevere in problem-solving. The step-by-step scaffolding meets the students where they are and incrementally brings them to mastery, with new material taught in digestible bites. The EPA strategy provides a mean to move students through a topic at an appropriate pace—not moved too quickly; students are given the time necessary to conceptually understand the concepts taught. The four elements described herein serve as a guide to help Algebra I teachers attain success for all students.

Keywords

algebra, mastery, pedagogy

1. Introduction

To the question "To what extent are your students mastering Algebra I content?" a secondary principal answered, "The word mastery is probably overstating the outcome; however, students have learned enough to pass the assessments and receive credit for the course." These types of comments are not rare; they mirror the decline in algebraic competency seen throughout the nation. We have a "D" on our nation's report card in mathematics where only 60-66% of our algebra grade level students have reached proficiency (NAEP, 2019) and where at best 50% of those enrolled in high school Algebra I courses have passed at a satisfactory level or higher" (Civil Rights Data Collection, 2018; NYSED Data site, 2019; Pappano, 2012; Rado, 2018). The results are worse when disaggregated by race/ethnicity. See Figure 1 below. More students take Algebra remediation in college than they do as high school seniors … the same issues from high school are found in college remedial algebra. … The matriculation rate from remedial math courses to college-level math courses is 22% (Laughbaum, 2017, p. 10).

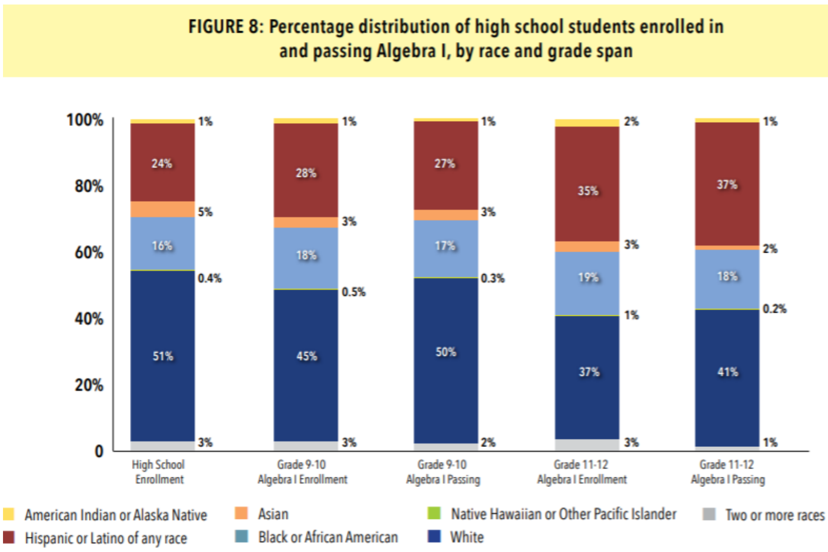

Figure 1. 2018 CRDC Data for Percentage Distribution of High School Students Enrolled in and Passing Algebra I by Race and Grade Span, p. 8

Realizing that Algebra 1 is a gatekeeper to not only higher mathematics but STEM careers in general (Laughbaum, 2017; Banchard & Muller, 2015; Stoelinga & Lynn, 2013), it is imperative that our students master the content matter. The purpose of this paper is to provide concrete examples of research-based pedagogical elements that may assist in Algebra content success for all students. These components are (a) basic skill development, (b) computational ease, (c) step-by-step scaffolding, and (d) the extensive use of the Explain-Practice-Assess (EPA) Strategy (Holmes, Spence, Finn, & McGee-Ingram, 2017). These pedagogical components are a result of reflective and reflexive experiences involving multi-years of teaching high school mathematics and evidenced-based practices (Lynch & Star, 2013; Marzano & Toth, 2014; Protheroe, Star, & Rittle-Johnson, 2009).

2. The Four Pedagogical Elements

2.1 Essential Component—Basic Skill Development

Most Algebra 1 classes assume a certain amount of prior knowledge. Even though most teachers would agree that this assumption is false, they teach as though it were true. In most classes, little time is devoted at the beginning of the course to provide instruction in redressing the gaps in order to bring all students to the same starting point. Various constraints prevent many teachers from determining and providing the background knowledge needed for each algebra topic covered. This situation is seldom remedied for a number of reasons, including time, insufficient funding to provide for additional support, well-organized, scaffolded remedial lessons, and the inability to clearly identify the basic skills that need remediation.

After considering the material to be covered in Algebra 1 (and, to a lesser extent, STEM careers), a list of 12 basic skills were deemed necessary for success. These are cancelling, integer arithmetic, graphing, factors, equivalent fractions, mixed numbers/improper fractions, word/mathematical expressions, place value/rounding, decimal arithmetic, fractions/decimal/percent conversions, scientific notation, and fraction arithmetic. These basic skills need to be addressed early in the course so that the tools/skills learned in the first few weeks can be applied from that point forward. Below is a table containing the basic skill, its description and an algebraic concept to which the skill applies (Table 1).

Table 1. Basic Skill Development Material

Basic Skill required	Covers	Algebraic Concept Application
Cancelling	Reducing ratios (lowest terms)	Unit conversion, multiplying and dividing monomial algebraic expressions, ratios and proportions, solving equations
Integer Arithmetic	Adding and subtracting, multiplying and dividing positive and negative numbers	All algebraic concepts introduced: rational expressions ($\frac{1}{x+1}+\frac{1}{3}$), solving systems of equations, probability
Graphing	Introduces basic graphing vocabulary, identification of points on coordinate axes and graphing	Solving systems of equations using graphing, linear programming
Factors	GCF and prime factorization	Multiplying and dividing fractions with variables $\frac{5x}{12y} \square \frac{3y}{35x}$, adding and subtracting fractions with variables $\frac{11x}{21}+\frac{9y}{28}$

Equivalent Fractions	Changing denominators	Adding and subtracting fractions with variables $\frac{11x}{21}+\frac{9y}{28}$, proportions $\frac{\$8}{1doz}=\frac{\$x}{8\,doz}$
Mixed Numbers and Improper Fractions	Mixed number ↔ improper fractions	Multiplying and dividing mixed numbers with variables
Words /Mathematical Expressions	Conversion between	Word problems
Place value and Rounding	Number sense	Simplifying answers
Decimal Arithmetic	Adding, subtracting, multiplying and dividing decimals	Word problems involving money, interest
Fraction/Decimal/Percent Conversions	Conversions between	Word problems, ratios
Scientific Notation	Standard form ↔scientific notation	Word problems involving very small and very large numbers, all science classes
Fraction Arithmetic and Lowest Common Multiple	Adding, subtracting, multiplying and dividing fractions	All algebra problems involving fractions ($\frac{3}{4}x=\frac{1}{8}$; $x=\frac{1}{6}$)

When these skills are strengthened early in the year, the remaining algebra topics can be covered much more successfully and efficiently. A class below grade level will be spending much more time at the beginning of the year, but after basic skills have been strengthened, the progress made through the rest of the year will be greatly improved.

When introducing basic skills, a detailed, scaffolded explanation is needed. These explanations should provide a step-by-step progression from the simplest problem to the more complex problem. The example problems should not be encumbered with difficult numbers. Emphasis should be put on understanding the concept and the process, not on cumbersome computations. The following fraction example (Figure 2) exemplifies this—going from the addition of two fractions with the same denominator and no reducing to adding two fractions with different denominators and reducing.

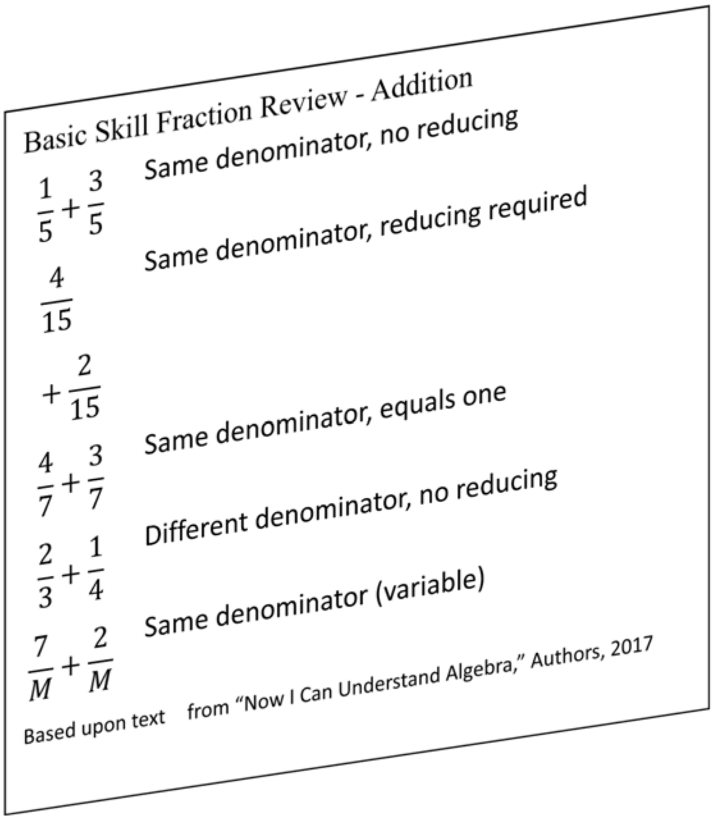

Figure 2. Basic Skill Fraction Review

Several problems of this same type should be given before proceeding to the next type. This way, teachers can easily identify the problem area—only one new concept is introduced at one time. The first practice should have the problems arranged in the same order, progressing from simplest to more complex. Future practices should have the word problems mixed up. This last format should be used on an assessment.

2.2 Essential Component—Ease of Computation

Ease of computation cannot be emphasized enough. Algebra problems, in general, should be designed with the easiest computation possible—especially when introducing a new topic. Emphasis should be placed on understanding the concept, appreciation of the logic involved, and understanding of the reasoning behind each step. The last thing that the student should be focusing on is messy arithmetic. Messy computations can be a headache for many students. They make students dislike doing algebra when in fact, the problem is the computation, not the algebraic concept. Easy computation also allows students to see, appreciate, and understand what the concept under discussion is. Messy computation stands in the way of all of this.

It's important to note that the concept is not "dummied down," the arithmetic is just simplified. Additionally, problems with easy arithmetic allow teachers to easily see and quickly correct where and when the student has gone awry.

An explanation made up of the following set of problems exemplifies this approach, where the simplicity of the numbers makes the complexity of the concept crystal clear. The following examples (Figure 3) promote a real understanding of the relationship between the factors and their product—a quadratic $(ax^2 + bx + c)$. In step one, where students are multiplying two factors, the way in which the factors determine a, b, and c, is emphasized—conceptual understanding. In step two, where the quadratic is being factored, the conceptual understanding previously learned is used to perform the reverse procedure. In each set of four problems, the value of c kept constant emphasizes the origin and promotes the understanding of the middle coefficient.

Notice that in each set of four problems below, the value of c kept constant emphasizes the origin and promotes the understanding of the middle coefficient.

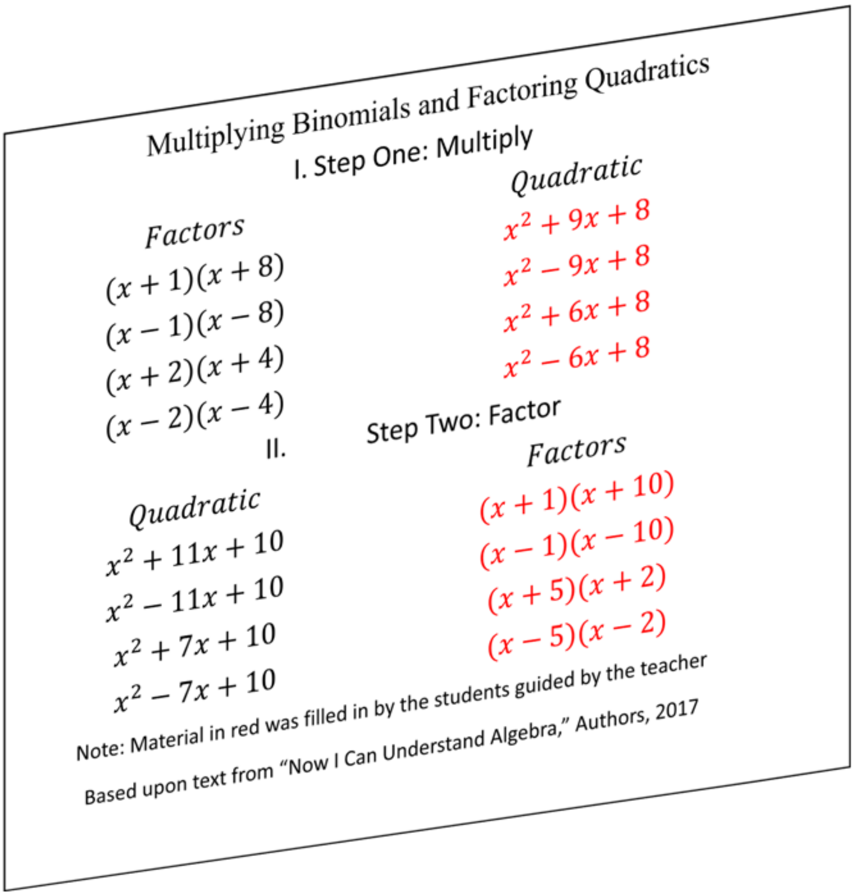

Figure 3. Multiplying Binomials and Factoring Quadratics

Finally, the elimination of messy computations will result in an environment that enables success, which in today's Algebra 1 classrooms is essential.

2.3 Essential Component—Step-by-step Scaffolding

The secret to successful scaffolding is only adding one new piece of information at a time. Students need to be given the chance to focus on and thoroughly understand each step of a procedure or layer in a process. This is especially true when introducing topics of varying levels of complexity, such as solving one equation, one unknown. In the following example (Table 2) the step-by-step scaffolding process is made evident. With each successive example, only one change is presented as a new concept is being introduced.

Table 2. Scaffolding Example

Scaffolding	**Added Step**
$x + 4 = 14$ $-4 \quad -4$ $x = 10$	One Step—isolate the variable by either adding/subtracting **or** multiplying/dividing.
$5x + 4 = 14$ $-4 \quad -4$ $\frac{5x}{5} = \frac{10}{5}$ $x = 2$	Two Steps—**isolate the variable** by first adding/subtracting **and** then multiplying/dividing.
$8x + 4 = 3x + 14$ $-4 \quad -4$ $8x = 3x + 10$ $-3x \quad -3x$ $\frac{5x}{5} = \frac{10}{5}$ $x = 2$	Three Steps—**isolate both the variable term and the constant** by adding/subtracting and then multiplying/dividing. Note: When introducing this procedure, it is an excellent idea to use variations of the same equation. This helps to accentuate the new step introduced in the scaffolding process.

$$2x + 1 + 6x + 3 = 20 + x + 2x - 6$$

$$8x + 4 = 3x + 14$$
$$-4 \qquad -4$$
$$8x = 3x + 10$$
$$-3x \quad -3x$$
$$\frac{5x}{5} = \frac{10}{5}$$
$$x = 2$$

Four Steps—**combine like terms** and then proceed as before.

$$2x + 1 + 3(2x + 1) = 20 + x + 2(x - 3)$$

$$2x + 1 + 6x + 3 = 20 + x + 2x - 6$$

$$8x + 4 = 3x + 14$$
$$-4 \qquad -4$$
$$8x = 3x + 10$$
$$-3x \quad -3x$$
$$\frac{5x}{5} = \frac{10}{5}$$
$$x = 2$$

Five Steps—**distribute** and **combine like terms** and then proceed as before.

This step-by-step scaffolding permit students to focus on each individual step. In multi-step problems, students can see the significance of each step as they are implemented in solving the problem. The rationale for each step is emphasized so that the solution is conceptually understood rather than algorithmically memorized.

When explaining a new concept, this approach allows teachers to identify at what step the student may have encountered problems. It is important to make sure that students are given ample time to practice each step in the scaffolding process before moving forward.

2.4 Essential Component—EPA Strategy

Extensive use of the EPA (Explain, Practice, Assess) Strategy should be implemented after each major topic/concept is explained. This is what truly gives students a chance to focus on, practice, understand, and internalize the material they are learning. In other words, the EPA Strategy promotes true mastery of a concept.

This is the epitome of what the Common Core Standards attempt to convey in evidence-based practices. Numerous practices (minimum of 3) followed by teacher corrections, allow students to make progress toward mastery. The dynamics of learning are in evidence here: practice the basic procedure, apply the procedure with multiple mistakes, try again and make fewer and different mistakes, and then reach proficiency (Powell, Fuchs, & Fuchs, 2013; VanDerHeyden & Allsopp, 2014).

When assessing student work, it is crucial to assess in the same manner in which the material had previously been handled. If multiple-choice questions were used in practice, multiple-choice should be used in the assessment. Assessments should mirror the practices in every way—in the way in which the questions are worded, in a logical sequence, in the progression of difficulty, and in topics presented. The following quiz (Table 3) would be appropriate following the treatment of one equation and one unknown given earlier (under scaffolding).

Table 3. Quiz Example

Scaffolding	Added Step	Quiz Questions
x + 4 = 14 -4 -4 x = 10	One Step—isolate the variable, by either adding/subtracting **or** multiplying/dividing.	1) $x + 8 = 13$ 2) $7x = 35$
5x + 4 = 14 -4 -4 $\frac{5x}{5} = \frac{10}{5}$ x = 2	Two Steps—**isolate the variable** by first adding/subtracting **and then** multiplying/dividing.	3) $7x + 10 = 45$ 4) $3x - 11 = 13$
8x + 4 = 3x + 14 -4 -4 8x = 3x + 10 -3x -3x $\frac{5x}{5} = \frac{10}{5}$ x = 2	Three Steps—**isolate both the variable term and the constant** by adding/subtracting and then multiplying/dividing. Note: When introducing this procedure, it is an excellent idea to use variations of the same equation. This helps to accentuate the new step introduced in the scaffolding process.	5) $6x - 8 = 4x + 10$ 6) $7x + 12 = 3x - 20$

$2x + 1 + 6x + 3 = 20 + x + 2x - 6$ $8x + 4 = 3x + 14$ $\quad -4 \qquad\; -4$ $8x = 3x + 10$ $-3x \quad -3x$ $\frac{5x}{5} = \frac{10}{5}$ $x = 2$	Four Steps—**combine like terms** and proceed as before.	7) $12x + 3 + 3x = 18 + 6x + 12$ 8) $5x - 4 + 10x + 12 = 12x + 20 - 4x + 30$
$2x + 1 + 3(2x + 1) = 20 + x + 2(x - 3)$ $2x + 1 + 6x + 3 = 20 + x + 2x - 6$ $8x + 4 = 3x + 14$ $\quad -4 \qquad\; -4$ $8x = 3x + 10$ $-3x \quad -3x$ $\frac{5x}{5} = \frac{10}{5}$ $x = 2$	Five Steps—**distribute** and **combine like terms** and proceed as before.	9) $6(x + 2) + 2(5x + 1) = 10(x + 2) + 30$ 10) $2(4x + 5) + 4x - 15 = 4(x + 9) + 3x + 4$

Note the simple arithmetic. All answers are whole numbers. This computational ease is especially important when crafting assessments. If you are assessing computational skills, the problems on the assessment should stress that. If you are assessing conceptual understanding of a process, computational difficulty must be minimized.

Comments on assessment tools refer to both brief quizzes up to the most comprehensive exams.

3. Summation

It is important to incorporate these four pedagogical components for success in teaching Algebra I classes (basic skill development, computational ease, step-by-step scaffolding, and extensive use of the EPA strategy). These components will make the journey to mastery achievable for even the reluctant learner. The reluctance of the learner is often seen as non-participation, but that may be that the student cannot participate. The reluctant learner is often reluctant because of an inability to participate. Features of the four essential components address possible student non-participation at every step. Basic skill development assures that all students begin with the requisite background and have the same opportunity for success. By covering the major skill topics indicated herein at the beginning of the course, students can meet with success right from the start, thus promoting their engagement throughout the course. By eliminating unnecessary computational complexity, students are more likely to participate and persevere in problem-solving. The step-by-step scaffolding meets the students where they are and incrementally brings them to mastery. New material is given to the students in digestible bites. The EPA strategy assures that students are not left behind or moved too quickly through a topic; in fact, the students are given the time necessary to conceptually understand the concepts taught.

The steps outlined and explained here serve as a guide for the teacher to attain success for his/her Algebra 1 students. Algebra 1 is an extremely valuable component of a student's education. It is not only the basic indicator of future success in math but all STEM-related fields as well.

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