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Original Paper

The Role of Complex Exponential Representations of Oscillation and Wave Equations in University Physics Teaching

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Abstract

Oscillations and waves are central topics in university physics and provide an important foundation for many subsequent courses. In physics and many engineering disciplines, oscillations and waves are commonly described using complex exponentials. This representation is simple and convenient and can substantially reduce the computational difficulty of problems involving linear operations on oscillations and waves. However, it has not received sufficient attention in university physics teaching. This paper uses complex exponential representations to explain the rotating-vector method and to address the superposition of oscillations and waves. Expressions for beats and standing waves are derived without

using sum-to-product trigonometric identities. On this basis, the paper discusses the role of complex exponential representations in university physics teaching.

Keywords

university physics, oscillations, waves, complex exponential representation

1. Introduction

Oscillations and waves play an important role in university physics teaching. These topics appear throughout many branches of physics, and the methods used to analyze them are widely applied in engineering. They also underpin the introduction of light waves and matter waves in the wave optics and modern physics components of university physics. For many science and engineering students, studying oscillations and waves provides a foundation for subsequent disciplinary courses. These topics are usually taught in the second semester of university physics, building on particle kinematics and dynamics. Simple harmonic motion and plane harmonic waves are the main objects of study. The corresponding oscillation and wave equations are derived [1-2], and quantities such as amplitude, period, frequency, phase, wavelength, and wave speed are defined. The properties of simple harmonic oscillations and harmonic waves are then discussed [3]. Many of these results can be naturally extended to alternating currents, electromagnetic waves, and matter waves. In university physics courses, simple harmonic motion and plane mechanical waves are usually described by real-valued displacement equations. These expressions directly characterize oscillations and waves and are easy to understand. However, they can be less convenient for problems such as superposition because they require trigonometric manipulations. Phase is a central issue in simple harmonic motion and harmonic waves [4-5], as well as a major focus and source of difficulty in university physics teaching. Phase-related problems are often treated using the rotating-vector method [6-8], in which a uniformly rotating vector represents simple harmonic motion. This is possible because the projection of the tip of a uniformly rotating vector onto any fixed direction undergoes simple harmonic motion. The method introduces an additional dimension, the y-axis of the rotating-vector diagram, for a particle undergoing one-dimensional simple harmonic motion. The meaning of this added axis can be difficult to explain, making its introduction seem somewhat artificial. If the rotating-vector method is presented only as a geometric technique for handling phase, students may overlook its connection to the phasor method widely used in engineering and advanced physics.

Simple harmonic motion and plane mechanical waves can also be represented using complex exponentials. This approach is widely used in advanced physics [9-10] and engineering [11-16]. Compared with real-valued displacement equations, complex exponential representations are more concise, can substantially simplify calculations involving linear operations, and provide a convenient way to handle phase. However, they are rarely discussed in university physics courses. This approach does not require advanced mathematical prerequisites such as complex analysis; it requires only basic concepts of complex numbers and Euler's formula. Students generally have some prior knowledge of complex

numbers, although a brief introduction to Euler's formula and its geometric interpretation is still needed. Introducing complex exponential representations in university physics courses is therefore feasible.

2. Complex Exponential Representations of Simple Harmonic Motion and Plane Harmonic Waves

For simple harmonic motion, the displacement equation is

$$x = A\cos(\omega t + \varphi) \tag{1}$$

Here, the positive real number A is the amplitude, ω is the angular frequency, and φ is the initial phase. We can then define the complex-valued function

$$\tilde{x} = \tilde{A}e^{i\omega t} \tag{2}$$

Here, $\tilde{A} = Ae^{i\varphi}$ is called the complex amplitude, and $\tilde{x}$ is the complex displacement relative to equilibrium. Its relation to the real displacement is $x = \mathrm{Re}(\tilde{x})$. The complex displacement $\tilde{x}$ contains all the information about the simple harmonic motion and uniquely determines its displacement function $x(t)$. The velocity and acceleration are given by $v = \dot{x} = \mathrm{Re}(\dot{\tilde{x}})$ and $a = \ddot{x} = \mathrm{Re}(\ddot{\tilde{x}})$, respectively. The one-to-one correspondence between $\tilde{x}$ and simple harmonic motion justifies using $\tilde{x}$ to represent it. Thus, $\tilde{x}$ is the complex exponential representation of simple harmonic motion. It is more concise than the real-valued displacement equation. Introducing this representation also clarifies the rotating-vector method: the tip of the rotating vector corresponds to the position of $\tilde{x}$ in the complex plane as it changes with time. The complex displacement $\tilde{x}$ describes a point moving in a circle about the origin in the complex plane with angular velocity equal to the angular frequency ω. The y-axis introduced by the rotating-vector method is the imaginary axis of the complex plane in which $\tilde{x}$ is represented. The phase $\omega t + \varphi$ is the argument of $\tilde{x}$. Thus, once the complex exponential representation of simple harmonic motion is understood, the rotating-vector method follows naturally. Similarly, the displacement equation of a plane harmonic wave can be written in complex exponential form as

$$\tilde{y} = \tilde{A}e^{i(\omega t - kx)} \tag{3}$$

Here, $k = \dfrac{2\pi}{\lambda}$ is the angular wavenumber of the plane harmonic wave, λ is its wavelength, and $\tilde{y}$ is the complex displacement of the particle at position x. Again, the complex exponential representation is more concise than the real-valued displacement equation.

3. Linear Operations on Simple Harmonic Motion and Plane Harmonic Waves

Another advantage of complex exponential representations of oscillations and waves is the ease with which linear operations with real coefficients can be performed. Taking the real part commutes with operations such as superposition with real coefficients and differentiation with respect to a real variable. Performing these operations on a complex exponential representation and then taking the real part therefore gives the corresponding result for the real physical quantity. The general procedure is to

express the oscillation or wave in complex exponential form, perform the linear operations, and then take the real part. A typical example is the superposition of simple harmonic oscillations with different frequencies. Consider a particle simultaneously undergoing two simple harmonic oscillations in the same direction, with equal amplitudes and closely spaced frequencies. Their displacement equations are $x_1 = A\cos(\omega_1 t)$ and $x_2 = A\cos(\omega_2 t)$. The resultant displacement is their sum, $x = x_1 + x_2$. Most university physics textbooks use a sum-to-product identity to express x as the product of two trigonometric functions and thereby introduce beats. For students unfamiliar with this identity, using it directly may make the derivation harder to follow. With complex exponential representations, the superposition can instead be evaluated using Euler's formula. The two oscillations are represented by $\tilde{x}_1 = A\mathrm{e}^{i\omega_1 t}$ and $\tilde{x}_2 = A\mathrm{e}^{i\omega_2 t}$, respectively. The resultant complex displacement is $\tilde{x} = A\left(\mathrm{e}^{i\omega_1 t} + \mathrm{e}^{i\omega_2 t}\right)$. Factoring out $\mathrm{e}^{i\frac{\omega_1+\omega_2}{2}t}$ and applying Euler's formula gives

$$\tilde{x} = 2A\mathrm{e}^{i\frac{\omega_1+\omega_2}{2}t}\cos\left(\frac{\omega_1-\omega_2}{2}t\right) \tag{4}$$

Taking the real part gives

$$x = \mathrm{Re}(\tilde{x}) = 2A\cos\left(\frac{\omega_1+\omega_2}{2}t\right)\cos\left(\frac{\omega_1-\omega_2}{2}t\right) \tag{5}$$

This result agrees with that obtained using the sum-to-product identity. When the two frequencies are close, Eq. (5) describes beats, with a slowly varying oscillation amplitude. A standing-wave expression can be derived in a similar way. In the same homogeneous medium, a standing wave can be formed by superposing two plane harmonic waves with the same direction of particle oscillation, equal amplitudes, and equal frequencies, traveling in opposite directions along the x-axis. Their complex exponential representations are $\tilde{y}_1 = A\mathrm{e}^{i(\omega t - kx + \varphi_1)}$ and $\tilde{y}_2 = A\mathrm{e}^{i(\omega t + kx + \varphi_2)}$. The resultant complex displacement is $\tilde{y} = A\mathrm{e}^{i\omega t}\left[\mathrm{e}^{i(-kx+\varphi_1)} + \mathrm{e}^{i(kx+\varphi_2)}\right]$. Factoring out $\mathrm{e}^{i\frac{\varphi_1+\varphi_2}{2}}$ gives

$$\tilde{y} = 2A\mathrm{e}^{i\left(\omega t + \frac{\varphi_1+\varphi_2}{2}\right)}\cos\left(kx - \frac{\varphi_1-\varphi_2}{2}\right) \tag{6}$$

Taking the real part gives the standing-wave expression

$$y = \mathrm{Re}(\tilde{y}) = 2A\cos\left(\omega t + \frac{\varphi_1+\varphi_2}{2}\right)\cos\left(kx - \frac{\varphi_1-\varphi_2}{2}\right) \tag{7}$$

Again, this agrees with the standing-wave expression obtained using the sum-to-product identity in most university physics textbooks.

4. Applications of Complex Exponential Representations in Subsequent Courses

4.1 Applications in Modern Physics

Matter waves are also described using exponentials in the modern physics component of university physics. For a one-dimensional, nonrelativistic free particle in a state of definite momentum, the wave

function can be written as $\psi=\psi_0 e^{\frac{2\pi i}{h}(px-Et)}$, where p, E, and h denote the particle momentum, energy, and Planck's constant, respectively. It is important to distinguish this wave function from a classical complex displacement: a quantum-mechanical wave function is itself a complex probability amplitude, and taking only its real part does not give the corresponding physical quantity. Some university physics courses also introduce the Schrödinger equation and use it to solve problems involving one-dimensional potential wells and barriers. These topics make extensive use of exponential forms of wave functions, which students may find difficult to understand without prior exposure to the exponential representations of oscillations and waves. Exponential representations are also commonly used to describe periodic physical quantities in engineering courses. For example, circuit analysis uses complex exponentials to describe alternating currents [11-12]. Helping students understand the complex exponential representation of physical quantities in university physics can therefore provide a foundation for subsequent disciplinary courses.

4.2 Applications in Circuit Analysis

In linear circuits under sinusoidal steady-state conditions, currents and voltages can be represented by phasors, while circuit elements are described by complex impedances. With the time factor $e^{i\omega t}$, the impedances of a resistor, an inductor, and a capacitor are $Z_R=R$, $Z_L=i\omega L$, and $Z_C=\frac{1}{i\omega C}$, respectively. For a series RLC circuit, the total impedance is $Z=R+iX$, where $X=\omega L-\frac{1}{\omega C}$ is the total reactance. This method shares the mathematical basis of the complex exponential representation of simple harmonic motion. Thus, understanding the physical meaning of $\tilde{x}=\tilde{A}e^{i\omega t}$ and learning how to use it in university physics can help students progress to complex impedance and phasor methods in subsequent disciplinary courses.

4.3 Applications in Electromagnetic Waves and Optics

In advanced physics and electrodynamics, the electric field and magnetic field of a plane electromagnetic wave are commonly expressed in complex exponential form. For example, the electric field can be written as $\tilde{\mathbf{E}}=\tilde{\mathbf{E}}_0 e^{i(\omega t-\mathbf{k}\cdot\mathbf{r})}$, with the actual electric field given by the real part of this expression. This representation combines the complex amplitude, propagation direction (specified by $\mathbf{k}$), and angular frequency (ω) in a single expression. It greatly facilitates the analysis of electromagnetic wave propagation, reflection, and refraction in different media and underpins wave optics and Fourier optics. Prior familiarity with this representation helps students understand analytical expressions and phase relations in optical interference and diffraction.

5. Considerations and Prospects for Teaching Practice

Although the preceding discussion demonstrates several advantages of complex exponential representations, their adoption in teaching still involves challenges and practical considerations. 1. Established teaching and textbook practices: Many university physics teachers are accustomed to traditional trigonometric representations, and mainstream textbooks often treat complex exponential

representations as supplementary rather than core material. This limits their use in the classroom. 2. Students' understanding of complex numbers: Although the complex exponential representation of simple harmonic motion involves basic concepts of complex numbers, some students in the early stages of university study may find the relation between complex numbers and actual physical quantities confusing. Teachers should therefore emphasize the auxiliary role of the imaginary axis (the y-axis). It does not represent an additional direction of motion in physical space; rather, it is an auxiliary axis used to represent the imaginary component of complex displacement and is a natural dimension of the complex plane. To address these challenges, a gradual transition is recommended in university physics teaching. After presenting the traditional trigonometric approach, teachers can introduce Euler's formula and explicitly compare it with the rotating-vector method. Reinterpreting the rotating vector through the complex exponential representation ($\tilde{x} = \tilde{A}e^{i\omega t}$) makes clear that the y-axis in the rotating-vector diagram is the imaginary axis of the complex plane used to represent complex displacement. This clarifies the relation between the mathematical representation and its physical projection and makes the introduction of complex exponentials more natural.

6. Conclusion

Complex exponentials provide a concise and efficient representation of oscillations and waves. They offer clear advantages for problems involving linear operations and allow the expressions for beats and standing waves to be derived without sum-to-product trigonometric identities. The complex exponential representation of simple harmonic motion also leads naturally to the rotating-vector method and helps clarify its meaning. Furthermore, complex exponential representations are widely used in advanced physics, including modern physics, and in many engineering courses. They therefore deserve greater attention in university physics teaching. University physics textbooks and syllabi should include more material on complex exponential representations where appropriate to the needs of different disciplines. Their mathematical elegance and computational convenience can help establish a foundation in mathematical physics for subsequent study of electromagnetism, quantum mechanics, signal processing, and related subjects. Their specific effects on student learning still need to be evaluated through teaching practice.

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