Original Paper

Mathematical Abilities of Preschool Children

Ljiljanka Kvesić[1*], Slavica Brkić[2] & Anita Imre[2]

[1] Department of Mathematics, University of Mostar, Mostar, Bosnia and Herzegovina

[2] Department of Physics, University of Mostar, Mostar, Bosnia and Herzegovina

[*] Ljiljanka Kvesić, Department of Mathematics, University of Mostar, Mostar, Bosnia and Herzegovina

Abstract

Regardless of the fact that people are born with innate sense of number, mathematical thinking requires certain intellectual effort for which many children are not ready. While children investigate and discover new issues in everyday life, they meet the world of mathematics although they are not aware of it. As mathematics is becoming more and more important in today's age of technology, it is very important that children are introduced to the spells of mathematics before they start attending school, and to continue to learn mathematics with that knowledge throughout school education. As well as reading, mathematics is a subject necessary for adequate functioning in society. What is more, mathematics is a subject that develops logical thinking and perception, thus mathematical teaching of children ought to be on more accessible level than it is currently. Parents and educators have access to various games and activities that involve children into mathematical thinking and creative resolving, which develops their self-confidence.

Through the research that has been conducted with both preschool and primary school children using various mathematical examples appropriate for their age, it has been found out that the children of younger age have early math skills, whereas, regarding primary school children, their later math achievement is fading over time or their conclusions are grounded upon the "expected". The traditional way of teaching mathematics and extensive material can cause poor mathematical achievement and cause a well-known fear of mathematics.

Keywords

preschool and school children, mathematics skills, sense of space, number and quantity, math anxiety

1. Introduction

The turn of the century has seen a dramatic increase in attention to the mathematics education of young children. Why is there such a surge of attention to mathematics in early childhood (Clements, 2004)? In

a global economy the vast majority of jobs require more sophisticated skills than in the past. American educators and business leaders have expressed strong concern about pupils' mathematics achievements (Mullis, 1997). Preschool children discover natural phenomena of their surroundings; sense of space and time, geometry and spatial sense, quantity and number sense is formed in their mind. Concepts are formed on the basis of their experience-children learn mathematics without knowing mathematical form. As late as the 1980s, the theoretical debate focused on whether mathematical skills or concepts should be taught first. One argument for the skills-first position was that young children were not capable of understanding mathematics, learning abstract mathematical concepts, or logical reasoning and, thus, needed to be told or shown simple skills and practice these skills until they were memorized by rote (Clements, 2014). Preschool education is the first level of the general education. Mathematics is often studied as a pure science, which is wrong. Teaching mathematics should be continually linked to specific examples from nature or in some fields of science. Mathematics helps us understand the world—and we use the world to understand mathematics (2). The world is interconnected. Everyday mathematics shows these connections and possibilities. The earlier young learners can put these skills to practice, the more likely we will remain an innovation society and economy. Despite young children starting school having, for the most part, a well-developed, informal competence in mathematics, it is apparent that the classroom is also a place where math anxiety can develop and flourish. Research studies (e.g., Vinson, 2001) suggest that, in combination with the parental and societal factors, math anxiety may have its roots in teaching and teachers, with math anxious teachers resulting in math anxious students at times.

Essentially, teachers must have an awareness and understanding of math anxiety, and develop an ability to assist math anxious students (Whyte, 2012).

1.1 Sense of Space, Number and Quantity

In a classic passage, Jean Piaget described a young child playing with pebbles and in so doing, discovering principles of mathematics:

"He lined them up in a row, counted them from left to right, and got ten. Then, just for fun, he counted them from right to left to see what number he would get, and was astonished that he got ten again. He put the pebbles in a circle and counted them, and once again there were ten. He went around the circle in the other way and got ten again. And no matter how he put the pebbles down, when he counted them, the number came to ten. He discovered here what is known in mathematics as commutativity, that is, the sum is independent of the order" (Piaget, 2007).

Much research on cognitive development focuses exclusively on children's knowledge, without asking where such knowledge comes from (Maratsos, 2007). Experience has taught them that winters are colder than summers; that the Sun always rises on one side, and sets on the other side of the horizon; that objects thrown upwards always fall back to the ground (Brkić, 2018). Much of children's knowledge is derived not from their direct experiences with the environment but rather from the input of others (Gelman, 2009). Importantly, the view that children learn from those around them does not

168

mean that children simply, passively take in what they are exposed to (Harris & Koenig 2006; Callanan 2006). Treating children as passively taking in input also would not predict the well-demonstrated phenomenon that children resist counter evidence. A classic example comes from Piagetian training studies, in which children persist in supplying incorrect answers to seemingly simple questions, even following instruction (Ginsberg & Opper 1969). Similar memory and processing biases are found in children's folk theories of physics, biology, and psychology (Schulz, 2007). Clearly, then, young children actively process the information around them, and are not passive conduits into which information pours. In the 1980s, it was revealed that pupils had certain ideas about many physical concepts and conceptual relations about physical phenomena that mostly did not correspond to the correct physical ideas (Driver, 1973). These wrong intuitive ideas are spontaneously developed by all pupils regardless of their background, upbringing or previous education, and they present a serious obstacle to learning physics. The research concerning intuitive ideas had no effect on the teaching until the work of R. Driver and L. Viennot in the 1980s (Driver, 1973; Viennot, 1979). Thereafter, there was a whole series of the investigations of wrong ideas which were called misconceptions. Traditional forms of teaching ignore the existence of misconceptions; they deal with the ready-made knowledge, trying to convey it to their pupils. Parents and educators have access to various games and activities that involve children into mathematical thinking and creative resolving, which develops their self-confidence: development of logical thinking, development of sense of space, development of visualization, development of creative thinking, development of sense of number and quantity. Many studies have shown that infants are able to differ small groups of objects ("1 vis a vis 2"), 1-2 years: They use words to denote small quantity (one dog, two cats, three candles,….), 2-3 years: Counting objects by touching them or pointing to them their fingers, 3-4 years: Quick recognition of small groups of objects without counting (1, 2 or 3 balls), 4-5 years: They begin to add and subtract small numbers (1 or 2 toys). Very often small children are able to list numbers by heart, but they fail to understand their meaning. First direct cognitive contact of a child with a number is one candle on a birthday cake. Using jigsaw puzzles with numbers, children learn about mathematic relations: matching a certain number of objects to corresponding numbers, from left to right, one by one, above-under, ability to make a sequence of equal objects, recognition of numbers in written form, ability to recognize and sort out objects by their size and shape, comparison of objects and shapes. Even though children are born with mathematical gene, it is necessary to develop mathematical skills (Devlin, 2000).

Importantly, the view that children learn from those around them does not mean that children simply, passively take in what they are exposed to. As a result of these efforts, we now know that early math skills are the strongest early predictors of children's math achievement years later.

2. Method

2.1 The Purpose of Research

The purpose of our research was to discover what mathematical abilities of preschool children are like, and whether these innate and acquired abilities help primary education pupils to adopt mathematical content easier.

Teaching and learning with games approach at the early mathematics stage is one of the strategies that have been recommended by researchers and child education experts. Literature review shows that problems of number concept and number operation mastery at the early mathematics stage can be overcome with organized games' activities (Clark & Roche, 2010; Siegler & Ramani, 2008; Yong, 2008). Children are capable of building number relationship and expanding arithmetic because all numbers are the repeated addition of one (Chin, 2015).

Preschool and school age pupils solved picture tests appropriate to their age. The testing was done at the kindergarten "Sportić" Mostar and also at the primary schools "Vijonica", Čitluk and "Kočerin", Široki Brijeg. The test included 92 preschool children and 124 primary education pupils.

In our research of preschool age children mathematical abilities, we used picture tests appropriate to the children's age. With the help of educators, the questions were read to each child. The children circled the picture they considered to be the right answer to the posed question.

The first test related to the sense of space.

2.2 Test and Results

2.2.1 Test for Preschool Age

A sense of space:

▶ What is left and what is right?

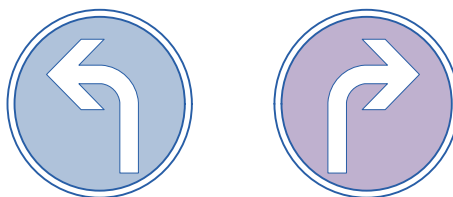▶ What is above and what is under?

▶ What is closer and what is further?

Figure 1. Left and Right
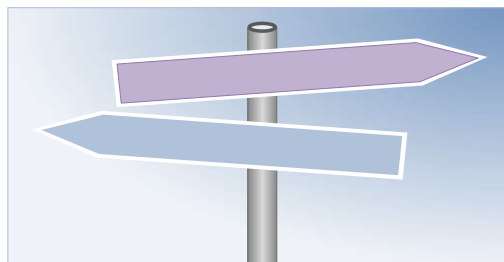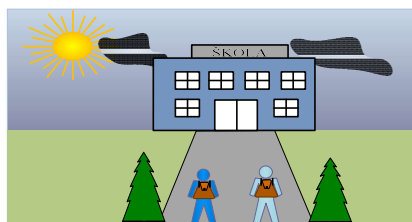
Figure 2. Above and Under

Figure 3. Closer and Further

Table 1. Correct Answers in Tests Number 1, 2 and 3

	(-/100)%
Test 1	80
Test 2	90
Test 3	62

Table 1 displays the results of a picture test done by preschool age children. Most of the preschool age children (4-5 years) know what left or right is, what above and below is, and what is closer and what is further. The sense of space is a key element for development of geometry and it can be used in areas which seem not to be connected with space. Many great mathematical discoveries occurred as a cause of change of aspect, in other words, mathematicians approach a problem in a different way and start observing it spatially. The evidence of the last Fermat principle was also a result of such an approach to the problem (Devlin, 2000).

2.2.2 Test for Preschool Age and School Children (I Grade and II Grade)

Another test was done by preschool age pupils and the children attending first and second grade of primary school. The children were supposed to recognize shapes (Figures 4, 5, 6), to order objects of the same shape correctly (Figure 7), to do additions of the same shapes (Figure 8), to order numbers correctly (Figure 10), to recognize geometrical objects and shapes (Figure 11, 12), to fill in the table with missing letters and numbers according to the template (Figure 15).
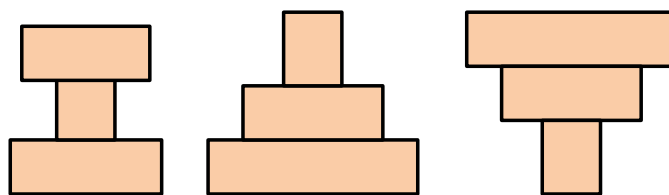
1) Which drawing reminds you of a birthday cake?

Figure 4. Birthday Cake

Table 2. Correct Answers in Test Number 1 for Preschool (4-5 Years) and School Children (6-7 Years)

	(-/100)%
Preschool	75
I grade	97
II grade	97

2) Name geometric shape!
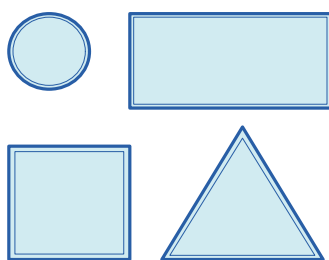
Figure 5. Geometric Shapes

Table 3. Correct Answers in Test Number 2 for Preschool and School Children

	(-/100)%
Preschool	41
I grade	5
II grade	33

3) Which geometric shape is different than the other three?
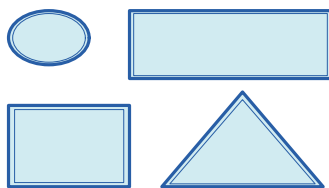
Figure 6. Geometric Shapes

Table 4. Correct Answers in Test Number 3 for Preschool and School Children

	(-/100)%
Preschool	72
I grade	12
II grade	100

In three last picture tests, the preschool and school age children were supposed to recognize, name or notice the difference between displayed shapes. We reached interesting results concerning geometric shapes as circle, triangle, square and rectangle. Preschool age children were more successful in naming the shapes and recognizing the differences than the school age children. The school age children confused geometric shapes and objects. Preschool children are obviously not familiar with the names of some geometric objects, so they didn't confuse square and cube, which happened with the school age children (Figure 5). First grade children who are already familiar with the concepts of geometric images obviously failed to understand the question, which resulted in their choice of triangle instead of circle (Figure 6).

4) We add a candle to a cake each year. Is the order of the candles regular?
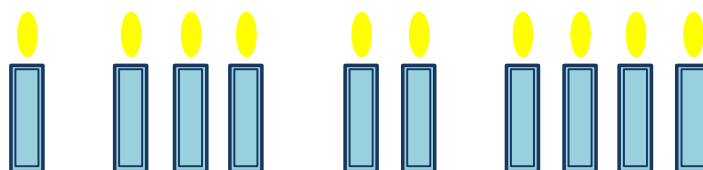
Figure 7. Candle on a Birthday Cake

Table 5. Correct Answers in Test Number 4 for Preschool and School Children

	(-/100)%
Preschool	100
I grade	100
II grade	100

As already mentioned in the Introduction, children have ability to order objects and numbers although they might be unable to count. This question was the easiest one for all children (100 % of correct answers). Besides correct number ordering, we can see that celebration of birthdays is an important issue when growing up. They know very well the order of increasing the number of candles on birthday cake. This is an indicator that the curriculum of mathematics needs to be revised and completely related to children's experience. Luckily, this is easy to do, as the foundations of mathematics are a part of everyday life. Connecting mathematics with their everyday experiences gives children a creative

encouragement for development of mathematical skills. Recognize that children begin school with a great deal of knowledge and intuition on which to build. Research has shown that children begin school with an intuitive knowledge of mathematics and abundant common sense.

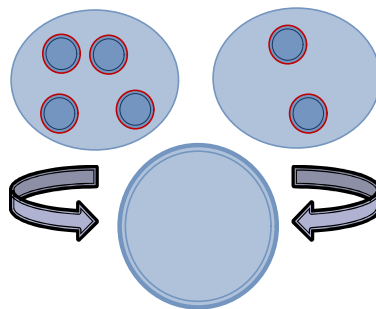5) Draw total number of small circles in the empty circle.

Figure 8. Joining Small Circles

Table 6. Correct Answers in Test Number 5 for Preschool and School Children

	(-/100)%
Preschool	98
I grade	98
II grade	61

6) Connect circles with corresponding number of sticks. Fill the empty circle with the corresponding number of remaining sticks.
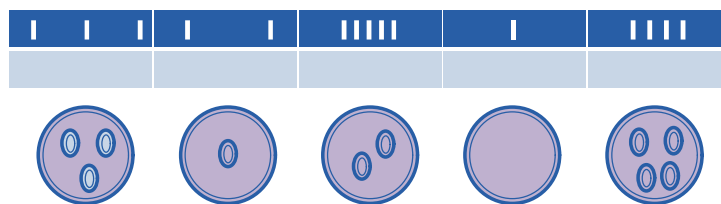

Figure 9. Connecting Circles and Sticks

Table 7. Correct Answers in Test Number 6 for Preschool and School Children

	(-/100)%
Preschool	42
I grade	98
II grade	61

174

Pupils of younger age were more successful than the second grade pupils in solving Test 5. The second grade pupils read the question superficially, considering it to be too easy and they filled the empty circle with three small circles (Figure 7). Test 6 was done best by the first grade pupils (Figure 8). It is possible that the question was not understood by the preschool age pupils, and it was not so interesting to second grade pupils. Their interest in mathematics is obviously fading away.

7) The drawing represents the game "school". On which drawings you notice a mistake? The game "school" is excellent for development of sense for space and balance, muscles motions and also for development of patience and resourcefulness. This game was played in ancient Rome!
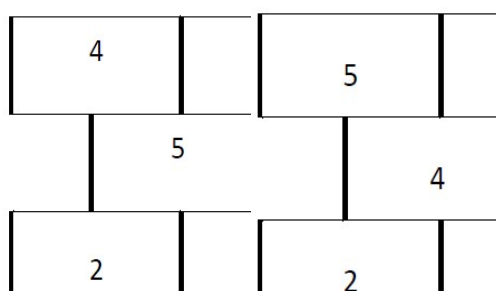

Figure 10. The Game "School"

Table 8. Correct Answers in Test Number 7 for Preschool and School Children

	(-/100)%
Preschool	98
I grade	80
II grade	98

Considering the "school" game, we can notice that all children are equally successful. Preschool age children are able to order numbers correctly, which they obviously have learned through game!
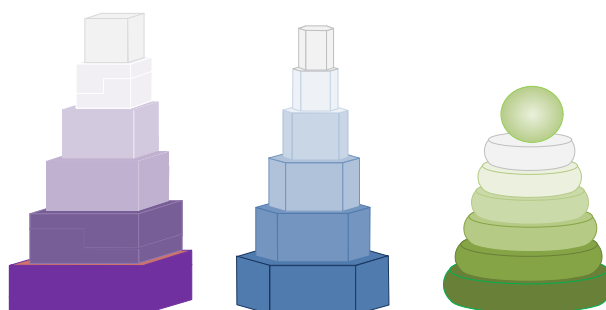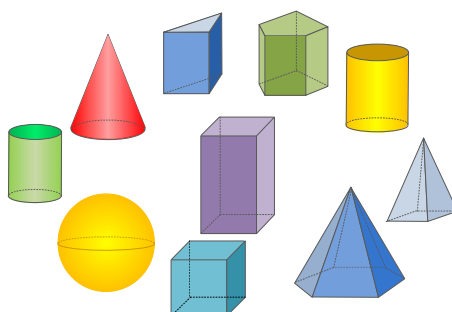
8) Sign an item that does not belong to the image!

Figure 11. Geometric Bodies and Shapes

Table 9. Correct answers in Test Number 8 for Preschool and School Children

	(-/100)%
Preschool	50
I grade	60
II grade	80

Second grade pupils recognized that the given compositions are disrupted by a ball, whereas the lower age children were indecisive or failed to understand what they were expected to do.

9) Connect the geometric bodies of the same shape!

Figure 12. Geometric Bodies and Shapes

Table 10. Correct Answers in Test Number 9 for Preschool and School Children

	(-/100)%
Preschool	10
I grade	18
II grade	12

Preschool children perceive the same shapes, while school-age children who have learned the names of geometric bodies classify, for example, all pyramids, although they are not of the same shape.

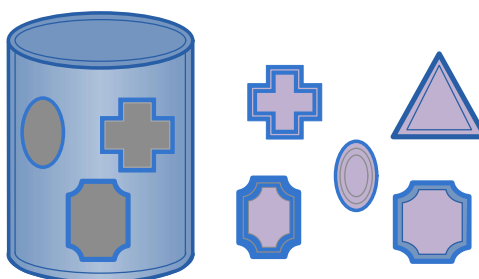10) Denote the objects that cannot be thrown into the box!

Figure 13. Bodies and Shapes

Table 11. Correct answers in Test Number 10 for Preschool and School Children

	(-/100)%
Preschool	48
I grade	75
II grade	55

11) Enter the numbers into Tables in the right order.

<div align="center">5 2 4 9 7 3</div>

Figure 14. The Correct Arrangement of Small Numbers

Table 12. Correct Answers in Test Number 11 for Preschool and School Children

	(-/100)%
Preschool	52
I grade	56
II grade	80

12). Letters are joined to numbers in the first table.

Complete empty spaces in two other tables according to the first Table.

Figure 15. Letters and Numbers

177

Table 13. Correct Answers in Test Number 12 for Preschool and School Children

	(-/100)%
Preschool	81
I grade	98
II grade	53

2.1.3 Test for School Children (I Grade, II Grade)

The third test was done by the pupils of first and second grade of primary education. The test comprised of two parts. The first part of the rest related to the knowledge of numbers from 1 to 20, and in the second part, the pupils were supposed to recognize the shapes of letters on the grounds of shapes of the offered objects. In the second part of the test the children were offered pictures presenting optical illusions. We often experienced an occasion when our own eyes cheat on us, so we tried to trick the children: "things are not always as they seem to be".

1) Enter the missing numbers into the Table!

Figure 16. Proper Ordering of Numbers from 1 to 20

Table 14. Correct Answers in Test Number 1 for School Children (6-7 Years)

	(-/100)%
I grade	98
II grade	87

2) Find out which numbers are in a wrong place.

1	2	9
5	6	7
3	10	11

Figure 17. Numbers in Wrong Places

Table 15. Correct Answers in Test Number 2 for School Children

	(-/100)%
I grade	26
II grade	40

Children mostly mastered numbers and counting to 20 (Figure 16). Test 2 has a surprisingly bad result (Figure 17), which contradicts the results of test 1 (Figure 16) and test 7 (Figure 10). This shows that children learn to count by heart and when they come across a mistake in number ordering their system falls apart. In the "school" game children connect numbers with their own experience and they know that a certain digit is in a wrong place.

3) *a*. Use a line to connect letters and similar objects.

*b*. Which letter do we associate with the remaining object?

R A L S K M O V I

Figure 18. Letters and Similar Objects

Table 15. Correct Answers in Test Number 3 for School Children

	(-/100)%
I grade	15
II grade	87

Optical illusions. The next four slides show optical illusions. We expect children to say what they "see"! The answers of the children in the first grade continued to look exactly as the second grade continued with what it determined. More or less they answered what they think is right.

4) Is the length and width of the image in the picture the same?
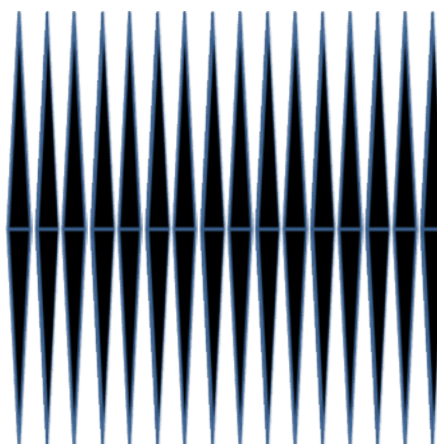
Figure 19. Optical Illusions

Table 16. Correct Answers in Test Number 1 for School Children

	(-/100)%
I grade	25
II grade	87

5) How many straight lines cross the zebra?
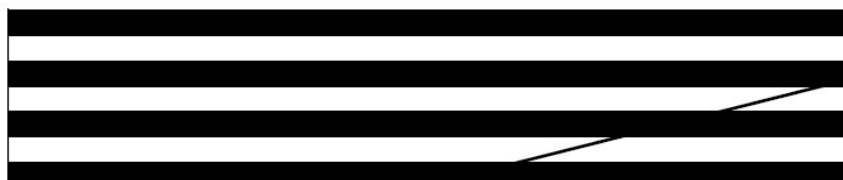
Figure 20. Optical Illusions

Table 17. Correct answers in Test Number 2 for School Children

	(-/100)%
I grade	40
II grade	68

6) Blue line (*a*) is screened by a horizontal obstacle. Which of the bottom lines (*b* or *c*) presents continuum of the upper blue line *a*?
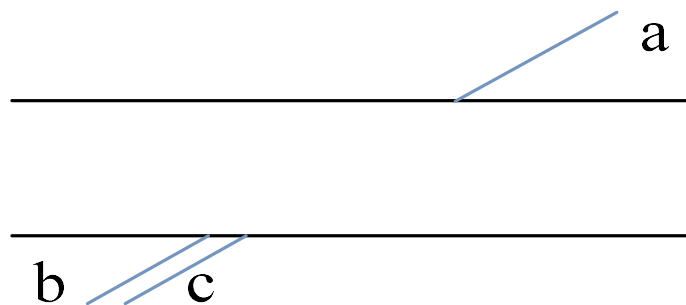
Figure 21. Optical Illusion

Table 18. Correct answers in Test Number 3 for School Children

	(-/100)%
I grade	42
II grade	75

7) Where do straight lines *a* and *b* cross straight line *p* in relation to lines *c* and *d*?

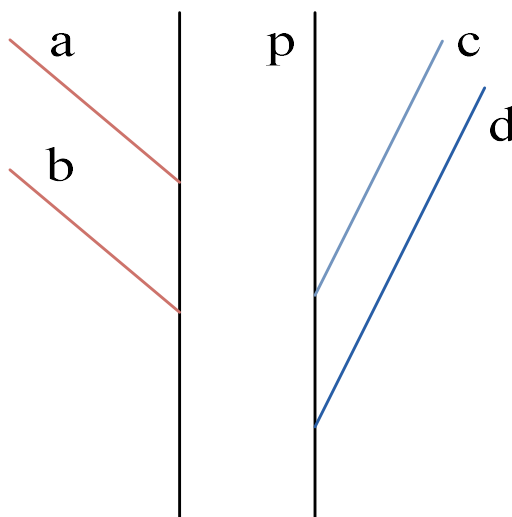

Figure 22.Optical Illusion

Table 19. Correct Answers in Test Number 7 for School Children

	(-/100)%
I grade	70
II grade	52

Optical illusions test was done only by the pupils of first and second grade of primary school. An inversion can be made here and we may say that the pupils with less correct answers were more successful, as they answered as they "saw"—that's why it is an illusion. The second grade pupils presumed that there was an illusion and they provided the right answers.

Solutions:

1) Image length and width are the same.

2) One line crosses the zebra.

3) Extended straight line *a* matches straight line b.

➢   Extended straight line *a* crosses straight line *p* in the same point as straight line *c*.

➢   Extended straight line *b* crosses straight line *p* in the same point as straight line *d*.

Our tests have proved that preschool children have sense for space, quantity and number. They sort things out in the right order, without previously having learnt it. We can see that the second-grade pupils had weaker results than the first-grade pupils, or the response was based upon the "expected" (optical illusions). Primary school children often lack the ability for mathematical education that is caused by traditional approach to math and superficial learning. Teacher´s creativity has a crucial role in math teaching. Primary education teachers must pay particular attention to the teaching of mathematics. Research studies (e.g., Vinson, 2001) suggest that, in combination with the parental and societal factors, math anxiety may have its roots in teaching and teachers, with math anxious teachers resulting in math anxious children at times. Teaching by math anxious teachers is characterized by an over-reliance on traditional instructional activities such as: drills, flash cards, and work sheets; assigning the same work for everyone; teaching to the textbook; insisting on only one correct way to complete a problem; concentrating more on basic skills rather than concepts; and, whole class instruction (Gurganus, 2007). At the beginning of primary education mathematics teacher should be game based. Children need to overcome math anxiety and the known fear of mathematics. School mathematics curriculum should incorporate challenging, interesting, real-world problems from children's environments in order to nurture higher-order thinking skills. Use distributed practice to build skills.

3. Discussion

It is clear that children acquire enormous amount of knowledge about the world and world phenomena from people who surround them, and they do not simply and passively accept what they are exposed to. Acquired knowledge comes out of own cognitive activities; a child makes his own choices based on his

innate intellect and acquired knowledge. There is a question of a nature of this inflow of information, and the nature of human mind to enable a child to use that contribution with his innate intellect. Intuition plays an important role in mathematicians' discussions of mathematical thought (McLeod, 1989). Intuitive knowledge in mathematics is knowledge which is self-evident, which carries with it characteristic feeling of certitude, and which goes beyond the facts that are available (Fischbein, 1987; Noddings & Shore, 1984). There is a scientific basis of rise in learning mathematics skills in preschool period and their relationship to later adoption of mathematics contents. Research has shown that the early gained mathematics skills are the strongest predictors of children's mathematics achievements in education which is to follow (Aunol, 2004). Our research partly points to contradictory results even in early primary education. Effects of early interventions on children´s math achievement reliably diminish over time, a finding known as the "fade-out" effect (Bailey, 2014). The adoption of mathematical teaching contents on preschool level (skills) won't be enough to contribute to long-term mathematics achievements significantly if the mathematics teaching in primary education is approached in such a way as to learn the contents only conceptually (traditional teaching) instead of building upon their innate and acquired skills in preschool age. The relationship of affective factors to mathematics learning and teaching is always influenced by the social context. The role of the social context is also receiving much like to see their research have a greater impact on real-world classrooms (Brown, 1988).

3.1 Encouraging, Motivation of Primary Education Pupils

Although we know that early math skills are the strongest early predictors of children's math achievement years later, math skills have to develop continuously. Teaching has to be interactive and creative so that primary education pupils could develop their innate and acquired ability for mathematics. Teaching should be of high quality and creativity. Teachers have to get feedback. The classroom atmosphere must be of high quality. Teachers should use mathematical games, entertaining tasks, mathematical pictures, drawings, songs and curiosities. To minimize math anxiety, teachers need to demonstrate and model a positive attitude, including: portraying an optimistic disposition and a love for mathematics that shows mathematics as a cultural tool; promoting the value of math by the way it contributes to society; and, getting beyond mathematical myths. The effects of stable factors are several times larger than the effects of children's earlier math achievement (Bailey, 2014). For those teachers that bring math anxiety with them into the classroom, it is imperative that their own fears and insecure feelings are confronted and controlled (Martinez, 1987).

4. Conclusion

Even though children are born with mathematical gene, it is necessary to develop mathematical skills. Preschool education is the first level of general education. Innate capacities and environmental evidence work together in development. Our research has shown that preschool children possess a developed sense of space, quantity and number. Mathematical skills are developed during earliest

child's age. Children that failed to develop those skills during preschool age are likely to fall behind concerning development of mathematical skills in their future education. Our research has shown that early mathematical skills will not be sufficient to substantially increase their math achievement in school age. Due to a well-known fear of mathematics and insufficient development of mathematical gene, primary school children show weaker results than expected. Preschool math achievement alone will not be sufficient to substantially boost their math achievement in school years outcomes later. In order to extend effective early math instruction, children need subsequent math instruction of higher quality for the long-term math achievements. In order to overcome fear of mathematics, a teacher has to make an effort to create a positive attitude towards mathematics. Successful pupils need successful teachers that will develop creative and critical thinking in pupils

Acknowledgement

References

Bailey, H. D. (2014). *What's the Point of Teaching Math in Preschool*? Retrieved from https://www.brookings.edu/research/whats-the-point-of-teaching-math-in-preschool/

Brkić, S., Kvesić, L. J., & Zubac, M. (2018). Physical Preconception of Primary Education Pupils *International Journal for Innovation Education and Research*, *6*(5).

Brown, J. S., Collins, A., & Duguid, P. (1988). *Situated cognition and the culture of learning*. Palo Alto, CA; Institute for research and Learning. https://doi.org/10.21236/ADA204690

Callanan, M. A. (2006). *Cognitive development, culture, and conversation: Comments on Harris and Koenig's "Truth in Testimony: How Children Learn about Science and Religion" Child Dev*, *77*, 525-530. https://doi.org/10.1111/j.1467-8624.2006.00887.x

Chestnut Hill, M. A. (n.d.). *Center for the Study of Testing, Evaluation, and Educational Policy*. Boston College.

Chin, L. C., & Zakaria, E. (2015). Understanding of Number Concepts and Number Operations through Games in Early Mathematics Education. *Creative Education*, *6*, 1306-1315. https://doi.org/10.4236/ce.2015.612130

Clark, D., & Roche, A. (2010). The Power of Single Game to Address a Range of Important Ideas in Fraction Learning. *Australian Primary Mathematics Classroom*, *15*, 18-24.

Devlin, K., & Mathematics, G. (2008). *1st Edition, Kindle Edition by Keith Devlin*.

Douglas, H. C., Arthur, J. B., & Julie, S. (2014). *Background Research on Early Mathematics, Background Research for the National Governor's Association (NGA) Center Project on Early Mathematics*.

Driver, R. (1973). *Student's conception and the learning of science* (PhD Thesis).

184

Fischbein, E. (1987). *Intuition in science and mathematics*. Bostons Reidel.

Gelman, A. S. (2009). *Learning from Others: Children's Construction of Concepts, Annu Rev Psychol*, *60*, 115-140. https://doi.org/10.1146/annurev.psych.59.103006.093659

Ginsburg, H. P., & Opper, S. (1969). *Piaget's Theory of Intellectual Development*. Englewood Cliffs, NJ: Prentice-Hall.

Gurganus, S. P. (2007). *Math instruction for students with learning problems*. Boston: Pearson Education.

Harris P. L., & Koenig, M. A. (2006). Trust in testimony: How children learn about science and religion. *Child*, *77*, 505-524. https://doi.org/10.1111/j.1467-8624.2006.00886.x

Maratsos, M. P. (2007). Commentary. *Monographs of the Society for Research in Child Dev.*, *72*, 121-126. https://doi.org/10.1111/j.1540-5834.2007.00425.x

Martinez, J. G. R (1987). Preventing math anxiety: A prescription. *Academic Therapy*, *23*, 117-125. https://doi.org/10.1177/105345128702300201

McLeod, D. B., & Adams, V. M. (1989). *Affect and Mathematical Problem Solving, A new perspective*. Springer-Verlag Publishing. https://doi.org/10.1007/978-1-4612-3614-6

Mullis, I. V. S., Martin, M. O., Beaton, A. E., Gonzalez, E. J., Kelly, D. L., & Smith, T. A. (1997). *Mathematics achievement in the primary school years: IEA's third international mathematics and science study (TIMSS)*. Chestnut Hill, MA: Center for the Study of Testing, Evaluation, and Educational Policy, Boston College.

Noddings, N., & Shore, P. J. (1984). *Awakening the inner eye: Intuition in education*. New York: Teachers College Press

Piaget J. (1970). In *Genetic Epistemology*. Duckworth E, translator. New York: Columbia. https://doi.org/10.7312/piag91272

Schulz, L. E., Bonawitz, E. B., & Griffiths, T. L. (2007). Can being scared cause tummy aches? Naive theories, ambiguous evidence, and preschoolers' causal inferences. *Dev Psychol*, *43*, 1124-1139. https://doi.org/10.1037/0012-1649.43.5.1124

Siegler, R. S., & Ramani, G. B. (2008). Playing Linear Numerical Board Games Promotes Low-Income Children's Numerical Development. *Development Science*, *11*, 655-661. https://doi.org/10.1111/j.1467-7687.2008.00714.x

Viennot, L. (1979). Spontaneous reasoning in elementary dynamics. *European Journal of Science Education*, *1*. 203-221, https://doi.org/10.1080/0140528790010209

Vinson, B. M. (2001). A comparison of preservice teachers' mathematics anxiety before and after a methods class emphasizing manipulatives. *Early Childhood Education Journal*, *29*(2), 89-94. https://doi.org/10.1023/A:1012568711257

Whyte, J., & Anthony, G. (2012). Math's anxiety: The fear factor in the mathematics classroom. *New Zealand Journal of Teachers' Work*, *9*(1), 6-15.

Yong, J. P. (2008). *Using Teacher-Made Games to Support Young Children's Learning of Number*

Concepts: Combining Hands-On Manipulative and Computer Software (Master's Thesis Buffalo).
NY: The State University of New York.