

Original Paper

A Perspective of Mathematics Teaching Based on Knowledge Structuring under the Guidance of Core Competencies

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Abstract

With the continuous deepening of educational reform, cultivating core competencies has become the fundamental goal of primary and secondary education. Structuring knowledge offers a new perspective and pathway for mathematics teaching oriented toward these core competencies. Based on the requirements for developing core competencies, this paper systematically elaborates on the main characteristics and core functions of “mathematics teaching based on structured knowledge,” drawing from key viewpoints on “knowledge structuring” and “mathematics pedagogy”. It further presents several reflections on practical teaching applications, aiming to provide theoretical reference and practical guidance for current mathematics education reform, promoting a shift in mathematics instruction from fragmented knowledge transmission to systematic thinking and the development of essential skills.

Keywords

core competencies, knowledge structuring, perspective of mathematics teaching, cognitive structure, systematic thinking

1. Introduction

In the pursuit of education, educators of every era have continuously probed into and practiced the same core proposition: how to cultivate human beings more effectively. In the digital age, especially amid the rapid development and popularization of artificial intelligence, the practice of this proposition has become increasingly urgent and well-defined. The “knowledge-based” education model has been unable to fully respond to the needs of the development of the times. The cultivation of literacy with critical thinking, problem-solving ability, and innovation ability as the core has become a consensus in

the field of education. However, the current mathematics education generally has the prominent problem of “emphasizing knowledge and problem-solving skills, neglecting thinking development and ability improvement”. Relying on the accumulation of fragmented knowledge and repeated mechanical exercises, to a certain extent and within a certain period of time, there may be “illusions” that achieve good teaching results. However, when the time interval is prolonged or more complex tasks are faced, the “superficial and inert” (Zheng, 2004) knowledge scattered in every corner of the mind cannot activate students’ thinking. At the same time, fragmented and isolated learning methods will also inhibit students’ potential, and knowledge structuring is an important starting point for dealing with these problems.

Knowledge structuring is a good state of knowledge in the mind. It is the basis for students to extract information in time, solve problems and enter in-depth thinking and exploration. It is not only the foundation of cultivating students’ core literacy, but also an important means. The “Compulsory Education Mathematics Curriculum Standards (2022 Edition)” clearly states that the focus of curriculum content organization is to structurally integrate content and explore ways to develop students’ core literacy (Ministry of Education of the People’s Republic of China, 2022). As the leader in the teaching process, teachers master the rhythm and direction of teaching, and the formation of a knowledge-structured mathematics teaching concept is to promote students’ knowledge structure, and then develop into cognitive structure, so as to enhance students’ mathematics core literacy.

2. Main Viewpoints of Knowledge Structuring and Perspective of Mathematics Teaching

2.1 Knowledge Structuring

“Knowledge structuring is a knowledge system that combines all parts of knowledge into a logical, organized and functional knowledge system according to certain rules. An ordered whole and a functional system constitute its two fundamental characteristics”(Tong & Wang, 2024). Rooted in relevant theories of cognitive psychology, knowledge structuring emphasizes students’ active construction and assimilation of knowledge during learning, with correlation, logicity and hierarchy as its major features. “From the perspective of cognitive processing, knowledge structuring mainly consists of two aspects: first, correlating accumulated knowledge to form a well-hierarchized knowledge system; second, abstracting cognitive approaches and fundamental concepts from concrete knowledge, and then associating more knowledge by applying these obtained cognitive approaches and fundamental concepts to build a sound knowledge structure”(Chen, 2022). Knowledge structuring is not a linear end point. Structured process and structured results are mutually reinforcing cognitive cycle processes.

2.2 Perspective of Mathematics Teaching

The view of mathematics teaching refers to teachers’ overall view and fundamental understanding of the whole process of mathematics teaching. “Teachers’ view of mathematics teaching is composed of three parts: mathematics view, cognition view and teaching view, and each part is divided into several

elements” (Yu, 2009). A mathematics teacher from the understanding of the nature of the subject, to the control of the teaching design, and then to the landing of the concept of educating people, each link reflects his own view of mathematics teaching. It is the basis for teachers to carry out meaningful and effective teaching in mathematics classroom to form a scientific view of mathematics teaching that is in line with students’ development. This paper discusses the concept of mathematics teaching focusing on knowledge structure.

3. Characteristics of the Perspective of Mathematics Teaching Based on Knowledge Structuring

Under the guidance of core literacy, the mathematics teaching concept based on knowledge structuring shows the following characteristics.

3.1 Highlighting Connections among Knowledge

Establishing connections among knowledge constitutes the core of knowledge structuring. Connectedness is an essential inherent feature of the theoretical system of mathematics: “On the one hand, mathematics is developed and organized through logical approaches on the basis of axiomatic systems, and the truth of a mathematical proposition is often reduced to the truth of previous propositions; on the other hand, from the perspective of mathematical development, research contents at lower levels lay the foundation for higher-level research”(Bao & Zhou, 2009).Connections among knowledge serve as an entry point for students’ further learning and the source of vital mathematical thoughts such as reduction, analogy, induction and deduction. In mathematics teaching activities, teachers should identify the growth points of students’ knowledge based on their prior knowledge, construct cognitive paths between students’ cognitive starting points and cognitive objectives, and guide and motivate students toward active learning and exploration, “so as to truly realize teaching in accordance with students’ aptitude and the restructuring of learning structures”(Wu, 2021).

3.2 Emphasizing the Dynamic Development of Cognition

Cognitive structures are always in a process of dynamic development, in which knowledge structuring acts as both an end and a means. Structures are not fixed once formed. With continuous accumulation of knowledge and enrichment of experiential learning, new cognitive approaches and ideas can keep growing on the basis of existing cognitive frameworks when students encounter new tasks and contexts. Taking “connection” as the trigger of cognitive paths can promote the continuous adjustment and improvement of cognitive structures, so as to construct a more systematic, stable and efficient cognitive system. The core of the dynamic development of cognitive structures lies in flexibility, adaptability and developability. What is outstanding is that students can solve problems more flexibly and adapt to changing situations quickly. At the same time, dynamic development is not only the development of more complete knowledge structuring, but also the development of comprehensive ability.

3.3 Advocating Independent Thinking and Construction

Students are the main body in learning activities. Learning is not a process of passively accepting knowledge. Meaningful teaching needs to guide students to think and construct independently.

Teachers cannot replace students to think. Students' cognitive structure can only be constructed by students themselves. Effective teaching activities should be based on students' mastery of knowledge to promote the development of their abilities. When students can take the initiative to find and analyze the connection between knowledge points, explore the strategies and methods of problem solving, have the consciousness of summary and reflection, and can compress the new concepts and methods obtained into the knowledge structure, they have a solid foundation for facing new tasks and new situations.

3.4 Emphasizing Situational Creation and Transfer

"Mathematics learning is a complex activity. In addition to the learner's cognitive factors, this activity is also affected by environmental factors. Learners not only determine their own learning behavior based on the representation of inner symbols but also through interaction with the environment"(Yu, 2009). The formation of knowledge structuring depends on the creation of context, which can provide a correlation framework. Students can identify and analyze the internal relationship between knowledge in meaningful and coherent real situations, and guide students to understand and construct knowledge deeply, so as to form a clear and logical knowledge structure. "Literacy is highly situational, and its construction is always based on context, and can be transformed and constructed in the process of problem proposing and solving "(Zhang & Wang, 2022). Situational transfer is an important way to realize knowledge transfer by using knowledge structure. By extracting existing knowledge, adaptive adjustment and creative application are carried out in new and more complex situations, so as to cultivate the ability to solve problems in real situations. Teachers should pay attention to the organic combination of situation creation and migration, and help students build a systematic cognitive structure by creating situations; through situational migration, structured knowledge is transformed into problem-solving ability.

4. Functions of the Perspective of Mathematics Teaching Based on Knowledge Structuring

4.1 For Students: From "Learning to Know" to "Learning to Learn"

The transformation from "learning to know" to "learning to learn" is a qualitative change in mathematics learning. It goes beyond the memorization of scattered mathematical facts and the "stylized" problem-solving training, but is devoted to guiding students to gain insight into the internal logical connection of mathematics. In this process, students gradually improve the ability to discover problems, mobilize cognitive resources, transfer applications, and solve problems, so as to build a growing knowledge structure and sustainable self-learning ability.

4.1.1 Reducing Cognitive Load and Facilitating Long-term Memory

Knowledge structuring is the process of constructing a hierarchical and interconnected knowledge network. This process emphasizes the exploration of the internal logical relationship between knowledge, so as to grasp the essence and rules of knowledge and avoid one-sided understanding and mechanical application. Knowledge structuring greatly reduces the cognitive load and memory pressure caused by fragmented information. On the one hand, the mathematical thinking methods contained in

the structured system, such as analogy, reduction, and combination of numbers and shapes, are widely transferable, suitable for different situations and fields, and can effectively reduce cognitive load ; on the other hand, the relationships such as hierarchy, causality, analogy and meaning connection that form knowledge structure provide multiple coding ways for the memory of knowledge, which can promote the long-term memory and extraction of knowledge.

4.1.2 Promoting Deep Learning

“Deep learning is relative to shallow learning, emphasizing that learning is not mechanical repetition and memory” (Liu, Liu, & Gu, 2023). It is a meaningful learning based on understanding and pursuing transfer applications. Its characteristics include: deep participation in learning, adopt advanced learning strategy, focus on the development of higher-order knowledge, learning based on understanding for migration (Peng, 2019). From the perspective of deep learning, knowledge structuring is an important basis for deep learning. By integrating fragmented information into a vertically and horizontally related knowledge system, it vertically integrates the internal logic of the discipline and horizontally connects multiple fields to help students form a more stable and transferable cognitive framework. Knowledge structuring can not only deepen the understanding of the nature of knowledge, stimulate fine processing and critical thinking, but also optimize working memory, improve the availability and sensitivity of knowledge, so as to release cognitive resources and trigger production rules for higher-order thinking activities, pointing directly to core literacy-oriented deep learning.

4.1.3 Upgrading Metacognitive Monitoring Competencies

“Metacognitive monitoring refers to the individual in the whole process of cognitive activities, their ongoing cognitive activities as a sense of their ongoing active, conscious monitoring, control and regulation” (Yu, 2010). Metacognition is an important internal factor affecting learning. It can help learners monitor and adjust their cognitive processes, evaluate different cognitive strategies, and extract and apply prior knowledge, so as to adjust and correct inefficient learning methods or wrong learning directions in time, and formulate the best learning strategies (Bao & Zhou, 2009). Knowledge structuring upgrades metacognitive monitoring from vague empirical judgment to scientific cognitive management by constructing a systematic cognitive framework :

- (1) Providing cognitive maps for metacognitive monitoring: structured knowledge presents the hierarchical structure and logical relationship between information. Learners can clearly see the correlation path between knowledge, so as to accurately locate the current cognitive position and quickly and accurately evaluate their understanding state.
- (2) Realizing automatic triggering of monitoring: when knowledge is structured, monitoring can change from passive problem finding to active cognitive navigation, and finally the more structured, the more self-knowledge; the more self-knowledge, the more independent virtuous circle.

4.2 For Teachers: From “Teaching Knowledge” to “Cultivating Competencies”

The knowledge-structured mathematics teaching concept provides a clear path and a strong fulcrum for teachers to realize the transformation from “teaching knowledge” to “cultivating competencies”.

4.2.1 Providing Diagnostic Basis and Guidance for Teaching Design

Knowledge structuring can provide scientific diagnostic basis and guidance for teaching design by revealing the internal logic and system connection of subject knowledge. With the idea of knowledge structuring, the cognitive development state of learners can be more effectively captured, so as to reasonably infer and clarify their recent development zone and potential ability boundary. At the same time, through the analysis of structural elements such as knowledge nodes and knowledge associations, it is helpful to infer the degree of connection between learners' different knowledge points and identify the obstacles and bottlenecks they may face in mastering new knowledge (Pian & Yu, 2025). On this basis, instructional design can be targeted to design differentiated tasks based on cognitive starting points, build "scaffolding" around the zone of proximal development to promote thinking progression, and design variants and migration exercises for potential obstacles to break through learning difficulties.

4.2.2 Optimizing Teaching Strategies and Improving Classroom Efficiency

Under the guidance of the concept of knowledge structured teaching, teachers will consciously optimize and integrate various teaching strategies based on the key perspective of building students' cognitive network, so as to effectively improve classroom efficiency. According to the teaching content, teachers can reasonably use diversified teaching strategies such as special topics, variations, analogies, and large units to explore the essential characteristics and internal logic of knowledge from multiple dimensions, guide students to form a structured knowledge framework, and gradually promote cognition to be structured. At the same time, it also provides new ideas and practical methods for the teaching process. Teachers can carefully design structured situations, logically coherent structured problem strings, and goal-focused structured activities based on the core of knowledge-based knowledge structure, so that the classroom teaching process is closely connected, and students' knowledge mastery and thinking ability are promoted.

4.2.3 Transforming Evaluation Approaches and Attending to Thinking Processes

Under the guidance of core competencies, the student evaluation system has undergone fundamental changes: the focus has shifted from knowledge-oriented "outcomes" to thinking-oriented "processes", and thinking evaluation has become a critical issue in education.

However, the evaluation of mysterious and abstract "thinking" has always been an unsolved problem in the education field. Knowledge structuring can build a bridge for thinking evaluation. Piaget's knowledge view of genetic constructivism reveals that "knowledge and thinking have internal consistency", that is, thinking is based on knowledge, and the level of thinking depends on the state of knowledge structure; knowledge is the result of the operation of thinking, and the structural state of knowledge is determined by the level of thinking (Bai, Feng, Shen, & Li, 2020). This provides a theoretical basis for thinking evaluation. Knowledge structure and thinking structure spiral up and develop together in the cycle of state and movement. Therefore, knowledge structure can be used as the carrier of "thinking visualization". With the help of the externalization of knowledge structure, by

analyzing the organization and association of students' knowledge nodes, teachers can indirectly infer their thinking patterns and levels, and provide observable and operable basis for thinking evaluation (Zhang, Feng, Gu, & Li, 2020).

5. Reflections at the Practical Level

5.1 *Linking Prior Experiences and Connecting Diverse Perceptions*

"Analysis of students' learning situation lays the foundation for structured teaching"(Wu, 2021). In teaching practice, teachers need to carry out in-depth learning-situation investigations to clarify students' current learning status, and figure out what students have already acquired, what they still need to learn, what they are able to learn, and what obstacles and difficulties they are facing. Taking this as the starting point, teachers should anchor new knowledge on students' prior experiences, activate their thinking processes, and build scaffolds that connect prior and new knowledge and bridge individual knowledge points to holistic knowledge structures. Students are thereby guided to achieve cognitive leaps, structural growth and thinking advancement within the zone of proximal development. In this process, teachers should make efforts to open up students' diverse perceptual channels. To this end, students are encouraged to engage collaboratively through observing, hands-on practice, verbal expression and in-depth reflection, so that they can fully experience the formation of knowledge and the expansion of thinking. Meanwhile, teachers should give full play to "multiple representations". They should intentionally guide students to transform and interpret mathematical information of different forms for the same mathematical object. Acting as a bridge linking prior experiences, multiple representations deepen students' knowledge understanding, help them internalize the knowledge structure of mathematics discipline, and gradually turn it into a three-dimensional, interconnected and transferable cognitive structure (Xi & Wu, 2019).

5.2 *Constructing Structures Rather Than Imposing Them from Outside*

During teaching, teachers often fall into a trap: they design teaching content based on their own understanding of knowledge and present a nearly perfect mind-map in class summary, mistakenly regarding it as students' cognitive structure. This unconscious act of equating teachers' personal cognitive structures with teaching outcomes essentially requires students to accept a ready-made knowledge system — in other words, directly "giving" the structure to students. Nevertheless, every student possesses different knowledge foundations and unique thinking paths. Only structures actively built through students' participation, thinking, experience and construction can belong to themselves and be dynamic and creative. Otherwise, such structures can hardly be truly integrated into their cognitive schemas. Teachers cannot replace students' construction process, let alone deliver structures directly to them. In teaching practice, teachers ought to adopt effective strategies to guide students to experience the generation and evolution of knowledge. Through independent inquiry, cooperative communication and continuous reflection, students gradually construct their own dynamic knowledge structures, rather than being passively imposed with prefabricated structures.

5.3 Consolidating Structuring Awareness and Making Good Use of Structuring Strategies

The mathematics-teaching perspective of knowledge structuring ultimately requires teachers to internalize it into conscious teaching awareness and externalize it into a set of operable teaching strategies.

5.3.1 Consolidating Structuring Awareness

First of all, as educators, teachers should clarify the importance and necessity of “structured teaching” for developing students’ core competencies. Students’ structured knowledge, structured cognition and structured competencies lay a solid foundation for them to cope with complex tasks, difficulties and challenges. Secondly, teachers need to deeply understand that mathematical knowledge forms an organic whole. When preparing lessons, they should consciously analyze teaching content from a holistic framework perspective based on students’ learning situation, and identify the horizontal and vertical connections among core concepts and thinking methods, so as to “keep individual trees in mind and keep the whole forest in sight”.

5.3.2 Making Good Use of Structuring Strategies

Situational Carriers. In mathematics education, situations serve as vital carriers of knowledge and constitute the key to implementing structured teaching. By creating “authentic problem situations”, abstract mathematical knowledge can be made intuitive, operable and open-ended for critical thinking. In teaching, teachers should be adept at constructing and utilizing structured situations. On the one hand, taking situations as a “bridge”, teachers guide students to actively relate and mobilize their existing cognitive experiences to accomplish knowledge construction within the situations. On the one hand, the situation is used as a “bridge” to guide students to actively correlate and invoke existing cognitive experience, and complete the construction of knowledge in the situation; on the other hand, for the purpose of context, the transfer and application of knowledge are realized in the new context, and the understanding of knowledge and the improvement of structure are further deepened. Knowledge comes from the situation, and ultimately it is necessary to return to the situation and create a structured situation, which can guide students to complete the complete cognition of what knowledge is, why, where it comes from and where it goes.

Problem leading. “Problem is the heart of mathematics” (Dai, 1996), which is an important way to inspire students to think and activate students’ thinking. Teachers should give full play to the core role of “problems”, and be good at embedding mathematical content and cognitive routes into a series of logical and hierarchical “problem chains”. Teachers can connect scattered knowledge points into an organic whole by carefully designing the sequence of problems from basic, guiding to challenging and exploratory, outline the structure and context of knowledge, guide students to link “cognitive resources”, and take the initiative to think, explore and construct along the preset cognitive path. In the personal experience of solving a series of problems, teachers can internalize the structure and methods of knowledge and realize the coordinated development of knowledge system and cognitive ability.

Reinforcement through Variation. “Variation teaching” is not merely a teaching skill, but also a

teaching ideology and curriculum-design principle. Originating from existing and solved mathematical problems, it ingeniously generates new problems containing new knowledge by modifying problem situations, conditions, obstacles or objectives on the basis of similarities, connections and structures among knowledge and problems(Li, 2021).With the aid of “conceptual variation”, students can understand concepts from multiple perspectives, moving from the concrete to the abstract and from the special to the general. By eliminating contextual interference, they grasp essential attributes and deepen their comprehension of connotations, so as to establish essential connections among concepts. Through “procedural variation”, students can comprehend the origin, evolution and applicable conditions of knowledge. In the process of concept formation and problem-solving, they construct a system of practical activity experience and build essential links between prior and new knowledge, and ultimately refine their personal knowledge structures(Gu, Huang, & Marton, 2007).In teaching practice, teachers should make flexible use of “variation teaching”. The organic combination of conceptual variation and procedural variation helps teachers set up clear navigation maps of knowledge structures for students.

6. Conclusion

“Without structure, there can be no competencies. Fragmented knowledge must be catalyzed by structured learning so that it can be naturally integrated into students’ cognitive structures and gradually transformed into their competencies”(Wu, 2024). Knowledge structuring constructs fragmented information into a well-structured knowledge system through internal logic, core concepts and hierarchical relationships. In this way, students can not only clearly see the whole picture and context of knowledge, but also temper the deep ability of bypassing and migrating applications. This process is the only way for the generation of literacy: only when knowledge is systematically organized in the mind to form an adjustable, extensible and critical dynamic network, can it transcend mechanical memory and eventually be internalized into a way of thinking and key character that supports individuals to understand the world and solve problems.

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